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JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATION- December 2022

B.Tech BI/BT V Semester

COURSE CODE: 18B11BI511 MAX. MARKS: 35

COURSE NAME: DESIGN AND ANALYSIS OF ALGORITHM

COURSE CREDITS: 03 MAX. TIME: 2 Hr

Note: All questions are compulsory. Carrying of mobile phone during examinations will be treated as case of unfair means.

1. The following is a divide-and-conquer algorithm for finding the maximum value in an array $S[1..n]$. The main body of the algorithm consists of a call to maximum $(1, n)$.

function maximum (x, y)
comment return maximum in $S[x..y]$

1 if $y - x \leq 1$ then return $(\max(S[x], S[y]))$
2 else
3 max1: = maximum $(x, \lfloor (x + y)/2 \rfloor)$
4 max2: = maximum $(\lfloor (x + y)/2 \rfloor + 1, y)$
5 return $(\max(\text{max1}, \text{max2}))$

(a) Prove that algorithm is correct. You may assume that n is a power of 2 .
(b) Write down a recurrence relation for the worst-case number of comparisons used by maximum (1,n). Solve this recurrence relation. You may assume that n is a power of 2 .
(c) What is the running time of maximum (1,n) ? Explain your answer. **(Marks:7) CO-2**

2. (a) Consider a graph G = (V, E), where V = $\{v_1, v_2, ..., v_{100}\}$, E = $\{(v_i, v_j) \mid 1 \leq i < j \leq 100\}$, and weight of the edge (v_i, v_j) is |i - j|. Calculate the weight of the minimum spanning tree of G.

(b) Let G = (V, E) be a weighted undirected graph and let T be a Minimum Spanning Tree (MST) of G maintained using adjacency lists. Suppose a new weighted edge (u, v) ∈ V × V is added to G. Calculate the worst case time complexity of determining if T is still an MST of the resultant graph.

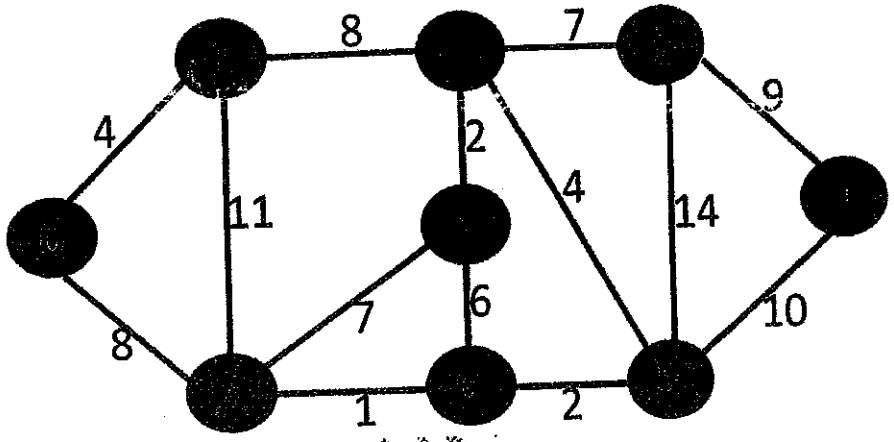
(Marks:7) CO-3

3. Assume that multiplying a matrix G_1 of dimension $p \times q$ with another matrix G_2 of dimension $q \times r$ requires pqr scalar multiplications. Computing the product of n matrices $G_1 G_2 G_3 .. G_n$ can be done by parenthesizing in different ways. Define $G_i G_{i+1}$ as

an explicitly computed pair for a given paranthesization if they are directly multiplied. For example, in the matrix multiplication chain $G_1G_2G_3G_4G_5G_6$ using parenthesization $(G_1(G_2G_3))(G_4(G_5G_6)), G_2G_3$ and G_5G_6 are the only explicitly computed pairs. Consider a matrix multiplication chain $F_1F_2F_3F_4F_5$, where matrices F_1, F_2, F_3, F_4 and F_5 are of dimensions $2 \times 25, 25 \times 3, 3 \times 16, 16 \times 1$ and 1×1000, respectively. In the parenthesization of $F_1F_2F_3F_4F_5$ that minimizes the total number of scalar multiplications, identify the explicitly computed pairs.

(Marks:7) CO-4

4. What is Dijkstra algorithm? Given a graph and a source vertex in the graph, find the **shortest paths** from the source to all vertices in the given graph.



(Marks:7) CO-4

5. Consider the following instance of knapsack problem:

Item	X1	X2	X3	X4	X5
Profit	15	12	9	16	17
Weight	2	5	3	4	6

The maximum weight of 12 is allowed in the knapsack. Find the value of maximum profit with the optimal solution of the fractional knapsack problem.

(Marks:7) CO-5