Scientia Iranica E (2021) 28(2), 1014–1029

SCIENTIA IRANICA

Sharif University of Technology
Scientia Iranica
Transactions E: Industrial Engineering
http://scientiairanica.sharif.edu

T-spherical fuzzy soft sets and its aggregation operators with application in decision-making

A. Guleria and R.K. Bajaj*

Department of Mathematics, Jaypee University of Information Technology, Waknaghat, Solan, Pin-173 234, Himachal Pradesh, India.

Received 25 February 2019; received in revised form 9 June 2019; accepted 25 June 2019

KEYWORDS
Soft set;
T-spherical fuzzy set;
Aggregation operators;
Score and accuracy function;
Decision making.

Abstract. In the present manuscript, a novel concept of T-spherical fuzzy soft set is introduced with various important operations and properties. In the field of information theory, an aggregation operator is a structured mathematical function that aggregates all the information received as input and provides a single output entity, found to be applicable for various important decision-making cases. Some averaging aggregation operators and geometric aggregation operators (weighted, ordered, and hybrid) for T-spherical fuzzy soft numbers have been proposed with their various properties. Further, utilizing the proposed aggregation operators of various types along with the properly defined score function/accuracy function, an algorithm for solving a decision-making problem has been provided. The proposed methodology has also been well illustrated through a numerical example. Some comparative remarks and advantages of the introduced notion of T-spherical fuzzy soft set and the proposed methodology have been listed to ensure better motivation and readability.

1. Introduction

Researchers in the field of Fuzzy Sets (FSs) and information are well aware of various generalized extensions of the notion of FSs [1] and Intuitionistic Fuzzy Sets (IFSs) [2,3] which have been taken place to model the uncertainties and the hesitancy inherent in many practical circumstances to have wider coverage for flexibility. Yager [4] revealed that the existing structures of FS and IFS were not capable enough to depict the human opinion in a more practical sense and introduced Pythagorean Fuzzy Sets (PyFSs), which effectively enlarged the span of information by introducing a new conditional constraint. The concept of membership/belongingness (yes), nonmembership/nonbelongingness (no), and indeterminacy/neutral (abstain) have been well described by the definition of IFSs as well as by the PyFSs. Consider an example of a voting system where voters can be categorized into four different classes: one who votes for (yes), one who votes against (no), one who neither votes for nor against (abstain), and one who refuses for voting (refusal). It may be noted that the concept of refusal has not been taken into consideration by any of the sets stated above. Cuong [5] introduced the concept of Picture Fuzzy Set (PFS) to handle such circumstances which would be sufficiently close to human nature of flexibility, where all the four parameters, that is, degree of membership, indeterminacy (neutral), nonmembership, and refusal have been taken into account. For a better understanding of the available literature, various generalizations of an FS are presented by a road map given in Figure 1.

Recently, Mahmood et al. [6] introduced the

*. *Corresponding author. Tel.: +91-1792-239235*
E-mail addresses: rakesh.bajaj@gmail.com (A. Guleria); rakesh.bajaj@juit.ac.in (R.K. Bajaj)

doi: 10.24200/sci.2019.53027.3018

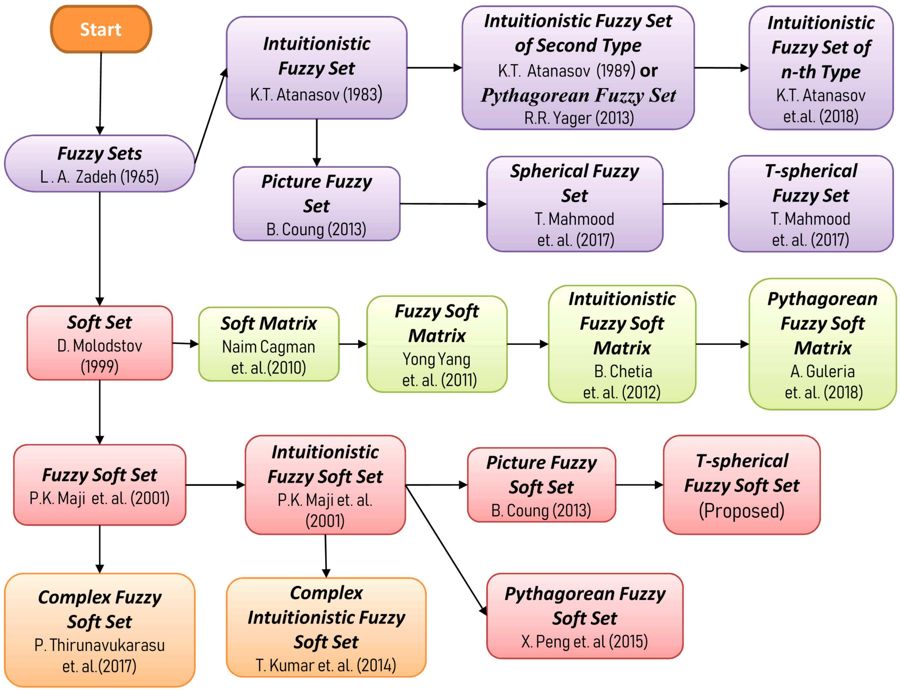

Figure 1. Extensions of fuzzy sets.

notion of Spherical Fuzzy Set (SFS) and T-Spherical Fuzzy Set (TSFS) that would give additional strength to the PFS by broadening the span for the membership of all the four parameters stated above. Next, Kifayat et al. [7] studied the geometrical comparison of FSs, IFSs, PyFSs, PFS with SFSs and TSFSs. Besides, the limitations of the existing similarity measures for IFSs and PFSs have been provided in view of the broader setup of the spherical fuzzy environment. Further, they proposed different types of similarity measures for TSFSs with their useful applications in various fields. Recently, Garg et al. [8] presented a new improved interactive aggregation operator for TSFSs with application to decision-making. Also, in recent years, researchers have gone through watchful deliberations to the existing theories and applied them to various application fields [6,7,9,10].

2. Literature review

Many researchers in the field of soft computing have widely utilized the concept of aggregation operators in which they have featured the concerns of each criterion/attribute or its ordered position. However, researchers have not addressed real-world circumstances where the interrelationships or connections among different criteria, e.g., priority, support feature, and inter-impact, reflect a significant role in the process of aggregation.

Some geometric aggregated operators for Intuitionistic Fuzzy Numbers (IFNs) were proposed by Xu and Yager [11]. Further, Yager [12] introduced the idea of power average aggregation operator that takes argument values to support each other in the process of aggregation. For IFNs, Xu and Yager [13] and Yu [14] studied the prioritized averaging/geometric aggregations. In addition to this, the new operator called Bonferroni mean, which could handle the inter-relationship between the input values, was proposed by Yager [15]. Subsequently, Xu and Yager [16] extended the existing Bonferroni mean to aggregating the IFNs. Liu and Li [17] incorporated power Bonferroni mean operators for the interval-valued IFNs and defined some standard operations with their operational laws. They proposed different types of important operators such as power, weighted power, power geometric, and weighted power geometric Bonferroni mean operators with various properties and applications in detail. Wang and Liu [18] extended the aggregation operators in the intuitionistic fuzzy environment using the Einstein norm operators.

Tao et al. [19] applied the Archimedean copulas and the associated co-copulas to the IFNs with some basic operations. They also proposed the copula weighted averaging aggregation operators and studied their basic properties. Further, they provided the modified maximizing deviation decision algorithm and illustrated its applicability to the decision-making problem. Garg [20] extended the generalized geometric as well as averaging operators over the (PyFS).

In the literature, there are many theories employed to deal with vagueness and uncertainties of many problems arising in engineering, economics, social science, etc. However, all the theories are subject to limitations because of the parametrization tool involved in it. In order to overcome these difficulties, Molodtsov [21] introduced the notion of soft set to deal with the uncertainties, being free from the inadequacy of parametrization and the various results established accordingly. Maji et al. [22] studied the theory of soft sets and defined soft binary operations such as conjunction, disjunction, union, intersection, equality, and complement of the soft sets. Various researchers have utilized the notion of fuzzy soft sets in the field of investment decision-making problems [23,24], optimization modeling [25], ranking in decision-making problems [26], etc.

Further, Maji et al. [27,28] successfully extended the soft set to Fuzzy Soft Set (FSS)/Intuitionistic Fuzzy Soft Set (IFSS) and studied their applications to decision-making problems. Peng et al. [29] introduced the Pythagorean fuzzy soft set along with various binary operations and also proposed an algorithm for decision-making. Next, Peng and Yang [30] proposed two novel approaches to solving stochastic MCDM based on regret and prospect theory for interval-valued FSSs. Also, Peng and Garg [31] devised three algorithms to solve a decision-making problem by utilizing WDBA, CODAS, and similarity measures. Further, algorithms for neutrosophic soft decision-making based on EDAS [32] and for the hesitant fuzzy soft decision-making problem based on WDBA/CODAS [33] have been studied in recent literature. Cuong [5] proposed a combination of PFS and soft set as Picture Fuzzy Soft Set (PFSS) and various operations and properties were also discussed.

Arora and Garg [9] and Garg et al. [10] studied the aggregated operators for IFSSs with their applications to decision-making. Wei [34] proposed some picture fuzzy aggregated operators viz., weighted, ordered-weighted and hybrid average operator/weighted, ordered-weighted and hybrid geometric operator by utilizing arithmetic/geometric operators. Liu and Zhang [35] introduced the picture fuzzy linguistic numbers with some basic operations and operational laws. They also proposed the Archimedean picture fuzzy weighted averaging operator and studied their basic properties. Further, they provided an algorithm for the decision-making process by utilizing the proposed averaging operator and illustrated the applicability through a numerical example.

The discussions on the existing literature reveal that we can enhance the flexibility of human opinions with revised conditional/spherical constraints by proposing a notion of T-spherical FSS as a new paradigm so that the parametrization may also be employed to deal with decision-making problems. Additionally, the study of various aggregation operators (averaging/geometric/hybrid) for T-spherical fuzzy soft is to be carried to propose the methodologies.

In the present work, we extend the concept of PFSS by proposing the T-Spherical Fuzzy Soft Set (TSFSS) along with various aggregation operators and applications. The organization of the paper is as follows. In Section 3, basic definitions and preliminaries are provided for a better understanding of the proposed extension and application. In Section 4, we have formally introduced T-spherical FSS, their basic set-theoretic operations, properties, and score/accuracy function. Different types of aggregation operators (averaging/geometric/hybrid) for T-spherical fuzzy soft numbers with their properties and results are studied in Section 5. In Section 6, a decision-making problem is formulated and an algorithm implementing the proposed aggregation operators is provided. A numerical example is also provided in Section 7 for illustrating all the necessary steps of the proposed methodology. Some comparative remarks and advantages of the proposed work are listed in Section 8. Finally, the paper concludes with a direction for some future work in Section 9.

3. Preliminaries

Some fundamental concepts in connection with SFS and TSFS, which are well known in the literature, are presented in this section. The following notions explain the generalization process from IFSs to TSFSs.

Let U be the universe of discourse with $\mu_A : U \rightarrow [0,1]$ and $\nu_A : U \rightarrow [0,1]$ which are the degrees of membership and non-membership, respectively. The set $A = \{< u, \mu_A(u), \nu_A(u) > \mid u \in U\}$ is called:

- ***Intuitionistic Fuzzy Set (IFS) [2].*** A in U if it satisfies the condition $0 \leq \mu_A(u) + \nu_A(u) \leq 1$ with the degree of indeterminacy given by $\pi_A(u) = 1 - \mu_A(u) - \nu_A(u)$;
- ***Pythagorean Fuzzy Set (PyFS) [4] or intuitionistic fuzzy set of second type [36].*** A in U if it satisfies the condition $0 \leq \mu_A^2(u) + \nu_A^2(u) \leq 1$ with the degree of indeterminacy given by $\pi_A(u) = \sqrt{1 - \mu_A^2(u) - \nu_A^2(u)}$.

To have further generalization, we consider the universe of discourse U with $\mu_A : U \rightarrow [0,1]$, $\eta_A : U \rightarrow [0,1]$ and $\nu_A : U \rightarrow [0,1]$ being the degree of membership, degree of neutral membership (abstain), and degree of non-membership respectively. The set $A = \{< u, \mu_A(u), \eta_A(u), \nu_A(u) > \mid u \in U\}$ is called as follows:

- ***Picture Fuzzy Set (PFS) [5].*** A in U if it satisfies the condition $\mu_A(u) + \eta_A(u) + \nu_A(u) \leq 1$ with the degree of refusal given by $r_A(u) = 1 - (\mu_A(u) + \eta_A(u) + \nu_A(u))$.

- ***Spherical Fuzzy Set (SFS) [6].*** A in U if it satisfies the condition $\mu_A^2(u) + \eta_A^2(u) + \nu_A^2(u) \leq 1$ with the degree of refusal given by:

$$r_A(u) = \sqrt{1 - (\mu_A^2(u) + \eta_A^2(u) + \nu_A^2(u))}.$$

- ***T-spherical fuzzy set [6].*** Let n be any natural number. A set A in U if it satisfies the condition $\mu_A^n(u)+\eta_A^n(u)+\nu_A^n(u) \leq 1$ with the degree of refusal given by:

$$r_A(u) = \sqrt[n]{1 - (\mu_A^n(u) + \eta_A^n(u) + \nu_A^n(u))}.$$

Particular cases:

- For $n = 2$, TSFS becomes SFS;
- For $n = 1$, TSFS becomes PFS;
- For $n = 2$ and $r_A = 0$, TSFS becomes PyFS or IFS of second type;
- For $n = 1$ and $r_A = 0$, TSFS becomes IFS.

Similarly, the generalization of the field of Soft Sets to PFSSs with explanatory examples is available in the literature and they are listed below.

Let $U = \{u_1, u_2, \ldots, u_m\}$ be the universe of discourse and $P = \{p_1, p_2, \ldots, p_n\}$ be the set of parameters. The pair (Φ, P) is called:

- *Soft Set* [21] over U iff $\Phi : P \to \mathcal{P}(U)$, where $\mathcal{P}(U)$ is the power set of U;
- *FSS* [37] over $\Phi(U)$, where Φ is a mapping given by $\Phi : P \to (F(U))$ and $F(U)$ denotes the set of all fuzzy sets of U;
- *Intuitionistic Fuzzy Soft Set (IFSS)* [22] over U if $\Phi : P \to IFS(U)$ and can be represented as:

$$(\Phi, P) = \{(p, \Phi(p)) : p \in P, \Phi(p) \in IFS(U)\},$$

  where $IFS(U)$ represents the set of all IFSs of U;
- *Pythagorean FSS* [29] over U if $\Phi : P \to PYFS(U)$ and can be represented as:

$$(\Phi, P) = \{(p, \Phi(p)) : p \in P, \Phi(p) \in PYFS(U)\},$$

  where $PYFS(U)$ represents the set of all PyFSs of U;
- *Picture Fuzzy Soft Set (PFSS)* [5] over U if $\Phi : P \to PFS(U)$ and can be represented as:

$$(\Phi, P) = \{(p, \Phi(p)) : p \in P, \Phi(p) \in PFS(U)\},$$

  where $PFS(U)$ represents the set of all PFSs of U.

4. *T*-spherical Fuzzy Soft Set (FSS)

In this section, we introduce T-spherical as an extension to PFSS [5]. Further, the score and accuracy function for the defined T-spherical FSS have been proposed along with various operations and different properties.

Definition 1 *(T-spherical FSS).* *Let U and $TSFS(U)$ be the domain of discourse and the collection of all T-spherical fuzzy sets over U, respectively. Let P be the set of parameters. The pair (Φ, P) is a T-spherical FSS over U iff $\Phi : P \to TSFS(U)$. For any parameter $p_k \in P$, Φ_{p_k} is a T-spherical FSS given by:*

$$\Phi_{p_k} = \{< u_i, \mu_k(u_i), \eta_k(u_i), \nu_k(u_i) >| \ u_i \in U\},$$

where $\mu_k(u_i), \eta_k(u_i)$, and $\nu_k(u_i)$ are the degrees of membership, neutral membership (abstain), and non-membership, respectively, with the condition:

$$0 \leq \mu_k^n(u_i) + \eta_k^n(u_i) + \nu_k^n(u_i) \leq 1,$$

and the degree of refusal:

$$r_k(u_i) = \sqrt[n]{1 - (\mu_k^n(u_i) + \eta_k^n(u_i) + \nu_k^n(u_i))};$$

where n is a natural number.

Example 1. *Consider the set of four houses, say, $H = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ and the set of parameters under consideration, say, $P = \{expensive(p_1), wooden(p_2), cheap\ (p_3), beautiful(p_4), good\ location(p_5)\}$. Then, the perception for the attractiveness of the houses may be described as a T-spherical FSS given by:*

$$(\Phi, P) = \{\Phi_{p_1}, \Phi_{p_2}, \Phi_{p_3}, \Phi_{p_4}, \Phi_{p_5}\},$$

where:

$$\Phi_{p_1} = \{< \gamma_1, 0.2, 0.5, 0.6 >, < \gamma_2, 0.3, 0.3, 0.5 >\};$$

$$\Phi_{p_2} = \{< \gamma_3, 0.4, 0.3, 0.4 >, < \gamma_4, 0.2, 0.2, 0.7 >, < \gamma_5, 0.9, 0.1, 0.1 >\};$$

$$\Phi_{p_3} = \{< \gamma_1, 0.3, 0.5, 0.2 >, < \gamma_4, 0.5, 0.3, 0.2 >\};$$

$$\Phi_{p_4} = \{< \gamma_1, 0.6, 0.3, 0.2 >, < \gamma_4, 0.9, 0.2, 0.1 >, < \gamma_5, 0.5, 0.4, 0.2 >\};$$

$$\Phi_{p_5} = \{< \gamma_2, 0.2, 0.1, 0.7 >, < \gamma_4, 0.6, 0.2, 0.2 >\}.$$

In particular, if we take $n = 2$, Definition 1 leads to the definition of *SFSS*.
Next, we propose some basic operations on TSFSs.

Operations on *T*-spherical FSS:
Let (Φ, Q) and (Ψ, R) be two T-spherical FSS on the same universe of discourse U. Let $Q, R \subseteq P$ be the set of parameters; then:

- Complement $(\Phi, Q)^c = (\Phi^c, Q)$ where $\Phi^c : Q \rightarrow TSFS(U)$ is a mapping given by $\Phi^c(p) = (\Phi(p))^c$, for all $p \in Q$;
- Subsethood $(\Phi, Q) \subseteq (\Psi, R)$, iff $Q \subseteq R$ and for all $p \in Q$, $\Phi(p) \subseteq \Psi(p)$;
- Equality $(\Phi, Q) = (\Psi, R)$, if $(\Phi, Q) \subseteq (\Psi, R)$ and $(\Psi, R) \subseteq (\Phi, Q)$;
- Union $(\Phi, Q) \cup (\Psi, R) = (H, S)$, where $S = Q \cup R$ for all $\xi \in S$ and:

$$H(\xi) = \begin{cases} \Phi(\xi) & \xi \in Q - R, \\ \Psi(\xi) & \xi \in R - Q, \\ \Phi(\xi) \cup \Psi(\xi) & \xi \in Q \cap R. \end{cases}$$

In other words, for all $\xi \in Q \cap R$:

$$H(\xi) = \Big\{ \Big(u, \max(\mu_{\Phi(\xi)}(u), \mu_{\Psi(\xi)}(u)), \min(\eta_{\Phi(\xi)}(u), \eta_{\Psi(\xi)}(u)), \min(\nu_{\Phi(\xi)}(u), \nu_{\Psi(\xi)}(u)) \Big) \Big\}.$$

- Intersection $(\Phi, Q) \cap (\Psi, R) = (H, S)$, where $S = Q \cap R$ for all $\xi \in S$ and:

$$H(\xi) = \begin{cases} \Phi(\xi) & \xi \in Q - R, \\ \Psi(\xi) & \xi \in R - Q, \\ \Phi(\xi) \cap \Psi(\xi) & \xi \in Q \cap R. \end{cases}$$

In other words, for all $\xi \in Q \cap R$:

$$H(\xi) = \Big\{ \Big(u, \min(\mu_{\Phi(\xi)}(u), \mu_{\Psi(\xi)}(u)), \min(\eta_{\Phi(\xi)}(u), \eta_{\Psi(\xi)}(u)), \max(\nu_{\Phi(\xi)}(u), \nu_{\Psi(\xi)}(u)) \Big) \Big\}.$$

Proposition 1. *Suppose that (Φ, Q) and (Ψ, R) are two T-spherical FSSs on the universal set U. Let $Q, R \subseteq P$ be two subsets of the set of parameters; then, as per their definitions, the following properties clearly hold:*

(i) $((\Phi, Q)^c)^c = (\Phi, Q)$;

(ii) $((\Phi, Q) \cap (\Psi, R))^c = (\Phi, Q)^c \cup (\Psi, R)^c$;

(iii) $((\Phi, Q) \cup (\Psi, R))^c = (\Phi, Q)^c \cap (\Psi, R)^c$.

Further, for simplicity and necessary computations, TSFSS can also be regarded as $T_u = (\mu_u, \eta_u, \nu_u)$ and called as T-Spherical Fuzzy Soft Number (TSFSN), where u is referential subscript used for establishing a connection between alternatives and parameters in computational examples. For application purposes, to rank these numbers, we propose the score and accuracy functions for the T-spherical fuzzy soft numbers as follows:

Definition 2. *Let $T_u = (\mu_u, \eta_u, \nu_u)$ be the T-spherical fuzzy soft number; then, we have:*

- *The score function is given as $S(T_u) = \mu_u^n - \eta_u^n - \nu_u^n$; $S(T_u) \in [-1, 1]$;*
- *The accuracy function is given as $H(T_u) = \mu_u^n + \eta_u^n + \nu_u^n$; $H(T_u) \in [0, 1]$.*

Based on the two functions defined above, the ordering of two T-spherical fuzzy soft numbers can be determined as follows.

Let $T_u = (\mu_u, \eta_u, \nu_u)$ and $T_v = (\mu_v, \eta_v, \nu_v)$ be two T-spherical fuzzy soft numbers; then, we have:

- $T_u \geq T_v$ if $S(T_u) \geq S(T_v)$;
- $T_u \leq T_v$ if $S(T_u) \leq S(T_v)$.

In case, if $S(T_u) = S(T_v)$ for any two TSFSN, then:

- $T_u \geq T_v$ if $H(T_u) \geq H(T_v)$;
- $T_u \leq T_v$ if $H(T_u) \leq H(T_v)$;
- $T_u \sim T_v$ if $H(T_u) = H(T_v)$.

5. Aggregation Operators for T-spherical fuzzy soft numbers

In the information fusion process, aggregation operators mathematically aggregate all the input information such as significance of criteria/attribute, prioritization, support and interrelationships of the individual data into a single one. The aggregation operators have been widely applied by various researchers to decision-making problems. In this section, we broadly propose two types of operators for T-spherical Fuzzy soft numbers - *averaging aggregation operators* and *geometric aggregation operators.*

5.1. Averaging Aggregation Operators

Here, we propose some averaging aggregation operators such as weighted averaging operator and ordered weighted/hybrid averaging operator with their properties for the proposed T-spherical fuzzy soft numbers. Some standard operations for the T-spherical fuzzy soft numbers are defined as follows.

Let $T_u = (\mu_u, \eta_u, \nu_u)$ and $T_v = (\mu_v, \eta_v, \nu_v)$ be two T-spherical fuzzy soft numbers and $\lambda > 0$ be any real number. Then, the following operations are defined over the two T-spherical fuzzy soft numbers as:

(a) $$T_u \oplus T_v = \left(\sqrt[n]{\mu_{T_u}^n + \mu_{T_v}^n - \mu_{T_u}^n \mu_{T_v}^n}, \eta_{T_u}\eta_{T_v}, \nu_{T_u}\nu_{T_v} \right).$$

(b) $$T_u \otimes T_v = \left(\mu_{T_u}\mu_{T_v}, \sqrt[n]{\eta_{T_u}^n + \eta_{T_v}^n - \eta_{T_u}^n \eta_{T_v}^n}, \sqrt[n]{\nu_{T_u}^n + \nu_{T_v}^n - \nu_{T_u}^n \nu_{T_v}^n} \right).$$

(c) $\lambda T_u = (\sqrt[n]{1-(1-\mu^n_{T_u})^\lambda}, \eta^\lambda_{T_u}, \nu^\lambda_{T_u})$.

(d) $T^\lambda_u = \Bigg((\mu_{T_u})^\lambda, \sqrt[n]{1-(1-\eta^n_{T_u})^\lambda}, \sqrt[n]{1-(1-\nu^n_{T_u})^\lambda}\Bigg)$.

(e) $T^c_u = (\nu_u, \eta_u, \mu_u)$.

Theorem 1. *Let $T_u = (\mu_u, \eta_u, \nu_u)$ and $T_v = (\mu_v, \eta_v, \nu_v)$ be two T-spherical fuzzy soft numbers and $\lambda, \lambda_1, \lambda_2 > 0$ be the real numbers. Then, the following operational laws hold:*

(i) $T_u \oplus T_v = T_v \oplus T_u$;

(ii) $T_u \otimes T_v = T_v \otimes T_u$;

(iii) $\lambda(T_u \oplus T_v) = \lambda T_v \oplus \lambda T_u$;

(iv) $(T_u \otimes T_v)^\lambda = T^\lambda_v \otimes T^\lambda_u$;

(v) $\lambda_1 T_u \oplus \lambda_2 T_u = (\lambda_1 + \lambda_2) T_u$;

(vi) $T^{\lambda_1}_u \otimes T^{\lambda_2}_u = T^{(\lambda_1+\lambda_2)}_u$;

(vii) $(T^{\lambda_1}_u)^{\lambda_2} = T^{\lambda_1\lambda_2}_u$.

Proof: The proof of these operational laws immediately follows from the definitions stated in (a)–(e).

Definition 3. *Suppose $\mathcal{P}$ is a collection of all T-spherical fuzzy soft numbers.*

Let $(T_{u_1}, T_{u_2}, ..., T_{u_m}) \in \mathcal{P}^m$. A mapping:

$$TSFSWA_\omega : \mathcal{P}^m \longrightarrow \mathcal{P}$$

is said to be a T-spherical fuzzy soft weighted averaging operator if:

$$TSFSWA_\omega(T_{u_1}, T_{u_2}, \ldots, T_{u_m}) = \oplus^m_{j=1}(\omega_j T_{u_j}), \quad (1)$$

where $\omega = (\omega_1, \omega_2, ..., \omega_m)^T$ is the weight vector corresponding to $(T_{u_j})^m_{j=1}$ such that $\omega_j \geq 0$, for all j; $\sum_{j=1}^m \omega_j = 1$.

Theorem 2. *The T-spherical fuzzy soft weighted averaging operator $TSFSWA_\omega$ aggregates all the input values and yields a TSFSN given by:*

$$TSFSWA_\omega(T_{u_1}, T_{u_2}, \ldots, T_{u_m}) = \left(\sqrt[n]{1-\prod_{j=1}^m(1-\mu^n_{T_{u_j}})^{\omega_j}}, \prod_{j=1}^m(\eta_{T_{u_j}})^{\omega_j}, \prod_{j=1}^m(\nu_{T_{u_j}})^{\omega_j}\right), \quad (2)$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$ is the weight vector corresponding to $(T_{u_j})^m_{j=1}$ such that $\omega_j \geq 0$, for all j; $\sum_{j=1}^m \omega_j = 1$.

Proof: Using the *principle of mathematical induction* on m, the result in Eq. (2) given in the above-stated theorem can be proved as follows:

For $m = 2$, we have:

$$TSFSWA_\omega(T_{u_1}, T_{u_2}) = \omega_1 T_{u_1} \oplus \omega_2 T_{u_2}.$$

It may be observed that both $\omega_1 T_{u_1}$ and $\omega_2 T_{u_2}$ are T-spherical fuzzy soft numbers. From the operation (c) defined in Subsection 5.1 for the T-spherical fuzzy soft numbers, we have:

$$\omega_1 T_{u_1} = (\sqrt[n]{1-(1-\mu^n_{T_{u_1}})^{\omega_1}}, \eta^{\omega_1}_{T_{u_1}}, \nu^{\omega_1}_{T_{u_1}});$$

and:

$$\omega_2 T_{u_2} = (\sqrt[n]{1-(1-\mu^n_{T_{u_2}})^{\omega_2}}, \eta^{\omega_2}_{T_{u_2}}, \nu^{\omega_2}_{T_{u_2}}).$$

Then, we have the equation shown in Box I. Hence, the result holds for $m = 2$. Next, we suppose that Eq. (2) holds for $m = k$, that is:

$$TSFSWA_\omega(T_{u_1}, T_{u_2}) = \omega_1 T_{u_1} \oplus \omega_2 T_{u_2}$$
$$= \left(\sqrt[n]{(2-(1-\mu^n_{T_{u_1}})^{\omega_1}-(1-\mu^n_{T_{u_2}})^{\omega_2})-\left((1-(1-\mu^n_{T_{u_1}})^{\omega_1})(1-(1-\mu^n_{T_{u_2}})^{\omega_2})\right)}, (\eta^n_{T_{u_1}})^{\omega_1}(\eta^n_{T_{u_2}})^{\omega_2}, (\nu^n_{T_{u_1}})^{\omega_1}(\nu^n_{T_{u_2}})^{\omega_2}\right)$$
$$= \left(\sqrt[n]{1-\left((1-\mu^n_{T_{u_1}})^{\omega_1}(1-\mu^n_{T_{u_2}})^{\omega_2}\right)}, (\eta^n_{T_{u_1}})^{\omega_1}(\eta^n_{T_{u_2}})^{\omega_2}, (\nu^n_{T_{u_1}})^{\omega_1}(\nu^n_{T_{u_2}})^{\omega_2}\right).$$

Box I

$$TSFSWA_\omega(T_{u_1},T_{u_2},\dots T_{u_{k+1}}) = \omega_1 T_{u_1} \oplus \omega_2 T_{u_2} \oplus \dots \oplus \omega_k T_{u_k} \oplus \omega_{k+1} T_{u_{k+1}}$$
$$= \Bigg(\sqrt[n]{1-\prod_{j=1}^{k}(1-\mu^n_{T_{u_j}})^{\omega_j} + 1 - (1-\mu^n_{T_{u_{k+1}}})^{\omega_{k+1}} - \big((1-\prod_{j=1}^{k}(1-\mu^n_{T_{u_j}})^{\omega_j})(1-(1-\mu^n_{T_{u_{k+1}}})^{\omega_{k+1}})\big)},$$
$$\prod_{j=1}^{k+1}(\eta^n_{T_{u_j}})^{\omega_j}, \prod_{j=1}^{k+1}(\nu^n_{T_{u_j}})^{\omega_j}\Bigg) = \Bigg(\sqrt[n]{1-\prod_{j=1}^{k+1}(1-\mu^n_{T_{u_j}})^{\omega_j}}, \prod_{j=1}^{k+1}(\eta^n_{T_{u_j}})^{\omega_j}, \prod_{j=1}^{k+1}(\nu^n_{T_{u_j}})^{\omega_j}\Bigg).$$

Box II

$$TSFSWA_\omega(T_{u_1},T_{u_2},\dots T_{u_k}) = \omega_1 T_{u_1} \oplus \omega_2 T_{u_2} \oplus \dots \oplus \omega_k T_{u_k} = \Bigg(\sqrt[n]{1-\prod_{j=1}^{k}(1-\mu^n_{T_{u_j}})^{\omega_j}},$$
$$\prod_{j=1}^{k}(\eta^n_{T_{u_j}})^{\omega_j}, \prod_{j=1}^{k}(\nu^n_{T_{u_j}})^{\omega_j}\Bigg).$$

Next, the result for $m = k+1$ is to be proved, that is, $TSFSWA_\omega(T_{u_1},T_{u_2},\dots T_{u_{k+1}})$ is also a TSFSN. By operational laws of T-spherical fuzzy soft numbers, we have the equation shown in Box II.

This clearly shows that the aggregated value is also a TSFSN. Therefore, Eq. (2) holds for all $m = k+1$. This finally proves the result.

Next, we state the following properties related to the T-spherical fuzzy soft weighted averaging operator which can easily be proved in view of the definitions:

(i) ***Idempotency***: If $T_{u_j} = T_u$; for all $j = 1,2,...,m$, then:
$$TSFSWA_\omega(T_{u_1},T_{u_2},\dots T_{u_m}) = T_u.$$

(ii) ***Boundedness***: If T_{u_j} $(j = 1,2,\dots,m)$ be the collection of TSFSNs, then:
$$\min_j T_{u_j} \le TSFSWA_\omega(T_{u_1},T_{u_2},\dots T_{u_m}) \le \max_j T_{u_j}.$$

(iii) ***Monotonicity***: Let T_{u_j} and $T_{u'_j}$ $(j = 1,2,...,m)$ be the two collections of T-spherical fuzzy soft numbers. If $T_{u_j} \le T_{u'_j}$, then:
$$TSFSWA_\omega(T_{u_1},T_{u_2},\dots T_{u_m}) \le TSFSWA_\omega (T_{u'_1},T_{u'_2},\dots T_{u'_m}).$$

Further, we propose a definition for the ordered weighted averaging operator for T-spherical fuzzy soft numbers as follows:

Definition 4. *Suppose $\mathcal{P}$ is a collection of all T-spherical fuzzy soft numbers.*

Let $(T_{u_1},T_{u_2},\dots,T_{u_m}) \in \mathcal{P}^m$. A mapping:
$$TSFSOWA_\omega : \mathcal{P}^m \longrightarrow \mathcal{P}$$
is called T-spherical fuzzy soft ordered weighted averaging operator if:
$$TSFSOWA_\omega(T_{u_1},T_{u_2},\dots,T_{u_m}) = \oplus_{j=1}^{m}(\omega_j T_{u_{\sigma(j)}}), \tag{3}$$
where $\omega = (\omega_1,\omega_2,\dots,\omega_m)^T$ is the weight vector corresponding to $(T_{u_j})_{j=1}^m$ such that $\omega_j \ge 0$, for all j; $\sum_{j=1}^{m}\omega_j = 1$ and $(\sigma(1),\sigma(2),\dots,\sigma(m))$ is a possible permutation of $(1,2,\dots,m)$, s.t. $T_{u_{\sigma(j+1)}} \le T_{u_{\sigma(j)}}$ for all $j = 1,2,\dots,m-1$.

Theorem 3. *The T-spherical fuzzy soft ordered weighted averaging operator aggregates all the input values and yields a TSFSN given by:*
$$TSFSOWA_\omega(T_{u_1},T_{u_2},\dots,T_{u_m}) = \Bigg(\sqrt[n]{1-\prod_{j=1}^{m}(1-\mu^n_{T_{u_{\sigma(j)}}})^{\omega_j}}, \prod_{j=1}^{m}(\eta_{T_{u_{\sigma(j)}}})^{\omega_j}, \prod_{j=1}^{m}(\nu_{T_{u_{\sigma(j)}}})^{\omega_j}\Bigg), \tag{4}$$
where $\omega = (\omega_1,\omega_2,\dots,\omega_m)^T$ is the weight vector corresponding to $(T_{u_j})_{j=1}^m$ such that $\omega_j \ge 0$, for all j; $\sum_{j=1}^{m}\omega_j = 1$ and $(\sigma(1),\sigma(2),\dots,\sigma(m))$ is a possible permutation of $(1,2,\dots,m)$, s.t. $T_{u_{\sigma(j+1)}} \le T_{u_{\sigma(j)}}$ for all $j = 1,2,\dots,m-1$.

Proof: The proof is similar to that of Theorem 2 and can easily be carried out.

Next, the following properties are related to the T-spherical fuzzy soft ordered weighted averaging operator which can easily be proved in view of the definitions:

(i) ***Idempotency***: If $T_{u_j} = T_u$, for all $j = 1, 2, ..., m$, then:

$$TSFSOWA_\omega(T_{u_1}, T_{u_2}, \dots T_{u_m}) = T_u.$$

(ii) ***Boundedness***: Let T_{u_j} $(j = 1, 2, \dots, m)$ be the collection of TSFSNs, then:

$$\min_j T_{u_j} \leq TSFSOWA_\omega(T_{u_1}, T_{u_2}, \dots T_{u_m}) \leq \max_j T_{u_j}.$$

(iii) ***Monotonicity***: Let T_{u_j} and $T_{u'_j}$ $(j = 1, 2, ..., m)$ be the two collections of T-spherical fuzzy soft numbers. If $T_{u_j} \leq T_{u'_j}$, then:

$$TSFSOWA_\omega(T_{u_1}, T_{u_2}, \dots T_{u_m}) \leq TSFSOWA_\omega(T_{u'_1}, T_{u'_2}, \dots T_{u'_m}),$$

where $T_{u'_j}$ is any permutation of T_{u_j} for $j = 1, 2, \dots, m$.

It may be noted that the T-spherical fuzzy soft weighted averaging operator takes weights of TSFSNs into account, while the T-spherical fuzzy soft ordered weighted averaging operator takes only the weights of the ordered positions of TSFSNs into account. This shows that both the operators take only one aspect into account. To overcome this issue and to incorporate both aspects together, we introduce T-Spherical Fuzzy Soft Hybrid Averaging (TSFSHA) operator and define it as follows.

Definition 5. *Suppose $\mathcal{P}$ is a collection of all T-spherical fuzzy soft numbers.*

Let $(T_{u_1}, T_{u_2}, \dots, T_{u_m}) \in \mathcal{P}^m$. A mapping:

$$TSFSHA_\omega : \mathcal{P}^m \longrightarrow \mathcal{P}$$

is called T-spherical fuzzy soft hybrid averaging operator if:

$$TSFSHA_{\omega,\gamma}(T_{u_1}, T_{u_2}, \dots, T_{u_m}) = \oplus_{j=1}^m(\gamma_j \widetilde{T}_{u_{\sigma(j)}}), \tag{5}$$

where $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_m)^T$ is the weight vector corresponding to $(\widetilde{T}_{u_{\sigma(j)}})_{j=1}^m$ s.t. $\gamma_j \geq 0$, for all j; $\sum\limits_{j=1}^m \gamma_j = 1$. $\widetilde{T}_{u_{\sigma(j)}}$ is the jth largest of the weighted TSFSNs $\widetilde{T}_{u_j}$, where $\widetilde{T}_{u_j} = (m\omega_j)T_{u_j}$ and m is the balancing coefficient with $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ being the weight vector of T_{u_j} with $\omega_j \geq 0$ for all j; $\sum\limits_{j=1}^m \omega_j = 1$.

Remarks:

- In case, we take uniformly distributed weights as $\gamma = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^T$; then, the T-spherical fuzzy soft hybrid averaging operator gives T-spherical fuzzy soft weighted averaging operator;
- However, if we take $\omega = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^T$, then the T-spherical fuzzy soft hybrid averaging operator gives T-spherical fuzzy soft ordered weighted averaging operator.

Theorem 4. *The T-spherical fuzzy soft hybrid averaging operator $TSFSHA_{\omega,\gamma}$ aggregates all the input values and yields a TSFSN given by:*

$$TSFSWA_{\omega,\gamma}(T_{u_1}, T_{u_2}, \dots, T_{u_m}) = \left(\sqrt[n]{1 - \prod_{j=1}^m (1 - \mu^n_{\widetilde{T}_{u_{\sigma(j)}}})^{\gamma_j}}, \prod_{j=1}^m (\eta_{\widetilde{T}_{u_{\sigma(j)}}})^{\gamma_j}, \prod_{j=1}^m (\nu_{\widetilde{T}_{u_{\sigma(j)}}})^{\gamma_j}\right). \tag{6}$$

Proof: The proof is similar to that of Theorem 2 and it can easily be carried out. Similarly, the properties of idempotency, boundedness and monotonicity related to the T-spherical fuzzy soft hybrid averaging operator can easily be listed and proved in view of the definitions.

5.2. Geometric aggregation operators

Here, we propose geometric aggregation operators (weighted, ordered, and hybrid) with their properties for the proposed T-spherical fuzzy soft numbers.

Definition 6. *Suppose $\mathcal{P}$ is a collection of all T-spherical fuzzy soft numbers.*

Let $(T_{u_1}, T_{u_2}, \dots, T_{u_m}) \in \mathcal{P}^m$. A mapping:

$$TSFSWG_\omega : \mathcal{P}^m \longrightarrow \mathcal{P}$$

is said to be a T-spherical fuzzy soft weighted geometric operator if:

$$TSFSWG_\omega(T_{u_1}, T_{u_2}, \dots, T_{u_m}) = \otimes_{j=1}^m (T_{u_j})^{\omega_j}, \tag{7}$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ is the weight vector corresponding to $(T_{u_j})_{j=1}^m$ such that $\omega_j \geq 0$ for all j; $\sum\limits_{j=1}^m \omega_j = 1$.

Theorem 5. *The T-spherical fuzzy soft weighted geometric operator $TSFSWG_\omega$ aggregates all the input values and yields a TSFSN given by:*

$$TSFSWA_\omega(T_{u_1},T_{u_2},\ldots,T_{u_m})$$

$$=\left(\prod_{j=1}^{m}(\mu_{T_{u_j}})^{\omega_j},\ \sqrt[n]{1-\prod_{j=1}^{m}(1-\eta^n_{T_{u_j}})^{\omega_j}},\right.$$

$$\left.\sqrt[n]{1-\prod_{j=1}^{m}(1-\nu^n_{T_{u_j}})^{\omega_j}}\right), \qquad (8)$$

where $\omega=(\omega_1,\omega_2,\ldots,\omega_m)^T$ *is the weight vector corresponding to* $(T_{u_j})_{j=1}^m$ *such that* $\omega_j\geq 0$, *for all* j; $\sum_{j=1}^{m}\omega_j=1$.

Proof: Using the *principle of mathematical induction* on m, the result in Eq. (8) given in the above-stated Theorem 5 can be proved as follows:

For $m=2$, we have:

$$TSFSWG_\omega(T_{u_1},T_{u_2})=T_{u_1}^{\omega_1}\oplus T_{u_2}^{\omega_2}.$$

It may be observed that both $T_{u_1}^{\omega_1}$ and $T_{u_2}^{\omega_2}$ are T-spherical fuzzy soft numbers. From the operation (c) defined in Subsection 5.1, for T-spherical fuzzy soft numbers, we have:

$$(T_{u_1})^{\omega_1}=(\mu^{\omega_1}_{T_{u_1}},\ \sqrt[n]{1-(1-\eta^n_{T_{u_1}})^{\omega_1}},$$

$$\sqrt[n]{1-(1-\nu^n_{T_{u_1}})^{\omega_1}});$$

and:

$$(T_{u_2})^{\omega_2}=(\mu^{\omega_2}_{T_{u_2}},\ \sqrt[n]{1-(1-\eta^n_{T_{u_2}})^{\omega_2}},$$

$$\sqrt[n]{1-(1-\nu^n_{T_{u_2}})^{\omega_2}}).$$

Then the equation shown in Box III is obtained. Hence, the result holds for $m=2$. Next, we suppose that Eq. (8) holds for $m=k$, that is:

$$TSFSGA_\omega(T_{u_1},T_{u_2},\ldots T_{u_k})=(T_{u_1})^{\omega_1}\otimes(T_{u_2})^{\omega_2}$$

$$\otimes\ldots\otimes(T_{u_k})^{\omega_k}=\left(\prod_{j=1}^{k}(\mu^n_{T_{u_j}})^{\omega_j},\right.$$

$$\left.\sqrt[n]{1-\prod_{j=1}^{k}(1-\eta^n_{T_{u_j}})^{\omega_j}},\ \sqrt[n]{1-\prod_{j=1}^{k}(1-\eta^n_{T_{u_j}})^{\omega_j}}\right).$$

Next, the result for $m=k+1$ is to be proved, that is, $TSFSGA_\omega(T_{u_1},T_{u_2},\ldots T_{u_{k+1}})$ is also a TSFSN. By operational laws of T-spherical fuzzy soft numbers, we have the equations shown in Box IV. This clearly shows that the aggregated value is also a TSFSN. Therefore, Eq. (8) holds for all $m=k+1$. This finally proves the result using the technique of mathematical induction.

Next, we list the following properties related to the T-spherical fuzzy soft weighted geometric operator, which can easily be proved in view of the definitions:

(i) ***Idempotency*:** If $T_{u_j}=T_u$; for all $j=1,2,\ldots,m$, then:

$$TSFSWG_\omega(T_{u_1},T_{u_2},\ldots T_{u_m})=T_u.$$

(ii) ***Boundedness*:** If T_{u_j} $(j=1,2,\ldots,m)$ be the collection of TSFSNs, then:

$$\min_j T_{u_j}\leq TSFSWG_\omega(T_{u_1},T_{u_2},\ldots T_{u_m})$$

$$\leq\max_j T_{u_j}.$$

(iii) ***Monotonicity*:** Let T_{u_j} and $T_{u'_j}$ $(j=1,2,\ldots,m)$ be the two collections of T-spherical fuzzy soft numbers. If $T_{u_j}\leq T_{u'_j}$, then:

$$TSFSWG_\omega(T_{u_1},T_{u_2},\ldots T_{u_m})$$

$$\leq TSFSWG_\omega(T_{u'_1},T_{u'_2},\ldots T_{u'_m}).$$

Further, we propose a definition for the ordered weighted geometric operator for T-spherical fuzzy soft numbers as follows:

$$TSFSWG_\omega(T_{u_1},T_{u_2})=(T_{u_1})^{\omega_1}\otimes(T_{u_2})^{\omega_2}$$

$$=\left((\mu^n_{T_{u_1}})^{\omega_1}(\mu^n_{T_{u_2}})^{\omega_2},\ \sqrt[n]{2-(1-\eta^n_{T_{u_1}})^{\omega_1}-(1-\eta^n_{T_{u_2}})^{\omega_2}-((1-(1-\eta^n_{T_{u_1}})^{\omega_1})(1-(1-\eta^n_{T_{u_2}})^{\omega_2}))},\right.$$

$$\left.\sqrt[n]{(2-(1-\nu^n_{T_{u_1}})^{\omega_1}-(1-\nu^n_{T_{u_2}})^{\omega_2})-((1-(1-\nu^n_{T_{u_1}})^{\omega_1})(1-(1-\nu^n_{T_{u_2}})^{\omega_2}))}\right)$$

$$=\left((\mu^n_{T_{u_1}})^{\omega_1}(\mu^n_{T_{u_2}})^{\omega_2},\ \sqrt[n]{1-\left((1-\eta^n_{T_{u_1}})^{\omega_1}(1-\eta^n_{T_{u_2}})^{\omega_2}\right)},\ \sqrt[n]{1-\left((1-\nu^n_{T_{u_1}})^{\omega_1}(1-\nu^n_{T_{u_2}})^{\omega_2}\right)}\right).$$

Box III

$$TSFSWG_\omega(T_{u_1},T_{u_2},\dots T_{u_{k+1}})=(T_{u_1})^{\omega_1}\otimes(T_{u_2})^{\omega_2}\otimes\dots\otimes(T_{u_k})^{\omega_k}\otimes(T_{u_{k+1}})^{\omega_{k+1}}$$

$$=\Bigg(\prod_{j=1}^{k+1}(\mu^n_{T_{u_j}})^{\omega_j},\ \sqrt[n]{1-\prod_{j=1}^{k}(1-\eta^n_{T_{u_j}})^{\omega_k}+1-(1-\eta^n_{T_{u_{k+1}}})^{\omega_{k+1}}-\big((1-\prod_{j=1}^{k}(1-\eta^n_{T_{u_j}})^{\omega_k})(1-(1-\eta^n_{T_{u_{k+1}}})^{\omega_{k+1}})\big)},$$

$$\sqrt[n]{1-\prod_{j=1}^{k}(1-\nu^n_{T_{u_j}})^{\omega_k}+1-(1-\nu^n_{T_{u_{k+1}}})^{\omega_{k+1}}-\big((1-\prod_{j=1}^{k}(1-\nu^n_{T_{u_j}})^{\omega_k})(1-(1-\nu^n_{T_{u_{k+1}}})^{\omega_{k+1}})\big)}\Bigg)$$

$$=\Bigg(\prod_{j=1}^{k+1}(\mu^n_{T_{u_j}})^{\omega_j},\ \sqrt[n]{1-\prod_{j=1}^{k+1}(1-\eta^n_{T_{u_j}})^{\omega_j}},\ \sqrt[n]{1-\prod_{j=1}^{k+1}(1-\nu^n_{T_{u_j}})^{\omega_j}}\Bigg).$$

Box IV

Definition 7. *Suppose $\mathcal{P}$ is a collection of all T-spherical fuzzy soft numbers.*

Let $(T_{u_1},T_{u_2},\dots,T_{u_m})\in\mathcal{P}^m$. A mapping:

$$TSFSOWG_\omega:\mathcal{P}^m\longrightarrow\mathcal{P}$$

is said to be T-spherical fuzzy soft ordered weighted geometric operator if:

$$TSFSOWG_\omega(T_{u_1},T_{u_2},\dots,T_{u_n})$$

$$=\otimes^n_{j=1}(T_{u_{\sigma(j)}})^{\omega_j},\tag{9}$$

where $\omega=(\omega_1,\omega_2,\dots,\omega_m)^T$ is the weight vector corresponding to $(T_{u_j})^m_{j=1}$ such that $\omega_j\geq 0$, for all j; $\sum\limits_{j=1}^m\omega_j=1$ and $(\sigma(1),\sigma(2),\dots,\sigma(m))$ is a possible permutation of $(1,2,\dots,m)$, s.t. $T_{u_{\sigma(j+1)}}\leq T_{u_{\sigma(j)}}$ for all $j=1,2,\dots,m-1$.

Theorem 6. *The T-spherical fuzzy soft ordered weighted geometric operator aggregates all the input values and yields a TSFSN given by:*

$$TSFSWG_\omega(T_{u_1},T_{u_2},\dots,T_{u_m})$$

$$=\Bigg(\prod_{j=1}^{m}(\mu_{T_{u_{\sigma(j)}}})^{\omega_j},\ \sqrt[n]{1-\prod_{j=1}^{m}(1-\eta^n_{T_{u_{\sigma(j)}}})^{\omega_j}},$$

$$\sqrt[n]{1-\prod_{j=1}^{m}(1-\eta^n_{T_{u_{\sigma(j)}}})^{\omega_j}}\Bigg);\tag{10}$$

where $\omega=(\omega_1,\omega_2,\dots,\omega_m)^T$ is the weight vector corresponding to $(T_{u_j})^m_{j=1}$ such that $\omega_j\geq 0$ for all j; $\sum\limits_{j=1}^m\omega_j=1$ and $(\sigma(1),\sigma(2),\dots,\sigma(m))$ is a permutation of $(1,2,\dots,m)$, such that $T_{u_{\sigma(j+1)}}\leq T_{u_{\sigma(j)}}$ for all $j=1,2,\dots,m-1$.

Proof: The proof is similar to the proof of Theorem 5 and can easily be carried out.

Similarly, the properties of *idempotency, boundedness, and monotonicity* related to the T-spherical fuzzy soft ordered weighted geometric operator can easily be listed and proved in view of the definitions. It may be noted that the T-spherical fuzzy soft weighted geometric operator takes only the weights of TSFSNs into account, while the T-spherical fuzzy soft ordered weighted geometric operator takes only the weights of the ordered positions of TSFSNs into account. This shows that both of the operators take only one aspect into account. In order to overcome this issue and incorporate both of the aspects together, we introduce the T-Spherical Fuzzy Soft Hybrid Geometric (TSFSHG) operator and define it as follows:

Definition 8. *Suppose $\mathcal{P}$ is a collection of all T-spherical fuzzy soft numbers.*

Let $(T_{u_1},T_{u_2},\dots,T_{u_m})\in\mathcal{P}^m$. A mapping:

$$TSFSHG_\omega:\mathcal{P}^m\longrightarrow\mathcal{P}$$

is said to be T-spherical fuzzy soft hybrid geometric operator, if:

$$TSFSHG_{\omega,\gamma}(T_{u_1},T_{u_2},\dots,T_{u_n})$$

$$=\otimes^n_{j=1}(\widetilde{T}_{u_{\sigma(j)}})^{\gamma_j},\tag{11}$$

where $\gamma=(\gamma_1,\gamma_2,\dots,\gamma_m)^T$ is the weight vector corresponding to $(\widetilde{T}_{u_{\sigma(j)}})^m_{j=1}$ s.t. $\gamma_j\geq 0$ for all j; $\sum\limits_{j=1}^m\gamma_j=1$. $\widetilde{T}_{u_{\sigma(j)}}$ is the jth largest of the weighted TSFSNs $\widetilde{T}_{u_j}$, where $\widetilde{T}_{u_j}=(m\omega_j)T_{u_j}$ and m is the balancing *coefficient with $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ being the weight vector of T_{u_j} with $\omega_j \geq 0$ for all j; $\sum_{j=1}^{m} \omega_j = 1$.*

Remarks

- In case, we take uniformly distributed weights as $\gamma = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^T$; then, the T-spherical fuzzy soft hybrid geometric operator gives a T-spherical fuzzy soft weighted geometric operator.
- However, if we take $\omega = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^T$, then the T-spherical fuzzy soft hybrid geometric operator gives a T-spherical fuzzy soft ordered weighted geometric operator.

Theorem 7. *The T-spherical fuzzy soft hybrid geometric operator $TSFSHG_{\omega,\gamma}$ aggregates all the input values and yields a TSFSN given by:*

$$TSFSHG_{\omega,\gamma}(T_{u_1}, T_{u_2}, \dots, T_{u_m})$$

$$= \left(\prod_{j=1}^{m} (\mu_{\widetilde{T}_{u_{\sigma(j)}}})^{\gamma_j}, \sqrt[n]{1 - \prod_{j=1}^{m} (1 - \eta^n_{\widetilde{T}_{u_{\sigma(j)}}})^{\gamma_j}}, \sqrt[n]{1 - \prod_{j=1}^{m} (1 - \eta^n_{\widetilde{T}_{u_{\sigma(j)}}})^{\gamma_j}} \right). \quad (12)$$

Proof: The proof is similar to that of Theorem 5 and can easily be carried out.

Similarly, the properties of *idempotency, boundedness, and monotonicity* related to the T-spherical fuzzy soft hybrid geometric operator can easily be listed and proved because of the definitions.

6. Decision-making model with T-spherical fuzzy soft information

We formulate a decision-making problem for our consideration given the T-spherical fuzzy soft information and propose a new methodology for its solution using the averaging and geometric operators.

Problem formulation

Let $\mathcal{C} = \{\mathcal{C}^1, \mathcal{C}^2, \dots, \mathcal{C}^q\}$ be the collection of q critera and $\mathcal{A} = \{\mathcal{A}^1, \mathcal{A}^2, \dots, \mathcal{A}^p\}$ be the collection of p alternatives. Based on the decision-maker's perception, we consider the weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_q)$ corresponding to each criterion $\mathcal{C}^j$ $(j = 1, 2, \dots, q)$, where $\omega_j \in [0, 1]$, for all j; $\sum_{j=1}^{q} \omega_j = 1$. Further, on the basis of information received from the decision makers in the form of T-spherical fuzzy soft numbers, we construct a decision matrix $R = [r_{ij}]_{p \times q} = [(\mu_{ij}, \eta_{ij}, \nu_{ij})]_{p \times q}$, where μ_{ij} represents the grade of membership of $\mathcal{A}^i$ positively satisfying the criterion $\mathcal{C}^j$; η_{ij} represents the degree of neutral/abstain membership of the alternative $\mathcal{A}^i$ for the criterion $\mathcal{C}^j$, and ν_{ij} represents non-membership degree of the $\mathcal{A}^i$ not satisfying the criterion $\mathcal{C}^j$, such that $\mu^n_{ij} + \eta^n_{ij} + \nu^n_{ij} \leq 1$; for all $i = 1, 2, \dots, p$; for all $j = 1, 2, \dots, q$ and n is a natural number (value to be chosen suitably). In view of the laid down criteria, we choose the best alternative from the available alternatives by using the flowchart of the proposed algorithm given in Figure 2.

Methodology

For the implementation of the proposed algorithm outlined above, we comprehensively present the necessary steps of the methodology below:

Step 1: Collect the information as a T-spherical fuzzy soft number related to each alternative satisfying the different criteria and arrange them as a T-spherical fuzzy soft decision matrix $R = [(\mu_{ij}, \eta_{ij}, \nu_{ij})]_{p \times q}$, given by the equation shown in Box V.

Step 2: In case the criteria are homogenous, we go to Step 3. Otherwise, if the criteria are heterogeneous, say, *cost criteria* and *benefit criteria*, we normalize the decision matrix using the equation:

$$r_{ij} = \begin{cases} T_{u_{ij}}; & \mathcal{C}^j \text{ is } \textit{cost criteria}, \\ T^c_{u_{ij}}; & \mathcal{C}^j \text{ is } \textit{benefit criteria}. \end{cases} \quad (13)$$

Step 3: Using either of the proposed aggregation operator (averaging operators or geometric operators), obtain the aggregated values (TSFSN) of all the

$$R_{p \times q} = \begin{array}{c} \\ \mathcal{A}^1 \\ \mathcal{A}^2 \\ \vdots \\ \mathcal{A}^p \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{C}^1 & \mathcal{C}^2 & \dots & \mathcal{C}^q \end{array} \\ \begin{pmatrix} (\mu_{11}, \eta_{11}, \nu_{11}) & (\mu_{12}, \eta_{12}, \nu_{12}) & \cdots & (\mu_{1q}, \eta_{1q}, \nu_{1q}) \\ (\mu_{21}, \eta_{21}, \nu_{21}) & (\mu_{22}, \eta_{22}, \nu_{22}) & \cdots & (\mu_{2q}, \eta_{2q}, \nu_{2q}) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{p1}, \eta_{p1}, \nu_{p1}) & (\mu_{p2}, \eta_{p2}, \nu_{p2}) & \cdots & (\mu_{pq}, \eta_{pq}, \nu_{pq}) \end{pmatrix} \end{array}.$$

Box V

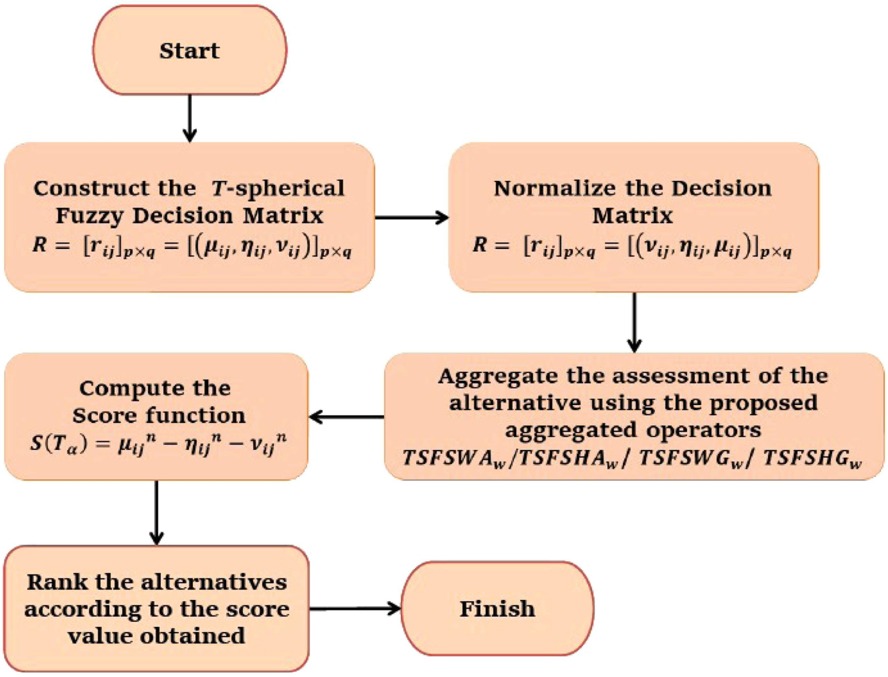

Figure 2. Flow chart of the proposed algorithm.

$$R = \begin{array}{c} \\ \mathcal{A}^1 \\ \mathcal{A}^2 \\ \mathcal{A}^3 \\ \mathcal{A}^4 \end{array} \begin{array}{c} \begin{array}{cccc} \quad\mathcal{C}^1 & \qquad\qquad\mathcal{C}^2 & \qquad\qquad\mathcal{C}^3 & \qquad\qquad\mathcal{C}^4 \end{array} \\ \begin{pmatrix} (0.2,0.2,0.6) & (0.5,0.3,0.2) & (0.5,0.2,0.3) & (0.4,0.3,0.2) \\ (0.3,0.4,0.4) & (0.6,0.3,0.1) & (0.5,0.3,0.2) & (0.2,0.1,0.7) \\ (0.4,0.5,0.2) & (0.6,0.3,0.2) & (0.7,0.2,0.2) & (0.3,0.3,0.5) \\ (0.3,0.2,0.6) & (0.2,0.2,0.6) & (0.2,0.3,0.6) & (0.4,0.2,0.4) \end{pmatrix} \end{array}.$$

Box VI

row-entries corresponding to each alternative $\mathcal{A}^i$ $(i = 1, 2, \dots, p)$ of the normalized decision matrix.

Step 4: By applying Definition (2), calculate the score value for each aggregated value obtained in Step 3 corresponding to each alternative $\mathcal{A}^i$ $(i = 1, 2, \dots, p)$.

Step 5: Ranking of the alternatives is finally done based on the score values obtained.

7. Numerical example

The proposed methodology for solving the multi-criteria decision-making problem is illustrated with a numerical example as follows.

Example: Assume that an Indian multi-national company is planning on some financial strategy for the upcoming year as per the group strategy objective. Four well-defined investment alternatives have been taken into consideration and labeled as $\mathcal{A}_1$: investment in "*South Indian Markets*"; $\mathcal{A}_2$: investment in "*East Indian Markets*"; $\mathcal{A}_3$: investment in "*North Indian Markets*"; and $\mathcal{A}_4$: investment in "*West Indian markets*". After a preliminary screening for evaluation purposes, it has been decided to proceed by taking four criteria, namely as $\mathcal{C}^1$: "*growth*"; $\mathcal{C}^2$: "*risk analysis*"; $\mathcal{C}^3$: "*the socio-political impact*", and $\mathcal{C}^4$: "*the environmental and other factors*". Suppose that based on the financial strategies adopted for the welfare of the company, the weight vector is $\omega = (0.2, 0.3, 0.1, 0.4)^T$.

Here, for the simplicity of the computation for the example under consideration, we take the value of n as 2 in the definitions. The computational steps for above-stated problem using the proposed algorithm are as follows:

Step 1. First, we construct the following spherical fuzzy soft decision matrix $R = [(r_{ij})] = [(\mu_{ij}, \eta_{ij}, \nu_{ij})], (i, j = 1, 2, 3, 4)$ for the four alternatives $\mathcal{A}^i$ $(i = 1, 2, 3, 4)$ and the four criteria $\mathcal{C}^j$ $(j = 1, 2, 3, 4)$ based on the information provided by the experts shown in Box VI.

Step 2. Since $\mathcal{C}^2$ and $\mathcal{C}^3$ are the cost criteria while $\mathcal{C}^1$

$$R = \begin{array}{c} \\ \mathcal{A}^1 \\ \mathcal{A}^2 \\ \mathcal{A}^3 \\ \mathcal{A}^4 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{C}^1 & \mathcal{C}^2 & \mathcal{C}^3 & \mathcal{C}^4 \end{array} \\ \begin{pmatrix} (0.6,0.2,0.2) & (0.5,0.3,0.2) & (0.5,0.2,0.3) & (0.2,0.3,0.4) \\ (0.4,0.4,0.3) & (0.6,0.3,0.1) & (0.5,0.3,0.2) & (0.7,0.1,0.2) \\ (0.2,0.5,0.4) & (0.6,0.3,0.2) & (0.7,0.2,0.2) & (0.5,0.3,0.3) \\ (0.6,0.2,0.3) & (0.2,0.2,0.6) & (0.2,0.3,0.6) & (0.4,0.2,0.4) \end{pmatrix} \end{array}.$$

Box VII

Table 1. Need to address the problem arising from Intuitionistic Fuzzy Sets (IFSs), Pythagorean Fuzzy Sets (PyFSs), and Picture Fuzzy Sets (PFSs).

R	C_1	C_2	C_3	C_4
C_1	$(1.0+0.0+0.0=1)$	$(0.40+0.20+0.69>1)$	$(0.36+0.19+0.79>1)$	$(0.56+0.17+0.62>1)$
C_2	$(0.68+0.20+0.44>1)$	$(1.0+0.0+0.0>1)$	$(0.40+0.24+0.56>1)$	$(0.51+0.29+0.61>1)$
C_3	$(0.76+0.20+0.42>1)$	$(0.54+0.24+0.42>1)$	$(1.0+0.0+0.0>1)$	$(0.48+0.14+0.77>1)$
C_4	$(0.49+0.17+0.68>1)$	$(0.59+0.29+0.53>1)$	$(0.77+0.17+0.38>1)$	$(1.0+0.0+0.0>1)$

and $\mathcal{C}^4$ are the benefit criteria, we have to normalize the decision matrix by using Eq. (13). Hence, we obtain the normalized decision matrix as shown in Box VII.

Step 3. Using the T-spherical fuzzy soft weighted average aggregating operator for the normalized decision matrix calculated in Step 2, the aggregated value for each alternative is presented below:

$$Aggregated\ Value : TSFSWA_{\omega}$$

$$\begin{array}{c} \mathcal{A}^1 \\ \mathcal{A}^2 \\ \mathcal{A}^3 \\ \mathcal{A}^4 \end{array} \begin{pmatrix} T_{u_1} : (0.444983, 0.26564, 0.274822) \\ T_{u_2} : (0.610765, 0.204767, 0.176173) \\ T_{u_3} : (0.526385, 0.319067, 0.270192) \\ T_{u_4} : (0.401021, 0.208276, 0.436105) \end{pmatrix}$$

Step 4. The score value for each aggregated value of the corresponding alternative is calculated by using Definition 2. The computed values are given as:

$$S(T_{u_1}) = 0.051918, \qquad S(T_{u_2}) = 0.300067,$$

$$S(T_{u_3}) = 0.102274, \qquad S(T_{u_4}) = -0.07275.$$

Step 5. Based on the values obtained in the last step, we observe that $S(T_{u_2}) > S(T_{u_3}) > S(T_{u_1}) > S(T_{u_4})$ and the ranking of the alternatives is done.

Thus, it has been found that the alternative $\mathcal{A}^2$ is the best one. Therefore, the best alternative strategy for the company is to invest in the East Indian Market.

8. Comparative remarks and advantages

The proposed notion of T-spherical FSS is a novel concept and an advanced extension of the classical fuzzy set. The T-spherical fuzzy sets have an added advantage to deal with a wider sense of applicability in uncertain situations. In detail, some important comparative remarks and advantages of utilizing T-spherical fuzzy set are listed below:

- The existing fuzzy sets, IFSs and PFSs have their own limitations which make them not capable to capture the full information specification, that is, there is a missing additional component of degree of refusal which is addressed by the spherical fuzzy sets/soft sets;
- When the uncertain or imprecise information takes the form of a fuzzy relation to ensure a kind of parametrization in the relation, we utilize the concept of T-spherical FSSs in natural sciences for therapeutic recommendations;
- The drawback in the existing literature of the fuzzy sets is that the condition does not allow experts/decision-makers to allocate the membership values of their own choice (refer Table 1). Somehow, this makes the decision-makers bounded for providing their input in a particular domain. However, the proposed T-spherical FSSs provide a generalization feature that has a strong impact on the application/decision-making process;
- The discussion over implementing the proposed T-spherical FSSs and the various aggregation operators for a financial strategic multi-criteria decision-making model/problem in Section 6 and Section 7 shows that the proposed work handled the generalized framework in an effective and consistent way.

9. Conclusions and future research directions

The novel concept of T-spherical FSS was successfully introduced along with various operations. Some important properties and the notion of score function/accuracy function for T-spherical fuzzy soft set were studied in brief. Averaging aggregation operators and geometric aggregation operators (weighted, ordered and hybrid) for T-spherical fuzzy soft numbers were proposed and well utilized along with their different properties in multi-criteria decision-making problems. Further, the proposed algorithm, that utilizes the aggregation operators was well implemented for solving a decision-making problem. A numerical example well presents the outline of the methodology. The proposed notion and the algorithms using the aggregation operators may further be utilized and extended in the future with the following possibilities:

- In literature, a variety of extensions of soft sets [21] to imprecise and incomplete information have been proposed. Given the generalizations and extensions of fuzzy sets shown by Figure 1 in the introduction section, we may further propose extending the notion of T-spherical fuzzy soft matrices based on [38–40] along with their various matrix operations, properties, and engineering applications;
- Since there is a type of parametrization tool involved in the soft sets and consequently in soft matrices, various related applications, such as stock management [41], medical diagnosis [42], and dimensionality reduction [43] have been studied recently. Hence, introducing the concept of T-spherical fuzzy soft matrices can lead to a new dimension in the soft set theory and related applications;
- The extended notion of T-spherical complex fuzzy soft sets and their aggregation operators may also be introduced based on the extension outlines discussed in [44,45].

References

1. Zadeh, L.A. "Fuzzy sets", *Information and Control*, **8**, pp. 338–353 (1965).
2. Atanassov, K.T. "Intuitionistic fuzzy sets", *Fuzzy Sets and Systems*, **20**(1), pp. 87–96 (1986).
3. Atanassov, K.T., and Vassilev, P. "On the intuitionistic fuzzy sets of n-th type", *Advances in Data Analysis with Computational Intelligence Methods, Studies in Computational Intelligence*, **738**, pp. 265–274 (2018).
4. Yager, R.R. "Pythagorean fuzzy subsets", In *Proceedings of Joint IFSA World Congress and NAFIPS Annual Meeting*, Edmonton, Canada, pp. 57–61 (2013).
5. Cuong, B. "Picture fuzzy sets-first results Part 1", *Seminar on Neuro-Fuzzy Systems with Applications*, Institute of Mathematics, Hanoi (2013).
6. Mahmood, T., Kifayat, U., Khan, Q., and Jan, N. "An approach toward decision making and medical diagnosis problems using the concept of spherical fuzzy sets", *Neural Computing and Applications*, **31**(11), pp. 7041–7053 (2019).
7. Kifayat, U., Mahmood, T., and Jan, N. "Similarity measures for T-spherical fuzzy sets with applications in pattern recognition", *Symmetry*, **10**(6), p. 193 (2018).
8. Garg, H., Munir, M., Ullah, K., Mahmood, T., and Jan, N. "Algorithm for T-spherical fuzzy multi-attribute decision making based on improved interactive aggregation operators", *Symmetry*, **10**, p. 670 (2018).
9. Arora, R. and Garg, H. "Robust aggregation operators for multi-criteria decision-making with intuitionistic fuzzy soft set environment", *Scientia Iranica*, **25**(2), pp. 931–942 (2018).
10. Garg, H., Agarwal, N., and Tripathi, A. "Some improved interactive aggregation operators under interval-valued intuitionistic fuzzy environment and its application to decision making process", *Scientia Iranica*, **24**(5), pp. 2581–2604 (2017).
11. Xu, Z.S. and Yager, R.R. "Some geometric aggregation operators based on intuitionistic fuzzy sets", *International Journal of General System*, **35**(4), pp. 417–433 (2006).
12. Yager, R.R. "Prioritized aggregation operators", *International Journal of Approximate Reasoning*, **48**(1), pp. 263–274 (2008).
13. Xu, Z. and Yager, R.R. "Power-geometric operators and their use in group decision making", *IEEE Transactions on Fuzzy Systems*, **18**(1), pp. 94–105 (2010).
14. Yu, D. "Intuitionistic fuzzy prioritized operators and their application in multi criteria group decision making", *Technological and Economic Development of Economy*, **19**(1), pp. 1–21 (2013).
15. Yager, R.R. "On generalized Bonferroni mean operators for multi criteria aggregation", *International Journal of Approximate Reasoning*, **50**(8), pp. 1279–1286 (2009).
16. Xu, Z.S. and Yager, R.R. "Intuitionistic fuzzy bonferroni means", *IEEE Transaction on Systems, Man and Cybernetics*, **41**(2), pp. 568–578 (2011).
17. Liu, P. and Li, H. "Interval valued intuitionistic fuzzy power Bonferroni aggregation operators and their application to group decision making", *Cognitive Computation*, **9**(4), pp. 494–512 (2017).
18. Wang, W. and Liu, X. "Intuitionistic fuzzy information aggregation using Einstein operations", *IEEE Transactions on Fuzzy Systems*, **20**(5), pp. 923–938 (2012).
19. Tao, Z., Han, B., and Chen, H. "On intuitionistic fuzzy copula aggregation operators in multiple attribute

decision making", *Cognitive Computation*, **10**(4), pp. 610–624 (2018).

20. Garg, H. "Generalized pythagorean fuzzy geometric aggregation operators using t–norm and t–conorm for multi criteria decision making", *International Journal of Intelligent Systems*, **32**(6), pp. 597–630 (2017).

21. Molodstov, D.A. "Soft set theory first result", *Computers and Mathematics with Application*, **27**, pp. 19–31 (1999).

22. Maji, P.K., Biswas, R., and Roy, A.R. "Intuitionistic fuzzy soft sets", *Journal of Fuzzy Mathematics*, **9**, pp. 677–692 (2001).

23. Kalaichelvi, A. and Haritha, M.P. "Application of fuzzy soft sets to investment decision making problem", *International Journal of Mathematical Sciences and Applications*, **1**(3), pp. 1583–1586 (2011).

24. Özgür, N.Y., and Taş, N. "A note on application of fuzzy soft sets to investment decision making problem", *Journal of New Theory*, **7**, pp. 1–10 (2015).

25. Irkin, R., Özgür, N.Y., and Taş, N. "Optimization of lactic acid bacteria viability using fuzzy soft set modelling", *International Journal of Optimization Control: Theories & Applications*, **8**(2), pp. 266–275 (2018).

26. Karaca, F. and Taş, N. "Decision making problem for life and non-life insurances", *Journal of BAUN*, Institute of Science and Technology, **20**(1), pp. 572–588 (2018).

27. Maji, P.K., Biswas, R., and Roy, A.R. "An application of soft sets in a decision making problem", *Computers and Mathematics with Applications*, **44**, pp. 1077–1083 (2002).

28. Maji, P.K., Biswas, R., and Roy, A.R. "Soft set theory", *Computers and Mathematics with Applications*, **45**, pp. 555–562 (2003).

29. Peng, X., Yang, Y., Song, J., and Jiang, Y. "Pythagorean fuzzy soft set and its application", *Computer Engineering*, **41**, pp. 224–229 (2015).

30. Peng, X. and Yang, Y. "Algorithms for interval valued fuzzy soft sets in stochastic multi-criteria decision making based on regret theory and prospect theory with combined weight", *Applied Soft Computing*, **54**, pp. 415–430 (2017).

31. Peng, X. and Garg, H. "Algorithms for interval valued fuzzy soft sets in emergency decision making based on WDBA and CODAS with new information measure", *Computers & Industrial Engineering*, **119**, pp. 439–452 (2018).

32. Peng, X. and Liu, C. "Algorithms for neutrosophic soft decision making based on EDAS, new similarity measure and level soft set", *Journal of Intelligent and Fuzzy Systems*, **32**, pp. 955–968 (2017).

33. Peng, X. and Li, W. "Algorithms for hesitant fuzzy soft decision making based on revised aggregation operators, WDBA and CODAS", *Journal of Intelligent and Fuzzy Systems*, pre-press, pp. 1–17 (2019).

34. Wei, G. "Picture fuzzy aggregation opertors and their application to multiple attribute decision making", *Journal of Intelligent and Fuzzy Systems*, **33**, pp. 713–724 (2017).

35. Liu, P. and Zhang, X. "A novel picture fuzzy linguistic aggregation operator and its application to group decision making", *Cognitive Computation*, **10**(2), pp. 242–259 (2018).

36. Atanassov, K.T. "Geometrical interpretation of the elements of the intuitionistic fuzzy objects", *Preprint IMMFAIS1–89, Sofia, 1989. Reprinted: Int. J. Bioautomation*, **20**(1), pp. S27–S42 (2016).

37. Maji, P.K., Biswas, R., and Roy, A.R. "Fuzzy soft sets", *Journal of Fuzzy Mathematics*, **9**(3), pp. 589–602 (2001).

38. Chetia, B. and Das, P.K. "Some results of intuitionistic fuzzy soft matrix theory", *Advances in Applied Science Research*, **3**, pp. 412–423 (2012).

39. Naim, C. and Serdar, E. "Soft matrix theory and its decision making", *Computers and Mathematics with Applications*, **59**, pp. 3308–3314 (2010).

40. Yong, Y. and Chenli, J. "Fuzzy soft matrices and their applications", *Lecture Notes in Computer Science*, LNAI7002, pp. 618–627 (2011).

41. Taş, N., Özgür, N.Y., and Demir, P. "An application of soft set and fuzzy soft set theories to stock management", Süleyman Demirel University, *Journal of Natural and Applied Sciences*, **21**(3), pp. 791–796 (2017).

42. Guleria, A. and Bajaj, R.K. "On Pythagorean fuzzy soft matrices, operations and their applications in decision making and medical diagnosis", *Soft Computing*, **23**(17), pp. 7889-7900 (2019).

43. Bajaj, R.K. and Guleria, A. "Dimensionality reduction technique in decision making using pythagorean fuzzy soft matrices", *Recent Patents on Computer Science*, **12**, pp. 1–8 (2019).

44. Kumar, T. and Bajaj, R.K. "On complex intuitionistic fuzzy soft sets with distance measures and entropies", *Journal of Mathematics*, **2014**, Article ID–972198, 12 pages (2014).

45. Thirunavukarasu, P., Suresh, R., and Ashokkumar V. "Theory of complex fuzzy soft set and its applications", *International Journal for Innovative Research in Science and Technology*, **3**(10), pp. 13–18 (2017).

Biographies

Abhishek Guleria, MPhil, is currently working as a Research Scholar at the Department of Mathematics, Jaypee University of Information Technology, Waknaghat, Solan, HP, India. He has received his BSc degree in Mathematics from CSK Himachal Pradesh Krishi Vishwavidyalaya, Palampur (Himachal Pradesh) and the MSc from Himachal Pradesh University, Summer Hill, Shimla (H.P.), India in 2013 and 2015, respectively. He received his MPhil (Mathematics) from Himachal Pradesh University, Summer Hill, Shimla (H.P.), India in 2017. His research interests are fuzzy information measures, decision making, pattern recognition, and soft computing techniques and applications.

Rakesh Kumar Bajaj, PhD, has received his BSc degree with honors in Mathematics from Banaras Hindu University, Varanasi, India and the MSc from the Indian Institute of Technology, Kanpur, India in 2000 and 2002, respectively. He received his PhD (Mathematics) from Jaypee University of Information Technology (JUIT), Waknaghat in 2009. He has been working as an Associate Professor at the Department of Mathematics, Jaypee University of Information Technology, Waknaghat, Solan, HP, India since 2003. His interests include fuzzy information measures, pattern recognition, fuzzy clustering, decision-making, fuzzy statistics, and fuzzy mathematics in image processing.