JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

T-1, EXAMINATION- 2023

Ph.D.: Mathematics

COURSE CODE (CREDITS): 13P1WMA232 (3) MAX. MARKS: 15

COURSE NAME: MATHEMATICAL ANALYSIS

COURSE INSTRUCTORS: MDS MAX. TIME: 1:00 Hrs.

Note: *All questions are compulsory. Marks are indicated against each question in square brackets.*

Quest.(1) If (X, d) is a metric space, then show that another metric on X is defined by

$$d^*(x,y) = \frac{d(x,y)}{1 + d(x,y)}$$

and X is bounded in the metric d^*. [3]

Quest (2) Discuss and sketch the open balls of unit radius about (0,0) for each of the following metrics for $\mathbb{R}^2$ [3]

(a) $d_1(z_1, z_2) = (\sum_{i=1}^{2}(x_i - y_i)^2)^{1/2}$
(b) $d_2(z_1, z_2) = \sum_{i=1}^{2} |(x_i - y_i)|$
(c) $d_3(z_1, z_2) = \max_{i=1,2}\{|(x_i - y_i)|\}$

where $z_i = (x_i, y_i) \in \mathbb{R}^2$, $\forall\, i = 1, 2$.

Quest (3) Find the limit superior and limit inferior for the sequence $\langle S_n \rangle_{n\in\mathbb{N}}$, where [2]

$$S_n = 4 + (-1)^n + \frac{1}{n}$$

Quest (4) State and prove the *Bolzano-Weierstrass Theorem.* [3]

Quest (5) (a) Show that every convergent sequence is bounded. Converse is true or not, justify your answer.

(b) Test the convergence of the sequence $\langle S_n \rangle_{n\in\mathbb{N}}$, where [2+2]

$$S_n = \frac{1}{2}, \quad S_{n+1} = \frac{2S_n + 1}{3}, \quad \forall n \in \mathbb{N}.$$