Jaypee University of Information Technology, Waknaghat

Test-3 Examinations - December 2022

B.Tech - III Semester (CSE/IT)

Course Code/Credits: 18B11MA313/3
Course Title: Probability and Statistics
Course Instructors: RAD, SST

Max. Marks: 35

Max. Time: 2 hours

Instructions: Answer all the questions. Define **random variables** along with range where applicable. Scientific calculators are allowed. Necessary statistical tables are supplied.

1. All athletes at the Olympic games are tested for performance-enhancing steroid/drug use. The imperfect test gives positive results (indicating drug use) for 90% of all "dirty" athletes who used steroids but also (and incorrectly) for 2% of "clean" athletes who do not use steroids. Suppose that 5% of all registered athletes use steroids. (4 Marks) [CO-1]

   (a) What is the probability that a randomly selected athlete is tested negative?

   (b) If an athlete is tested negative, what is the probability that he/she uses steroids?

2. Let **X** and **Y** be the number of hardware failures in two computer labs in a given month. The joint distribution of **X** and **Y** is given in the table below. (5 Marks) [CO-2]

y \ x	0	1	2
0	0.52	0.20	0.04
1	0.14	0.02	0.01
2	0.06	0.01	0.00

   (a) Compute the marginal distribution of **X**.

   (b) Compute the conditional distribution of **Y** given **X**=1.

   (c) Compute the probability of at least one hardware failure.

3. The speed of a file transfer from a server on campus to a personal computer at a student's home on a weekday evening is normally distributed with a mean of 60 kilobits per second and a standard deviation of 4 kilobits per second. Assume 1 megabyte = 1000 kilobyte. (4 Marks) [CO-3]

   (a) Find the probability that the file will transfer at a speed of less than 58 kilobits per second.

   (b) If the file is 4 megabyte, what is the average time it will take to transfer the file?

4. A computer manager needs to know how efficiency of her new computer program depends on the size of incoming data. Efficiency will be measured by the number of processed requests per hour. Applying the program to data sets of different sizes, she obtains the following: (6 Marks) [CO-4]

Data size (gigabytes) (x)	6	7	7	8	10	10	15
Processed requests (y)	40	55	50	41	17	26	16

   (a) Find the regression line of $\bar{y}$ on x.

   (b) What would be the number of processed requests for a data size of 16 gigabytes?

5. Consider equations of two *regression* lines: $2x + 3y = 6$ and $3x + 2y = 12$. (5 Marks) [CO-4]

   (a) Determine the *correlation coefficient* r_{xy}.

   (b) Find $\frac{\sigma_y}{\sigma_x}$.

6. A forester wants to compare two different mist blowers (Type A and Type B) for consistent application. He wants to use the mist blower with the smaller variance, which means more consistent application. He wants to test that the variance of Type A (0.087 gal.2) is significantly greater than the variance of Type B (0.073 gal.2). (5 Marks) [CO-5]

Type A	Type B
$s_1^2 = 0.087$	$s_2^2 = 0.073$
$n_1 = 16$	$n_2 = 21$

(a) State the null and alternative hypotheses.

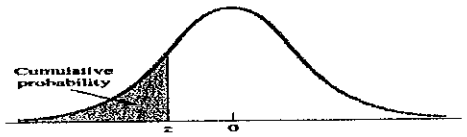
(b) At 5% significance level, does there exist enough evidence to support the claim that the variance of Type A is significantly greater than the variance of Type B?

7. The number of times a machine broke down each week was observed over a period of 100 weeks and recorded in the accompanying table. It was found that the average number of breakdowns per week over this period was 2.4. (6 Marks) [CO-5]

Number of breakdowns	0	1	2	3	4	5 or more
Number of weeks	9	23	34	23	3	8

Test the null hypothesis that the population distribution of breakdowns is Poisson. Use $\alpha = 0.10$.

(Standard) Normal Probability Table:



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

χ^2 Probability Table:

Right tail

Degrees of freedom	α									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801