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A Constitutive Model for Creep Rupture

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The paper presents a viscoplastic formulation including damage for creep and creep rupture. The model incorporates a damage evolution law which includes both the viscoplastic strain rate as well as stress rate. Primary, secondary and tertiary creep are incorporated in this formulation. Model parameters are obtained for polycarbonate using both creep test and the constant strain rate test data. It is found that the model can give a reasonable predictions for creep and creep rupture. However the model will need to be modified for the constant strain rate case. The paper also presents a multi-axial formulation for the model. The formulation is general enough so we will be able to apply it to a complex problem of landslides in soil masses which is stated in the paper as our plan for future research.

Keywords continuum damage mechanics, viscoplasticity, creep, creep rupture, constitutive equations, landslides

1. INTRODUCTION

The time dependent behavior of materials as diverse as polymers, metals, soils and rock is well known. These materials under moderate stresses experience primary creep that is followed by secondary creep, during which the strain rate is either constant or decreases with time. Under high stresses, the secondary creep is followed by tertiary creep, during which the strain rate increases with time and may lead to rupture. There are well established models for secondary creep, which include both analytical as well as empirical formulations [1]. Recently some models have been developed which include both secondary as well as tertiary creep leading to rupture [2].

There has been considerable interest in the application of damage continuum mechanics in modeling material response at high stresses and temperatures. Applications of damage mechanics include materials as diverse as metals, polymers, concrete, soil and rock [3–15].

This paper presents a viscoplastic-damage formulation for creep and creep rupture starting from one dimensional conditions and leading to multi-axial conditions. A damage law which includes both strain rate as well as stress rates is adopted in which data from both creep tests with constant stress and data from constant strain tests are used to arrive at model parameters. The model takes into account the damage during primary creep in addition to secondary and tertiary creep.

The presentation forms a starting point for a much broader effort the authors have undertaken—the prediction of landslides in soil deposits. In northern India, particularly in the Himalayas, landslides pose a major problem. The highways on mountain roads normally have on one side unsupported slopes which fail from time to time at a constant stress level. Sometimes a slope can take as much as 150 years to fail after the construction of the highway. The problem is clearly one of creep rupture.

Traditionally a soil mass is treated as a continuum wherein a rupture surface is not predefined. Hence, the problem falls within the domain of continuum damage mechanics. For ease in modeling, the material used in this presentation is polycarbonate which behaves much the same way as saturated clays—the soils known to be a main culprit in landslides.

2. THE DAMAGE APPROACH

In the damage approach the macroscopic behavior of the material is modeled by introducing internal state variables which in some sense describe the physical state of the material [16]. The approach differs from that of a material scientist since here we make no attempt to model the mechanisms which take place at the microscopic level as a material deforms. The approach also differs from the classical fracture mechanics where the damage propagates along nearly a preconceived path. Here the damage is assumed to be uniformly distributed.

The first attempt to formulate damage constitutive equations by a phenomenological approach was made by Kachanov [17,18]. He postulated that the effective cross section of a structural member subjected to a uniaxial load deteriorated gradually.

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Following Kachanov's effective stress approach, if we define a damage parameter D as

$$D = \frac{A_f}{A} \tag{1}$$

where A_f is the damaged part of the apparent cross sectional area A, then, the area of cross section effectively carrying the load,

$$A_e = A - A_f \tag{2}$$

Therefore, the effective stress due to the axial load P can be written as,

$$\sigma_e = \frac{P}{A_e} = \frac{P/A}{A_e/A} = \frac{\sigma}{\left(A - A_f\right)/A} \tag{3}$$

$$\text{i.e.,}\quad \sigma_e = \frac{\sigma}{1 - D}; \qquad 0 \leq D \leq 1 \tag{4}$$

where $D = 0$ for the state of no damage and $D = 1$ for total damage. It will be assumed that no damage occurs below the elastic limit. Therefore, the viscoplastic constitutive equations can be expressed as,

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} \quad \sigma \leq \sigma_0 \tag{5}$$

$$\dot{\varepsilon} = \frac{\dot{\sigma}_e}{E} + \left(\frac{\sigma_e - \sigma_0}{\mu}\right)^n \quad \sigma > \sigma_0 \tag{6}$$

Substituting for the definition of the effective stress σ_e, Eq. 4, we get the viscoplastic constitutive equation accounting for damage.

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E\,(1 - D)} + \frac{\sigma\,\dot{D}}{E\,(1 - D)^2} + \left(\frac{\sigma - \sigma_0\,(1 - D)}{\mu\,(1 - D)}\right)^n \tag{7}$$

In this work, we shall neglect the second term accounting for the effect of damage rate on the strain rate, to have

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E\,(1 - D)} + \left(\frac{\sigma - \sigma_0\,(1 - D)}{\mu\,(1 - D)}\right)^n \tag{8}$$

or

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E'} + \left(\frac{\sigma - \sigma_0'}{\mu'}\right)^n \tag{9}$$

where primes are used to denote the current values of the material parameters. Thus,

$$\begin{aligned} E' &= E\,(1 - D) \\ \sigma_0' &= \sigma_0\,(1 - D) \\ \mu' &= \mu\,(1 - D) \end{aligned} \tag{10}$$

The rate of progressive degradation of each parameter is then written as

$$\begin{aligned} \dot{E}' &= -E\,\dot{D} \\ \dot{\sigma}_0' &= -\sigma_0\,\dot{D} \\ \mu' &= -\mu\,\dot{D} \end{aligned} \tag{11}$$

A rule for the evolution of damage remains to be postulated. Here, a knowledge of materials science can be a guide in identifying processes that are responsible for the deterioration of stiffness parameters. Two distinct processes may induce damage in viscoplastic materials. An increase in stress generates stress concentrations along the grain boundaries inside a material. And, if the stress rate is high, the stress concentrations do not have time to relax and a crack growth is triggered everywhere in the material resulting in the overall degradation of its stiffness. On the other hand, if the strain rate is too high the viscoelastic threshold along the crystal planes is exceeded and the gliding of grains takes place resulting in viscoplastic deformations. A damage rule of the following form, due to Mroz and Angellilo [19] seems appropriate.

$$\dot{D} = a\,\dot{\varepsilon}^V + b\,\dot{\sigma} \tag{12}$$

where a and b are two material constants. The first term of the damage rule accounts for the degradation due to the growth of viscoplastic strains (viscous damage), whereas the second term accounts for microcracking initiated by the increasing stress (stress damage). During creep under constant stress, the second term vanishes, whereas during relaxation under constant strain, the first term may be ignored. No damage occurs within the elastic domain and for negative stress rates.

2.1. Constitutive Equations for Creep Rupture

Figure 1 shows a typical creep curve. The curve is generally divided into three parts.

i Instantaneous deformation followed by primary creep.
ii Steady state creep, where the creep rate is essentially constant.
iii Tertiary creep, where the creep rate increases rapidly leading to creep rupture.

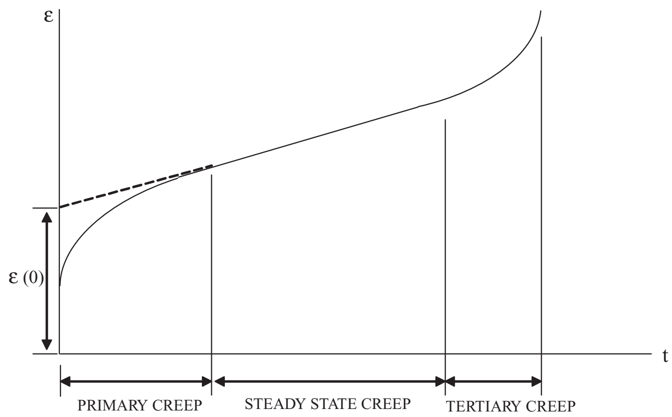

FIG. 1. A typical creep curve.

Due to the instantaneous application of stress σ_C, the initial strain can be computed as follows

$$\dot{D} = b\,\dot{\sigma}\left(\dot{\varepsilon}^V = 0\right) \tag{13}$$

Initial damage,

$$D(0) = b\int_{\sigma_0}^{\sigma_C} d\sigma = b(\sigma_C - \sigma_0) \tag{14}$$

Therefore,

$$\varepsilon(0) = \frac{\sigma_0}{E} + \frac{(\sigma_C - \sigma_0)}{E[1 - D(0)]} \tag{15}$$

We will assume that it is the initial damage which is responsible for the primary portion of a creep curve. Thus, applying the Odqvist's method, $\varepsilon(0)$ takes care of the strain incurred due to the primary creep, as shown in Figure 1.

Steady state and tertiary creep are modeled by Eqs. (8) and (12) Since $\dot{\sigma} = 0$,

$$\dot{\varepsilon}^V = \left[\frac{\sigma_C - \sigma_0(1 - D)}{\mu(1 - D)}\right]^n \tag{16}$$

or

$$\dot{D} = a\left[\frac{\sigma_C - \sigma_0(1 - D)}{\mu(1 - D)}\right]^n \tag{17}$$

Eq. (17) can be solved incrementally for the initial condition given by Eq. (14). Creep rate is then obtained for each increment from Eq. (16).

2.2. Constitutive Equations for Constant Strain Rate Tests

Since, $\dot{\varepsilon} = R$ (the rate of strain) we get from Eq. (8)

$$\frac{d\sigma}{dt} = E(1 - D)\left[R - \left\{\frac{\sigma - \sigma_0(1 - D)}{\mu(1 - D)}\right\}^n\right] \quad \sigma > \sigma_0 \tag{18}$$

Substituting for $\dot{\varepsilon}^V$ and $\dot{\sigma}$ into the damage rule, Eq. (12), we get

$$\frac{dD}{dt} = a\left[\frac{\sigma - \sigma_0(1 - D)}{\mu(1 - D)}\right]^n + tbE(1 - D)\left[R - \left\{\frac{\sigma - \sigma_0(1 - D)}{\mu(1 - D)}\right\}^n\right] \tag{19}$$

Eqs. (18) and (19) are ordinary differential equations of the form

$$\begin{aligned}\dot{\sigma} &= F(\sigma, D)\\ \dot{D} &= G(\sigma, D)\end{aligned} \tag{20}$$

respectively, and can be solved by the methods of numerical integration. A combination of Runge-Kutta and predictor-corrector methods is used in this work.

3. EVALUATION OF MATERIAL PARAMETERS AND PREDICTIONS

The model has 6 material constants which have been evaluated for polycorbonate. E and σ_0 are determined from the constant strain rate tests. For the polycarbonate data shown in Brinson [20] and Brinson et al. [21], $E = 350,000$ psi and $\sigma_0 = 4700$ psi. Next, the constant b is found from a constant strain rate test performed at a very high strain rate. If the strain rate is sufficiently high, the material suffers only stress damage. Since no viscous deformations are assumed to occur in such a case, we obtain plasticity like constitutive equations, i.e., elasticity equations with damage.

$$\dot{D} = b\,\dot{\sigma} \tag{21}$$

or,

$$D = b\int_{\sigma_0}^{\sigma} d\sigma = b(\sigma - \sigma_0) \tag{22}$$

and

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E(1 - D)} \tag{23}$$

Combining Eqs. (22) and (23)

$$\frac{d\sigma}{dt} + REb\sigma = RE(1 + b\sigma_0) \tag{24}$$

For the initial condition. $\sigma = 0$ at $t = 0$, the equation has the solution

$$\sigma = \frac{1 + b\sigma_0}{b} left(1 - e^{-\varepsilon E_0 b}) \qquad (25)$$

As the strain becomes large the stress tends to a horizontal asymptote

$$\sigma_f = \frac{1}{b} + \sigma_0 \qquad (26)$$

i.e.,

$$b = \frac{1}{\sigma_f - \sigma_0} \qquad (27)$$

And, when the stress reaches its asymptotic value, D attains a value of unity and the fracture occurs, Figure. 2.

For the polycarbonate, to find b, we used the stress-strain curve with the head rate of 20 in/min and the strain rate of 5.5×10^{-2} per minute. Since the asymptotic value of stress is difficult to estimate, Eq. (18) was fitted to the curve to find b = 0.000171.

Having determined E, σ_0 and b we proceed on to find the viscosity parameter μ. Again the stress-strain curve with the head rate of 20 in/min is used. We find the lowest value of μ which will fit the curve utilizing our main constitutive Eq (5) and (8) along with the damage rule, Eq. (12). At this stage the constant a is taken to be zero. μ was found to be nearly 10×10^6 psi.

Next, the exponent n is evaluated from the short term creep tests, Figure 3. The value of D is entirely due to the initial damage, $D = b(\sigma_C - \sigma_0)$, and its evolution during first 30 min. of creep tests is negligible. An average value of n for the three creep curves was found to be 1.25.

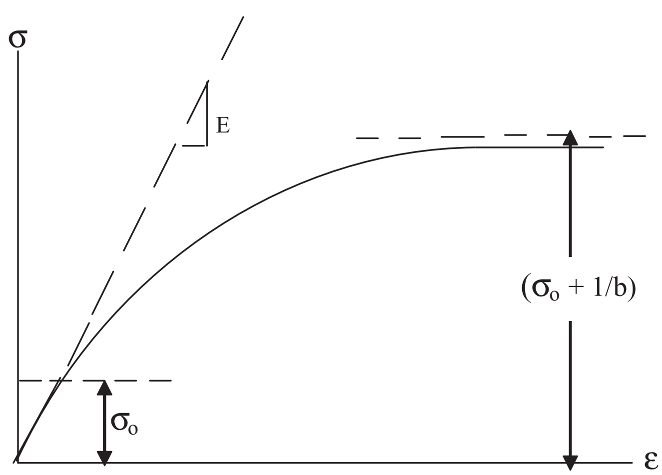

FIG. 2. Stress-strain response for elastic damage.

Now, a creep repture test is utilized to determine the damage constant a. Its value has to be 0.29 so that the creep rupture for a stress level of 7952 psi takes place in approximately 40 hours in accordance with the experimental observations [20, 21].

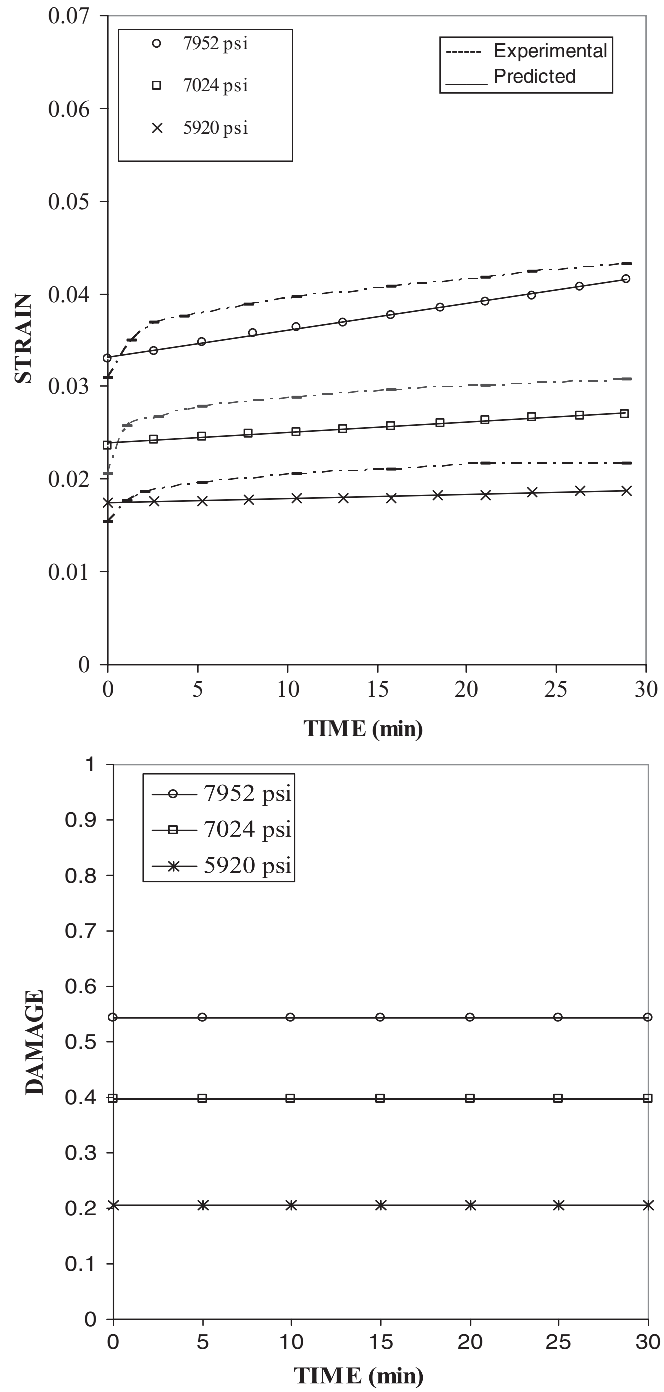

FIG. 3. Short term creep tests.

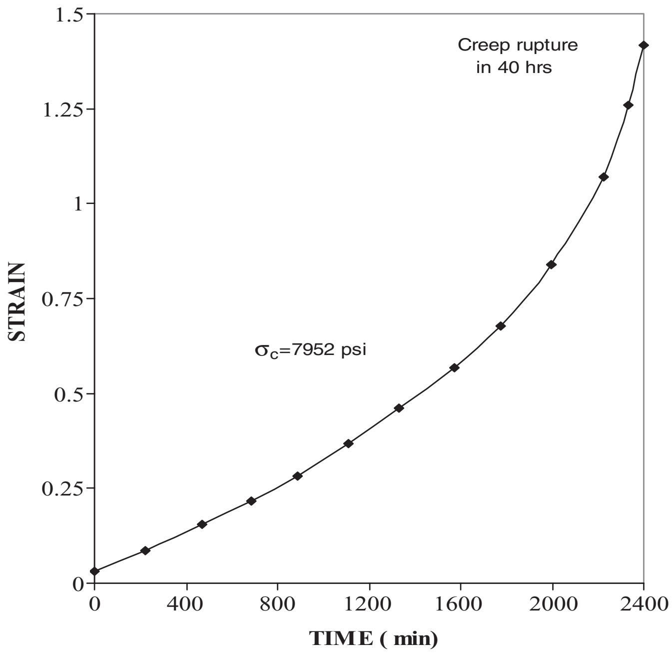

FIG. 4. Long term creep test.

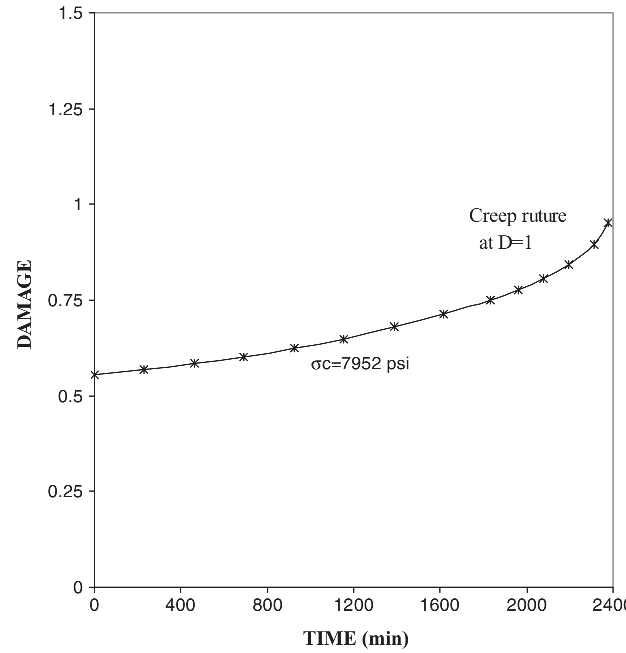

FIG. 5. Evolution of damage during creep.

A summary of the material parameters for the polycarbonate is as follows:

$$E = 350,000 \text{ psi}$$
$$\sigma_0 = 4700 \text{ psi}$$
$$\mu = 10 \times 10^6 \text{ psi}$$
$$n = 1.25$$
$$a = 0.29$$
$$b = 0.000171$$

With these parameters, the predictions for creep including damage are shown in Figure. 3 through 5. It can be observed that the model can give reasonable predictions of both short term creep as well as creep rupture. The model can also be used to predict constant strain rate response as illustrated in Figures 6 and 7 for the strain rate data of Table 1. However, the response is poor for low strain rates and the model will need to be modified for this case.

TABLE 1
Strain Rate data

Curve	Head Rate (in/min)	Strain Rate
R_1	20	5.5×10^{-2}
R_2	2	5.5×10^{-3}
R_3	.2	5.5×10^{-4}
R_4	.01	2.75×10^{-5}
R_5	.002	5.5×10^{-6}

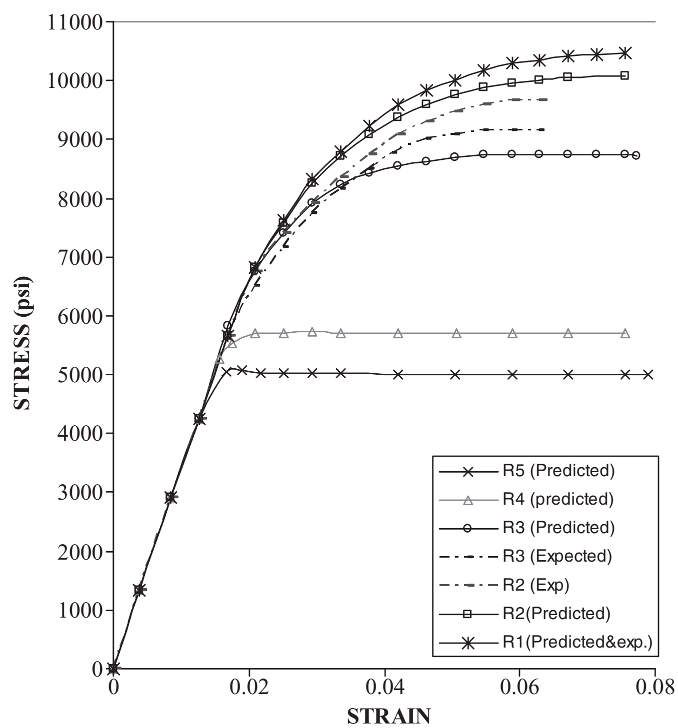

FIG. 6. Constant strain rate tests.

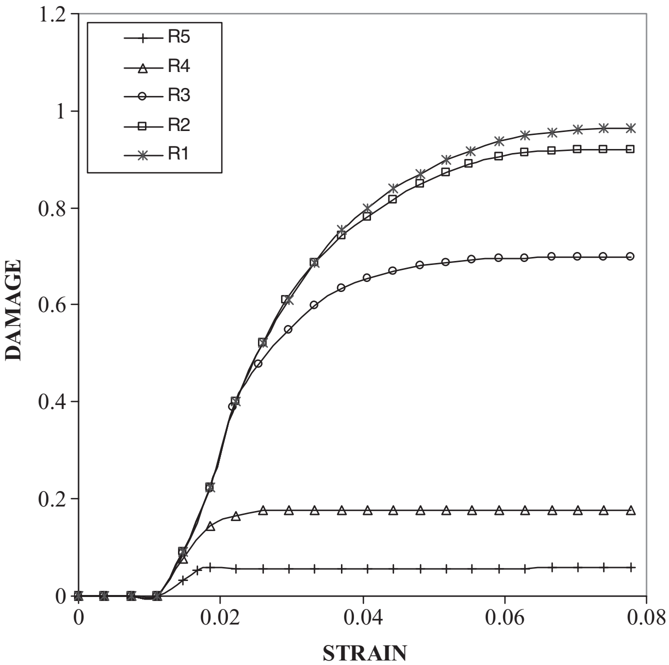

FIG. 7. Evolution of damage during constant strain rate tests.

4. CONSTITUTIVE EQUATIONS FOR MULTIAXIAL LOADING

Just as in the theory of plasticity, we postulate here a yield surface within which a material behaves elastically. And, after the stress path reaches the yield surface, viscoplastic strains begin to accrue. A notable difference between the theory of plasticity and that of viscoplasticity is, that in the latter, the stress state does not have to stay on the yield surface. In fact, it is the excess stress over the current yield surface which governs the viscoplastic strain rate. If we imagine a potential (or loading) surface passing through the current stress state, then the rate of viscoplastic strains is assumed to be proportional to the distance between the potential and yield surfaces measured along the stress path. In this work, we shall assume the potential and yield surfaces to be of the same shape. Thus, the viscoplastic flow rule can be expressed as,

$$\overset{\bullet v}{\varepsilon}_{ij} = \frac{< F(\sigma - \sigma_0') >^n}{(\mu')^n} \frac{\partial F(\sigma_{ij})}{\partial \sigma_{ij}} \tag{28}$$

where, $F(\sigma_{ij}) = \sigma_0'$, denotes the yield surface and , $F(\sigma - \sigma_0')$, the magnitude of stress vector between the current stress state and the yield surface. The brackets, $<>$, imply that only positive values of the expression contained are admissible. When the yield surface is of von Mises type,

$$F(\sigma_{ij}) = \sqrt{3\overline{J}_2} - \sigma_0' = 0 \tag{29}$$

where $\overline{J}_2$ is the second deviatoric stress invariant and, σ_0' is the current value of the uniaxial yield strength. The Flow rule, Eq. (28), in one-dimension reduces to

$$\overset{\bullet v}{\varepsilon} = \left(\frac{\sigma - \sigma_0'}{\mu'} \right)^n \quad \text{for } \sigma > \sigma_0' \tag{30}$$

This expression is used in Eq. (10). Note that,

$$\begin{aligned} \sigma_0' &= \sigma_0 (1 - D) \\ \mu' &= \mu (1 - D) \end{aligned} \tag{31}$$

More general expressions for degradation are possible such as

$$\sigma_0' = \sigma_0 - D(\sigma_0 - \sigma_r) \tag{32}$$

where σ_r is the residual value of the yield strength, or

$$\sigma_0' = \sigma_0 - \frac{D - D_0}{1 - D_0} (\sigma_0 - \sigma_r) \tag{33}$$

which accounts for the initial value of the damage parameter, D_0.

In order to define the evolution of damage, we postulate a damage surface in three-dimensional stress space:

$$\Phi(\sigma_{ij}) = c \tag{34}$$

The stress damage takes place only when the stress state lies on the damage surface and the stress path is directed exterior to the surface as in the theory of plasticity. The surface contracts during the deformation process. The evolution of damage can be defined as,

$$\begin{aligned} \dot{D} &= \dot{D}_\varepsilon + \dot{D}_\sigma \\ &= a \left(\delta_{ij} \overset{\bullet v}{\varepsilon}_{ij} \right) + b \left(\delta_{ij} \frac{\partial \Phi}{\partial \sigma_{ij}} \right) \dot{\Phi} \end{aligned} \tag{35}$$

where, the stress damage, $\dot{D}_\sigma$, is admissible when

$$\dot{\Phi} = \frac{\partial \Phi}{\partial \sigma_{ij}} \dot{\sigma}_{ij} > 0 \tag{36}$$

A parabolic damage surface of the form, $\overline{J}_2 + mJ_1 - c = 0$, seems appropriate inasmuch as damage has been found to be dependent on the hydrostatic pressure, and the form is free of corners and discontinuities.

If we assume that the damage surface is conical (Drucker-Prager type),

$$\Phi = \sqrt{3\overline{J}_2} + mJ_1 - c = 0 \tag{37}$$

where m can be taken as an integer number and,

$$c = c_0 + \int \dot{\Phi}\, dt \tag{38}$$

then, for a uniaxial test Eq. (38) reduces to

$$\dot{D} = a \dot{\varepsilon}^{V} + b(1+m)(1+m)\dot{\sigma} \tag{39}$$

which reduces to Eq. (12) only for the value of m equal to zero. In that case, the damage and yield surfaces would be similar.

No confusion should arise here between the concepts of hardening and damage. Material science recognizes the fact that two entirely different mechanisms are responsible for the two processes. Hardening occurs due to the inhomogenous nature of microstructure and the build up of elastic energy in its microelements, whereas damage takes place due to microcracking. Hardening is a function of total viscoplastic work, the damage is a function of stress rate and the viscous strain rate.

Perhaps the picture will be clearer if we assume that the material hardens until the peak stress level is reached. Thereafter, hardening stops and the damage begins leading to strain softening or necking phenomenon. When the damage and yielding are simultaneously activated, as is the case in this work, $c_0 = \sigma_0$ in Eq. (38).

For the von Mises yield surface, Eq. (29), and the Drucker-Prager damage surface, Eq. (37), the necessary equations for the model are summarized as follows.

$$\bar{J}_2 = \frac{1}{2} s_{ij} s_{ij} \tag{40}$$

$$\dot{\varepsilon}^{v}_{ij} = \frac{1}{\mu(1-D)} < (3\bar{J}_2)^{\frac{1}{2}} - \sigma_0(1-D) >^{n} \left(\frac{\sqrt{3}}{2\sqrt{\bar{J}_2}} s_{ij} \right) \tag{41}$$

$$\dot{\sigma}_{ij} = D^{e}_{ijk1} \left(\dot{\varepsilon}_{k1} - \dot{\varepsilon}^{v}_{k1} \right) \tag{42}$$

where, $[D^e]$ is the elasticity constitutive matrix.

$$\frac{\partial \Phi}{\partial \sigma_{ij}} = \frac{\sqrt{3}}{2\sqrt{\bar{J}_2}} s_{ij} + m\delta_{ij} \tag{43}$$

$$\dot{D} = \dot{D}_{\varepsilon} + \dot{D}_{\sigma}$$

i.e.,

$$\dot{D} = a\delta_{ij} \dot{\varepsilon}^{v}_{ij} + b \left(\delta_{ij} \frac{\partial \Phi}{\partial \sigma_{ij}} \right) \left(\frac{\partial \Phi}{\partial \sigma_{k1}} \dot{\sigma}_{k1} \right); \quad \dot{\Phi} > 0 \tag{44}$$

$$\dot{D}_{\sigma} = 0; \quad \text{for} \quad \dot{\Phi} \le 0 \tag{45}$$

$$= 0; \quad \text{for} \quad \left(3\bar{J}_2\right)^{\frac{1}{2}} < c \tag{46}$$

Eq. (41) can be incorporated into a finite element program by a procedure such as that of Zienkiewicz and Cormeau [22]. The formulation above lies within a more general theoretical framework recently presented by Einav et al. [23].

5. CONCLUSIONS

The paper presents a viscoplastic formulation including damage for creep and creep rupture. The model incorporates a damage evolution law which includes both the viscoplastic strain rate as well as stress rate. Primary, secondary and tertiary creep are incorporated in this formulation. The model parameters are obtained for polycarbonate using both creep test and constant strain rate test data. It is found that the model can give reasonable predictions for creep and creep rupture. However the model will need to be modified for the constant strain rate case.

A multi-axial generalization of the model is also presented. The three-dimensional framework is fairly comprehensive inasmuch as it can be applied to polymers as well as pressure dependent media such as geomaterials. However, a detailed implementation for the multi-axial model is not included.

It is hoped that the work presented forms a good starting point in our quest for a damage model for predicting landslides in soil deposits.

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