Jaypee University of Information Technology, Waknaghat

Test-2 Examination, May 2023

B.Tech - II Semester (CSE/IT/ECE/ECM/CE/CEC)

Course Code/Credits: 18B11MA211/4
Course Title: Engineering Mathematics-II
Course Instructors: RAD, KAS, NKT, SST

Max. Marks: 25

Max. Time: 1.30 hour

Instructions: All questions are compulsory. Marks are indicated against each question.

1. Discuss the convergence of the series $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{n}{n^2+1}$. (3 Marks) [CO-1]

2. Find the *radius of convergence* of the series $\sum_{n=0}^{\infty} \frac{(x-5)^n}{n^2}$. (2 Marks) [CO-1]

3. Answer the following questions: (4 Marks) [CO-1]

   (a) Consider the Fourier series $f(x) = 1 - \frac{1}{2}\cos x - 2\left\{\frac{\cos 2x}{1\cdot 3} - \frac{\cos 3x}{3\cdot 5} + \frac{\cos 4x}{5\cdot 7} - \dots\right\}$ of $f(x) = x\sin x$ in $(0,\ \pi)$. Deduce that $\frac{\pi - 2}{4} = \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \cdots$

   (b) Find the coefficient a_5 of $\cos(5x)$ in the Fourier cosine series of the function $f(x) = \sin(5x)$ in the interval $(0,\ 2\pi)$.

4. Find *complementary* solution of $(\mathbf{D}-2)^3(\mathbf{D}^2+9)y = x^2e^{2x} + x\sin(3x)$. (2 Marks) [CO-2]

5. Consider $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x + \ln x$. (4 Marks) [CO-2]

   (a) Convert the given equation in to a differential equation with constant coefficients.

   (b) Determine the *particular integral.*

6. Consider $(\mathbf{D}^2 - 3\mathbf{D} + 2)y = \cos(e^{-x})$. Let $y_h(x) = c_1e^x + c_2e^{2x}$ be the *homogeneous* solution.

   (a) Determine the `Wronskian` of $f(x) = e^x$ and $g(x) = e^{2x}$.

   (b) Find the *non-homogeneous* solution y_p by the method of *variation of parameter.*

   (3 Marks) [CO-2]

7. Find power series solution of $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - y = 0$ about $x = 0$. (4 Marks) [CO-3]

8. Consider the following questions: (3 Marks) [CO-3]

   (a) Express $\mathcal{J}_5(x)$ in terms of $\mathcal{J}_1(x)$ and $\mathcal{J}_2(x)$, where $\mathcal{J}_n(x)$ is the Bessel's function.

   (b) Express $4x^3 + 6x^2 + 7x + 2$ in terms of Legendre polynomials.

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