

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

T-2, EXAMINATION- 2023

B. Tech. II Semester (BI/BT)

COURSE CODE (CREDITS): 18B11MA212 (04)

MAX. MARKS: 25

COURSE NAME: BASIC MATHEMATICS-II

COURSE INSTRUCTORS: MDS

MAX. TIME: 90 Minutes.

Note: All questions are compulsory. Marks are indicated against each question in square brackets.

Quest.(1) Investigate the convergence of the following series (CO-1) [2.5+2.5]

(a)

$$\sum_{n=3}^{\infty} \frac{5n^3 - 3n}{n^2(n-2)(n^2+5)}$$

(b)

$$\sum_{n=1}^{\infty} \left(\frac{(n+3)^{n+1}}{n^{n+1}} - \frac{n+3}{n} \right)^n$$

Quest.(2) Find the velocity, speed, and acceleration of a particle at $t = 0$, whose motion in space is

given by the position vector $\vec{r}(t) = 2e^{-t}\hat{i} + 3\sin(2t)\hat{j} + 3\cos(2t)\hat{k}$. (CO-2) [3]

Quest.(3) If $f(x, y) = x \cos y + ye^x$, find the second-order derivatives $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, and $\frac{\partial^2 f}{\partial y \partial x}$.

(CO-2) [3]

Quest.(4) Making use of chain rule, express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of "r" and "s" if

(CO-2) [4]

$w = (x + y + z)^2$, $x = r - s$, $y = \cos(r + s)$, $z = \sin(r + s)$
and also find these values at $r = 1, s = -1$.

Quest.(5) If $u = \sin^{-1} \left(\frac{x^2 + y^2}{x^{5/2} + y^{5/2}} \right)$, then prove that

(CO-2) [3]

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{2} \tan u$$

Quest.(6) Compute $\text{div}(\vec{V})$, $\text{curl}(\vec{V})$ and verify that $\text{div}(\text{curl}(\vec{V})) = 0$, where

(CO-2) [4]

$$\vec{V} = xyz\hat{i} + (2x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$$

Quest.(7) Solve the linear differential equation

(CO-3) [3]

$$x^2 \frac{dy}{dx} + xy = 2x^2 e^{x^2}$$