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Free Vibration Analysis of Bridges Having Combined Skewness and Curvature

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Article Info
Volume 83
Page Number: 610 - 619
Publication Issue:
March - April 2020

Article History
Article Received: *24 July 2019*
Revised: *12 September 2019*
Accepted: *15 February 2020*
Publication: *12 March 2020*

Abstract:
This paper presents the numerical study on the modal behaviour of skew-curve concrete box-girder bridges. 3-D finite element models for different curvature angles ranging between 0° to 90° at an interval of 30° with combination of skew angles of 0°, 15°, 30°, 45° and 60° have been developed in finite element program CSiBridge. Modal analysis for each bridge configuration is performed to generate mode shapes, time periods and modal mass participating ratios in the three orthogonal directions. With change in bridge configuration, variations of the time period and modal mass participation ratio is presented in graphical form for vertical and horizontal vibration modes. Results showed that time period of the first in-plane vibration mode is usually decreasing with increase in curvature as the in-plane bending flexibility of the deck decreases due to arching action which occurs because of increase in curvature. However, the time period increases with increase in skew angle for first in-plane vibration mode. Time period of first longitudinal mode is usually increasing with increase in skewness and curvature. Modal mass participation ratios are also found to increase with increase in skewness and curvature for first in-plane vibration mode. These ratios show complicated and variable behaviour in higher modes with high skew-curve combinations. These results hold pivotal role in determining the seismic response of skew-curve bridges.

Keywords: *RC Box Girder Bridges, Skewness, Plan Curvature, Natural Time Period, Mode Shape, Mass Participation Factor, Free Vibration Analysis, Skew-Curve.*

1. Introduction

A structure facilitating passage over an obstacle such as any river, valley, railway, roads etc. without closing the way under it, is known as bridge structure. The required passage may be for different purposes such as railway, pedestrians and roads. Based on geometric alignment of the traffic route, bridges are generally of two types i.e. straight and curved bridges. The horizontal geometric alignment of the superstructure of a curved bridge is defined in terms of either curvature angle (β) or radius of curvature (R) for the given centreline length of the bridge deck as shown in Figure 1 (b). Similarly, based on the orientation of the support with respect to the traffic flow direction, bridges are classified as straight and skew bridges. Those bridge structures in which longitudinal axis of the deck is not perpendicular to the substructure (i.e. abutments and/or bents) alignment are termed as skew bridges. Degree of skewness is defined in terms of skew angle (θ) as shown in Figure 1 (a). Due to high traffic conjunctions and/or geometrical demands, many times it is needed to combine curved bridges on skewed supports, such bridges are called as “skew-curve” bridges.

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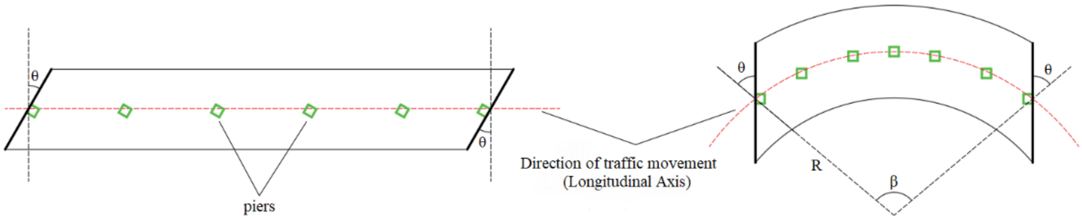


a. Skew Bridge b. Curved Bridge

Figure 1. Schematic Diagram for Skew Bridge and Curved Bridge

Seismic vulnerability of the bridges is one of the major issues that has taken the attention of many researchers. The seismic behaviour of skew and curved bridges individually has been presented in many studies but a very limited research has been performed on free vibration response of the skew-curve bridges. Modal response of the structure is very important in determination of the condition that whether resonance will happen under seismic excitations or not. For earthquake resistant design, obtaining knowledge on the complex behaviours of skew-curve bridges is of extreme importance. Modal analysis is the initial step of the seismic analysis of any structure which helps in determining the basic dynamic characteristics such as natural frequencies, time periods, mode shapes etc.

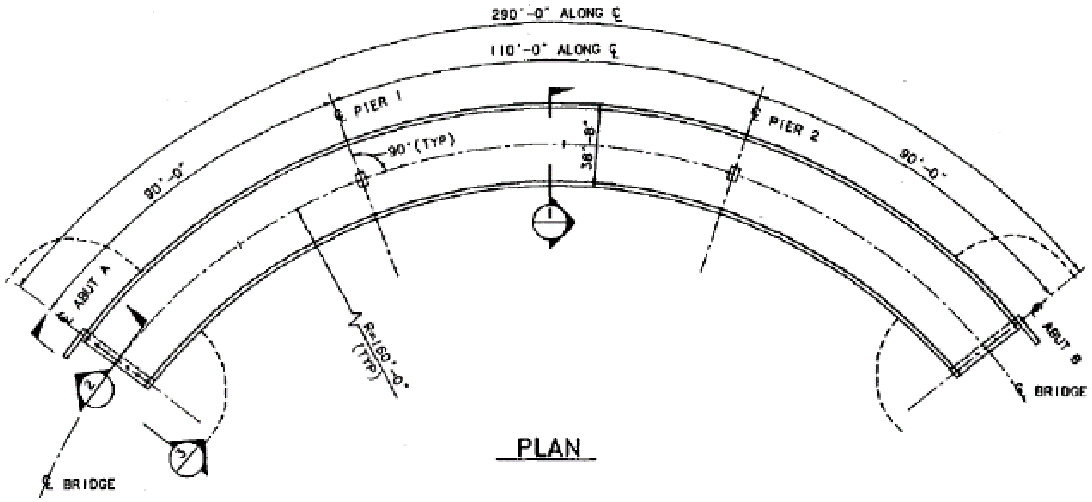
Free vibration analysis has been proven to show thatfor skew bridges having only one span, translational modes are naturallycoupled [1]. Skew bridges having unrigid connection between abutments and deck slab, shows planar rigid body rotations of the deck instead of naturally expected torsional deformations. Thus, torsional and flexural deformations of the bridge deck can be neglected [2 &3]. Although, vertical excitation has negligible effect on horizontal transverse response, however, with increase in super elevation of the bridge deck effect of coupling (vertical and horizontal transverse response) gets increased [4]. In case of thin-walled girder bridges having in-plan curvature, warping effect comes out to be negligible when radius of curvature exceeds 1.7 times the total length of the span [5].

Observations have been made during the Chile earthquake (2010) that one of the typical modes of the failure of horizontally curved bridges is unseating of the deck from abutments [6]. Therefore, larger support lengths should be provided to prevent girder unseating of skew bridges which are located in near-fault regions [7 & 8]. Probability of deck unseating is more at the acute corners of skewed bridge [9]. Skewed and horizontally curved bridges suffered more damages due to rotation of the superstructure or displacement towards the outside of the curve line.From the past studies, it has been concluded that skew and curved bridges have more complex behaviour under the effect of earthquake ground motion and even for free vibration response. These movements i.e. bi-directional movement and rotation of the bridge deck also causes seismic pounding of the bridge deck with abutments which increases drastically at the time of resonance.

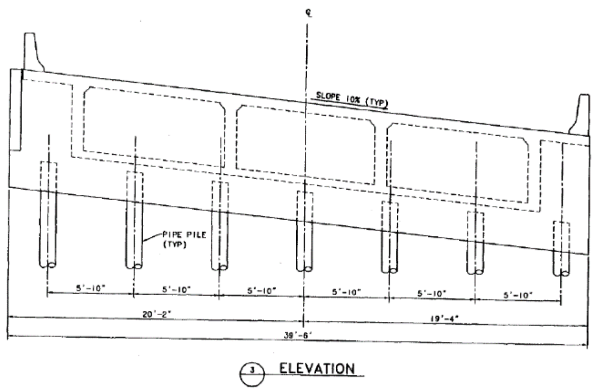
Nowadays, finite element analysis is a common way to perform numerical modal analysis of the bridge structures. In this study, numerical modal analysis of a skew-curve concrete box girder bridge is presented in terms of mode shapes, time period and modal participation mass ratios. Furthermore, different skew angles ranging between 0° to 60° at an interval of 15° have been introduced along with the variations in curvature angles ranging between 0° to 90° at an interval of 30°.

2. Benchmark Bridge

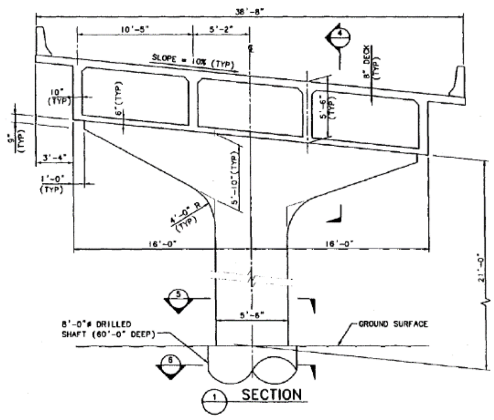
Benchmark bridge for this study has been taken from example no. 6 of the FHWA series (FHWA, 1996-a) [11]. The configuration of the bridge is a three-span, three cell, concrete box girder superstructure with span lengths of 27.43m(90ft), 33.53m(110ft), and 27.43m(90ft) respectively as shown in Figure 2. The superstructure of the bridge is horizontally curved at a curvature angle of 104° which was supported on the reinforced concrete columns founded on drilled shafts and on integral abutments founded on steel pipe piles. The intermediate bents have a crossbeam integral with the box girder and a circular column which is supported by 18.29m(60ft) deep drilled shaft foundation. Diameter of drilled shaft foundation and the column was 2.43m(8ft) and 1.67m(5.5ft) respectively. For the stated soil type, modulus of subgrade reaction has been taken as $6000 kN/m^3$. The diaphragm-type abutments are supported by seven 12m deep pipe piles.



a. Structural Layout of the Bridge



b. Abutment details of the Bridge



c. Bent details of the Bridge

Figure 2. Detailed drawings of the bridge [11]

3. Numerical Modelling of Bridges

To facilitate a numerical study of the modal response of skew-curve highway bridges, 3-Dimensional finite element models were developed using CSiBridge. Numerical model has been prepared based on detailed drawings of the benchmark bridge [10 & 11]. The changes have been made in benchmark bridge to generate the models with various skew and curvature angles, but with the same overall dimensions. Models have been generated with four different curvature angles (β) i.e. 0°, 30°, 60°, and 90°. For each unique curvature angle, to generate skew-curve bridges, skew angles (θ) have been varied as 0°, 15°, 30°, 45° and 60°. A total 20 number of bridge configurations have been modelled out of which Figure 3 (a) shows 3-D finite element model for a combination of 90° β and 0° θ. Length of centreline has been kept constant i.e. equal to 88.4m(290ft) for every bridge configuration. For all bridge models, assumption has been made that

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superstructure elements are linear-elastic in nature. A superelevation of 10% has been provided for the bridge deck. Three interior diaphragms of 0.23m(9in) thickness and two thick exterior diaphragms of thickness 0.76m(2.5ft) have been provided at the middle of each span and at the abutments respectively. Material assigned is 27.58Mpa concrete for superstructure elements and 24.82MPa for substructure elements. Area elements have been used for modelling of deck, soffit, girders and diaphragms while bent columns, piles and drilled shaft foundation have been modelled using 3-D frame elements. The tapered portion of the piers has been modelled using non-prismatic frame elements with linearly varying cross-section as shown in Figure 3 (b).

a. 3-D model of the bridge for curvature angle (β) = 90° and skew angle (θ) = 0°

b. Piers

Figure 3. 3-D model of the bridge

Spring elements have been used for accounting the effect of soil stiffness on the piles and drilled shaft foundation. Lateral passive resistance of the foundation soil along the foundation of the piers i.e. drilled shaft has been represented using 15 pairs of soil springs. 13 pairs of soil springs have been used to model the lateral stiffness of the foundation soil along the piles. Gap link elements have been used for modelling of the abutment gap. The stiffness of gap element activates only when the gap closes. Multi-linear plastic link element has been employed to simulate the response of backfill soil. Maximum meshing size has been assigned as 0.3m, thus total number of shell elements have been found ranging between 35000 to 100000 which varies with the change in bridge configuration.Discrete soil spring stiffness $k_i(kN/m)$ can be determined for a given tributary length H_i(m) of pile using the following equation;

$$k_i = k_h.D.H_i$$

Where, $k_h$is the soil modulus in kN/m^3 and can be calculated using the following equation;

$$k_h = n_h\left(\frac{z}{D}\right)$$

In above equation; $n_h$is depth-independent subgrade reaction coefficient in kN/m^3 , z is depth below ground surface in *meters*. $D$is diameter of the pile in *meters.*

4. Modal Analysis of Skew-Curve Bridges

Normally, the bridge structures experience continuous dynamic motion due to ambient excitations such as wind, traffic earthquake and their combination which occur naturally. These small ambient vibrations are normally close to the natural frequencies of the structure and are terminated by energy dissipation in the real structure.When the load is carried away, the structure attains free vibration. So, it is possible that the external excitation could match a natural frequency of the bridge structure which will cause resonance. When the input load excitation frequency matches one of the natural frequencies of the structure then resonance of frequencies occurs. Therefore, it is necessary to study the natural frequency of the bridge structures which will help in better seismic design.

Each mode consists of a mode shape and a set of modal properties such as time periods, frequencies, modal participating mass ratios and modal participation factors etc.Structure's deformed shape at a specific natural frequency of vibration is known as its normal mode shape of vibration. Each mode shape is associated with a specific natural frequency. Natural frequencies of the structure are those frequencies at which the structure naturally tends to vibrate if subjected to any disturbance and the time taken to complete a cycle of vibration is termed as time period. Participation factor indicates how strongly a given mode contributes to the response.

Generally, vibration modes of the bridge are categorised as; vertical vibration modes and horizontal vibration modes. Horizontal translation modes are further categorised as; longitudinal and transverse vibration modes. Transverse vibration modes are also termed as in-plane vibration modes due to in-plane movement of deck (perpendicular to longitudinal axis) while in longitudinal mode, bridge moves along its longitudinal axis. In case of vertical vibration modes, either out of plane deformation or torsional deformation in longitudinal direction is generally observed, which are attributed by out-of-plane bending modes and the longitudinal-torsional mode.

In order to explain the modal behaviour of bridges, the parameters such as mode-shapes, time periods and mass participation factors have been studied.

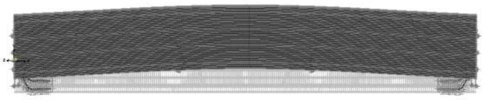
4.1 Validation of Results

In this study, skew-curve bridges with varying configuration of curvature and skew angle have been analyzed for modal analysis. To validate the results, a 3-D model is generated with same configuration as taken in the thesis [10]. For the validation of the results from present analysis, time periods of $\beta = 0°$ & $\theta = 0°$ (straight bridge) has been compared in the Table 1 which shows a very good coherence with the source thesis [10]. Figure 4 shows an excellent agreement between mode shapes obtained from present analysis with mode shapes described in thesis [10] for bridge configuration having curvature angle (β) = 0° & skew angle (θ) = 0°.

Table 1. Comparison of the Time Periods of first In-Plane, Longitudinal and Out of Plane Mode from present analysis with standard results [10]

Mode	**Time Period in Present Analysis** (seconds)	**Time Period in [10]** (seconds)	**Difference** (in percentage)
1st In-Plane Mode	0.499	0.517	-3.48
1st Longitudinal Mode	0.322	0.319	0.99
1st Out of Plane Mode	0.254	0.259	-1.65

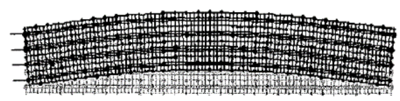
Mode Shapes in Present Analysis



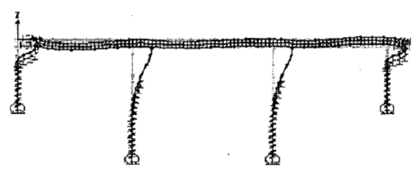
1st In-Plane Mode Shape

1st Longitudinal Mode Shape

Mode Shapes in [10]



1st In-Plane Mode Shape



1st Longitudinal Mode Shape

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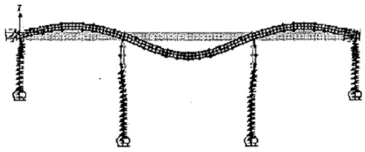
1st Out of Plane Mode Shape



1st Out of Plane Mode Shape

Figure 4. Comparison of mode shapes of present analysis with thesis [10] for bridge configuration having curvature angle (β) = 0° & skew angle (θ) = 0°

4.2 Modal Behaviour of Skew-Curve Bridges

Main objective of the study is to examine the behaviour of skew-curve bridges under free vibrations. Modal analysis has been performed on the generated bridge models to find out the mode shapes, time periods, natural frequencies and modal participating mass ratios.

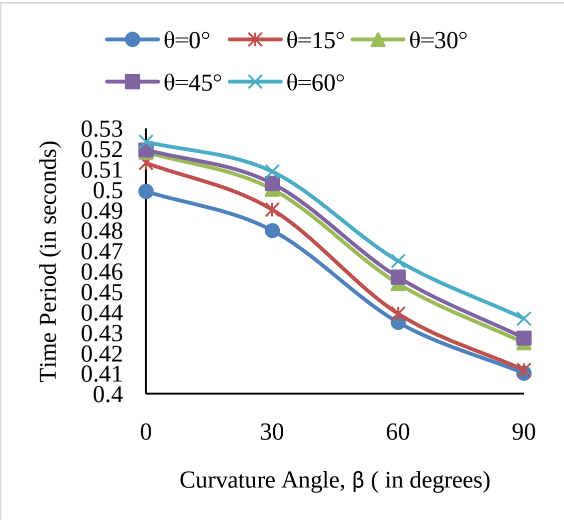
It has been observed that longitudinal modes cause the deck to move as a rigid body along the longitudinal direction with lateral substructure deformation. Variations of the time period and modal mass participating ratios of first longitudinal mode shape are shown in Figure 5 (b) and Figure 7 (b) respectively for different combinations of skew and curvature angle.

The transverse vibration modes are associated with the in-plane bending of the deck and lateral deformation of the substructure. Variations of the time period and modal mass participating ratios of first transverse mode shape are shown in Figure 5 (a) and Figure 7 (a) respectively for various bridge configurations.

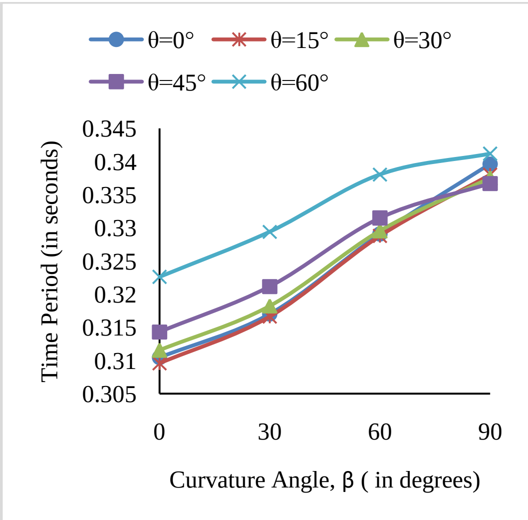
Vertical vibration modes generate out of plane bending in bridge deck and longitudinal deformation in substructure, its associated time period and modal mass participating ratios are shown in Figure 6 and Figure 8 respectively for all bridges.

4.2.1 Effect of Skewness and Curvature on Time Periods

From past studies, it has been noticed that skewness as well as curvature significantly affects the natural time period of a bridge. This study attempts to analyze the combined effect of curvature and skewness on a three span RC box-girder bridge. Time periods of horizontal and vertical vibrational modes for all bridge configuration have been presented in Figure 5 and 6 respectively, which shows highest time period for in-plane vibration mode, closely followed by longitudinal and out of plane vibration modes irrespective of curvature and skewness present in the bridge.



a. Variations in time periods of 1st In-Plane Mode for various bridge configurations



b. Variations in time periods of 1st Longitudinal Mode for various bridge

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configurations

Figure 5. Time Periods of the Horizontal Vibrational Modes

It is evident from Figure 5 that time period of the first in-plane vibration mode is 0.499seconds for straight bridge configuration. In general, the time period of first in-plane mode decreases with increase in curvature angle because of decrease in the flexibility of the bridge-deck for in-plane bending due to arching action which increases with increase in curvature. For bridges having no skewness, the maximum percentage difference of -17.84% (decrease) has been observed with increase in curvature angle upto 90°. However, with increase in skew angle time period for first in-plane mode generally increases as shown in Figure 5 (a). Maximum percentage difference of 1.2% (increase) has been noticed with increase in skew angle upto 60° for bridges having no curvature. Hence, the least time period is noticed for the bridge with curvature angle of 90° and skew angle of 0°. In the case of skew-curve bridges, the time period has been noticed to decrease with increase in deck-curvature for a particular skew angle. Thus, it can be said that induction of skewness in curved bridges will help them to increase their time period.

Time period of first longitudinal vibration mode is 0.310seconds (Figure 5 (b)) for straight bridge configuration. Usually, time period of this mode has been noticed to increase with increase in skew and curvature angle. The maximum time period is 0.341seconds which has been observed for the bridge having curvature angle of 90° and skew angle of 60° and the difference of 9.9% has been observed between the time periods of this bridge and straight bridge. However, role of skewness diminishes in controlling the longitudinal time period for highly curved bridges.

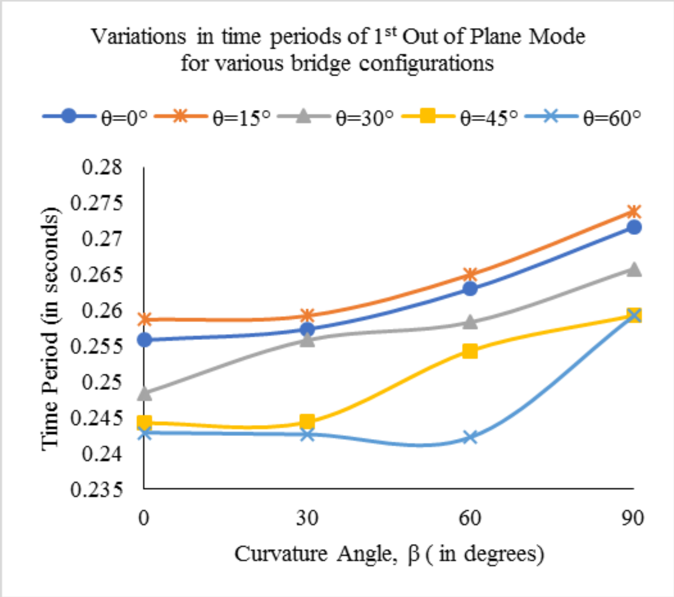

Figure 6. Time Periods of Vertical Vibration Modes

For the straight bridge configuration, the time period of first out of plane vibration mode has been observed as 0.255seconds (Figure 6) which is significantly lower than in-plane and longitudinal modes. For any particular skew angle, with increase in curvature angle a general pattern of increase in time period has been noticed whereas the time period decreases with increase in skew angle. Maximum observed value of time period is 0.273seconds which have been observed for bridge having curvature angle of 90° and skew angle of 15°. Generally, time period for out of plane vibration lags behind the first mode of in-plane and longitudinal vibrations, however for skew-curve bridges as geometry becomes complex, in few cases time period of second in-plane mode also preceded first out of plane mode.

4.2.2 Effect of Skewness and Curvature on Mass Participation Ratios

Mass participation ratio shows that how much amount of the total mass of the structure is participating in exciting the specific mode of the structure. This ratio can be calculated by squaring the participation factor of respective mode and

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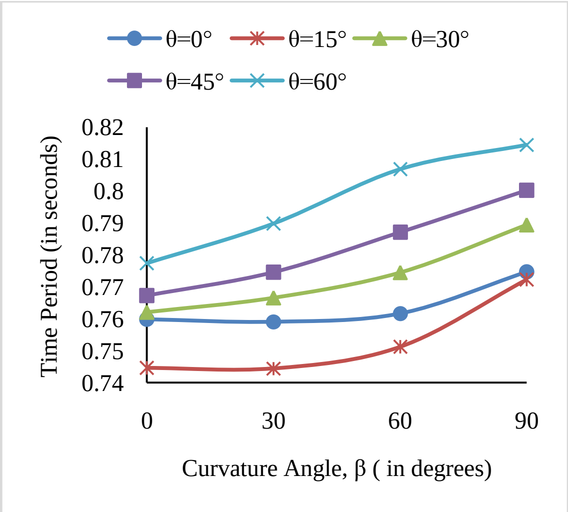
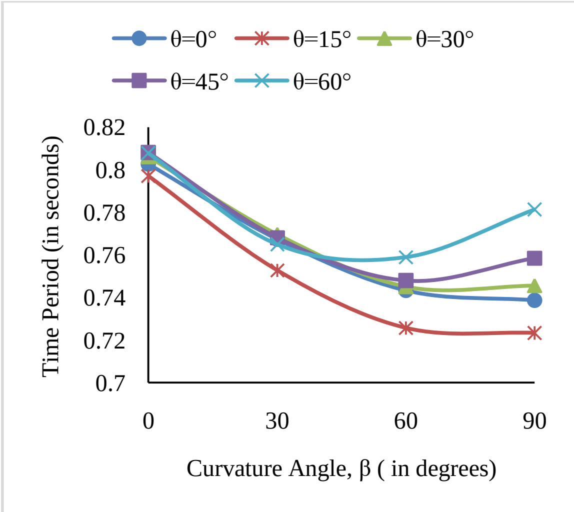
divide this value with the sum of joint masses of the structure [12]. Mass participation ratios of the first in-plane, first longitudinal and third out of plane vibration mode have been discussed in this section.

First and second out of plane vibration modes have very less values of mass participation factor which indicates that these modes are rarely participating in exciting the mass of the structure. Therefore, results of modal mass participation ratios of third out of plane vibration modes have been discussed instead of first and second out of plane vibration modes. Variations of the modal mass participation ratios with change in configuration of the bridges is shown in Figure 7 and 8.



a. Mass Participation Ratios of first In-Plane Mode for various bridge configurations



b. Mass Participation Ratios of first Longitudinal Mode for various bridge configurations

Figure 7. Mass Participation Ratios for Horizontal Vibration Modes

Mass excited by the first in-plane vibration mode is about 76% for straight bridge configuration which generally increases with increase in skew and curvature angle as shown in Figure 7 (a). Maximum contribution of in-plane vibration mode is 81% which have occurred for the bridge model with combination of curvature angle of 90° with 60° skew angle. Hence, with increase in skewness and curvature, the first in-plane mode will significantly affect to the mass of the structure, as compared to straight bridge. This indicates that in highly skew-curve bridges, higher modes of in-plane vibration mode do not significantly impact the effective mass.

In case of straight bridge, 80% of the mass is excited by the first longitudinal vibration mode (Figure 7(b)). Usually, the mass participation ratios have been noted to decrease with increase in curvature angle, whereas it increases with increase in skew angle for this vibration mode. This means that the contribution of first longitudinal mode in exciting the mass of the structure will decrease as the curvature increases. The least mass participation was observed for the bridge configuration with curvature angle of 90° and skew angle of 15°.

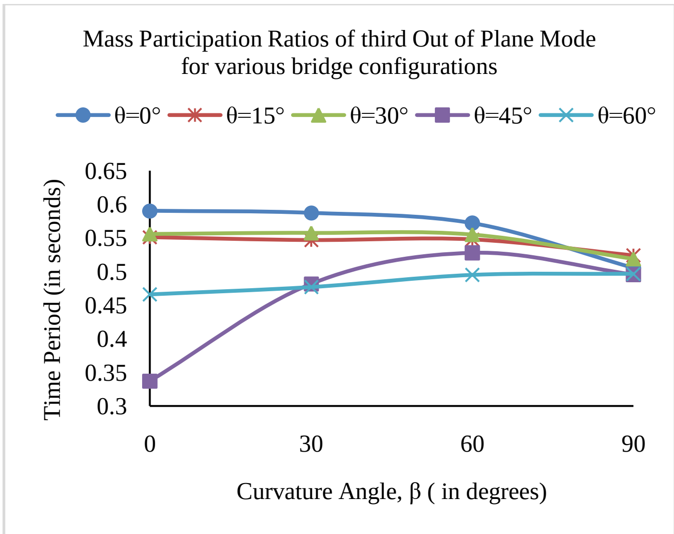


Figure 8 Mass Participation Ratios for Vertical Vibration Modes

First and second out of plane vibration modes have very less value of mass participation ratio which shows that these vibration modes have very less contribution in exciting the mass of the bridge structures. However, the third out of plane vibration mode has been found to participate significantly in exciting the mass of the bridge structure for straight bridge as well as for skew-curve bridge configurations. For straight bridge configuration, mass participation ratio of third out of plane vibration mode is 59%. However, for the combination of curvature and skew angle, the variations in mass participation ratios have not been observed in symmetric order. Moreover, in general wit increase in skew or curvature angle, the mass participation ratio of third out of plane vibration mode has been observed to decrease or have very less changes.

5. Conclusion

In this study, the modal analysis for the various skew-curve bridge configurations have been carried out using finite element analysis program CSiBridge. To facilitate the study of modal behaviour of skew-curve bridges, the changes have been made in the benchmark bridge which was a horizontally curved concrete box-girder bridge. For different curvature angles of 0°, 30°, 60° and 90°; the skew angle is varied in range of 0° to 60° at an interval of 15°. From the results of modal analysis, the following conclusions are drawn:

- Free vibration analysis indicates time period of horizontal vibration modes precedes time period of vertical vibration modes irrespective of skewness and curvature of the bridge.
- Increase in curvature of the bridge deck leads to decrease the time period of first horizontal (in-plane) vibration mode of the structure whereas increase in skew angle, increases the time period.
- For first longitudinal vibration mode, the time period usually increases with increase in both skewness and curvature which signifies the decrease in natural frequency of the bridge structures. Therefore, stiffness of the structure should be increased to increase the natural frequency of the structure.
- Increase in skew angle causes the time period to decrease usually in first out of plane vibration mode whereas increase in curvature angle causes increase in time period of first out of plane vibration mode.
- Mass participation ratio of first in-plane vibration mode commonly increases with increase in skewness and curvature which signifies that this mode will contribute more significantly than in straight bridge configuration to excite the mass of the structure
- Mass participation ratio of first longitudinal vibration mode decreases with the increase in deck-curvature whereas usually increases with increase in skew angle.
- Commonly, the mass participation ratio of third out of plane vibration mode have very less changes with increase in curvature angle for any constant skew angle.

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