Jaypee University of Information Technology, Waknaghat

Test-3 Examination, June 2023

B.Tech - II Semester (CSE/IT/ECE/ECM/CE/CEC)

Course Code/Credits: 18B11MA211/4 Max. Marks: 35
Course Title: Engineering Mathematics-II
Course Instructors: RAD, KAS, NKT, SST Max. Time: 2 Hrs.

Instructions: All questions are compulsory. Marks are indicated against each question.

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1. Solve $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} - 4y = x^4$. (5 Marks) [CO-2]

2. Consider the following questions: (5 Marks) [CO-3]

   (a) Prove that $n\mathcal{P}_n(x) = x\mathcal{P}_n'(x) - \mathcal{P}_{n-1}'(x)$, where $\mathcal{P}_n(x)$ is the *Legendre* polynomial.

   (b) Find the value of *Bessel* function $\mathcal{J}_{3/2}(x)$ in terms of *sine* and *cosine* functions.

3. Consider **1-D** heat equation governing the temperature distribution in a rod of length ℓ:

$$\frac{\partial \mathrm{u}}{\partial \mathrm{t}} = \mathrm{c}^2\frac{\partial^2 \mathrm{u}}{\partial x^2}$$

   where c^2 is the *thermal diffusivity* in (length)^2/time. (4 Marks) [CO-4]

   (a) Find the solution $\mathrm{u}(x,\ \mathrm{t})$ subject to the following boundary and initial conditions:

   - $\mathrm{u}(0,\ \mathrm{t}) = 0,\quad \mathrm{u}(\ell,\ \mathrm{t}) = 0$ for all $\mathrm{t} > 0$
   - $\mathrm{u}(x,\ 0) = f(x)$

   (b) Find the temperature $\mathrm{u}(x,\ \mathrm{t})$ in a rod of length $\ell = 1$ if the initial temperature is

$$f(x) = \begin{cases} x & ,\ 0 < x < 1/2 \\ 1 - x & ,\ 1/2 < x < 1 \end{cases}$$

4. Consider the following questions: (4 Marks) [CO-5]

   (a) Show that $\lim\limits_{z\to 0} \dfrac{x^2y}{x^4 + y^2}$ does not exist.

   (b) Determine whether $f(z) = \begin{cases} z^2 + \mathrm{i}z + 2 & ,\ z \neq \mathrm{i} \\ \mathrm{i} & ,\ z = \mathrm{i} \end{cases}$ is continuous. Redefine, if necessary to make it continuous.

5. Show that the following function (4 Marks) [CO-5]

$$\mathrm{f}(z) = \begin{cases} \dfrac{10\mathrm{i}}{z} & ,\ z \neq 0 \\ 0 & ,\ z = 0 \end{cases}$$

   satisfies Cauchy-Riemann equations at the origin but does not have a derivative there at.

6. Evaluate the following complex integrals without parameterizing $\mathcal{C}$: (4 Marks) [CO-6]

   (a) $\oint_{\mathcal{C}} \frac{e^z}{z+2\mathrm{i}}\,\mathrm{d}z$, where $\mathcal{C}$ is the circle $|z-1|=1$

   (b) $\oint_{\mathcal{C}} \frac{4-3z}{z(z-1)(z-2)}\,\mathrm{d}z$, where $\mathcal{C}$ is the circle $|z|=\frac{3}{2}$

7. Find the Laurent series for $f(z)=\frac{1}{z(2z+1)}$ valid in $0<|z|<1/2$. (4 Marks) [CO-6]

8. Show that $\int_0^{2\pi} \frac{1}{(5-3\cos(\theta))^2}\,\mathrm{d}\theta = \frac{5\pi}{32}$. (5 Marks) [CO-6]

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