Enrolment No.: ....

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION-2022

Ph.D.-I Semester (Mathematics)

COURSE CODE (CREDITS): 17P1WMA231 (3) MAX. MARKS: 25

COURSE NAME: ADVANCED LINEAR ALGEBRA

COURSE INSTRUCTOR: Pradeep Kumar Pandey MAX. TIME: 1 Hour 30 Min

Note: *All questions are compulsory. Marks are indicated against each question in square brackets. Mobile Phones, smart watches, calculators, and any other electronic gadgets etc. are prohibited during the Examination.*

Q1. Diagonalize the following matrix, if it is diagonalizable (in the otherwise case give reasons in support of your answer): [5] [CO-2]

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Also, write the algebraic and geometric multiplicities of all the eigenvalues of A.
(Hint: One eigenvalue of the matrix A is 3).

Q2. State the *Cauchy-Schwarz* inequality for the inner product spaces. Moreover, using the *Cauchy-Schwarz* inequality for the vectors $x = [a \;\; b]^t$ and $y = [1 \;\; 1]^t$ in the inner product space $\mathbb{R}^2$, show that $\left(\frac{a+b}{2}\right)^2 \leq \frac{a^2+b^2}{2}$. [3] [CO-3]

Q3. Consider the vector space $\mathbb{P}_3([-1,1])$ equipped with inner product defined by: [7] [CO-3]

$$\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)dx$$

Given the (ordered) basis $\{1, x, x^2, x^3\}$, use the Gram-Schmidt procedure to obtain an (ordered) orthogonal basis for $\mathbb{P}_3([-1,1])$.

Q4. Obtain the QR decomposition of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 0 & 2 & 0 \end{bmatrix}^t$. [5] [CO-4]

Q5. Test the consistency of the linear system $A\boldsymbol{x} = \boldsymbol{b}$, where $A = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}^t$, and $\boldsymbol{b} = [2 \quad 0 \quad 11]^t$, and obtain its least square solution. [5] [CO-4]

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