

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

T-2, EXAMINATION- 2023

Ph.D.: Mathematics

COURSE CODE (CREDITS): 13P1WMA232 (3)

MAX. MARKS: 25

COURSE NAME: MATHEMATICAL ANALYSIS

COURSE INSTRUCTORS: MDS

MAX. TIME: 90 Minutes.

Note: All questions are compulsory. Marks are indicated against each question in square brackets.

Quest.(1) Show that the Euclidean space \mathbb{R}^n , with Euclidean metric defined by

$$d(x, y) = \left(\sum_{j=1}^n (\xi_j - \eta_j)^2 \right)^{1/2}$$

where $x = (\xi_1, \xi_2, \dots, \xi_n)$, $y = (\eta_1, \eta_2, \dots, \eta_n)$, is complete. [5]

Quest.(2) Prove that every open ball is an open set. [3]

Quest.(3) Prove that the sequence $\{f_n\}$ of function where $f_n = \frac{x}{1+nx^2}$, converges uniformly on any closed interval I . [5]

Quest.(4) Show that the function [5]

$$f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

satisfies the Cauchy-Riemann equation at $z = 0$, but $f'(0)$ does not exist.

Quest.(5) Prove that $u(x, y) = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function $v(x, y)$, such that $f(z) = u + i v$ is analytic. Also express $f(z)$ in terms of z . [4]

Quest.(6) Evaluate the integral $\int_C \frac{2z+3}{z} dz$, where C is [3]

(a) Upper half of the circle $|z| = 2$ in the clockwise direction.

(b) Lower half of the circle $|z| = 2$ in the anti-clock direction.