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JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST - 2 EXAMINATION- October 2019

B.Tech. (CSE&IT) VII Semester

COURSE CODE: 10B1WCI733 MAX. MARKS: 25

COURSE NAME: Graph Algorithms and Applications

COURSE CREDITS: 3 MAX. TIME: 1Hr 30 Min

***Note:*** *All questions are compulsory.*

1. [5 Marks]
a. Prove or disprove: Every tree T has at most one perfect matching.
b. Prove or disprove: Prove that every maximal matching in a graph has at least $\alpha'(G)/2$ edges.

2. [5 Marks]
a. Two people play a game on graph G, alternately choosing distinct vertices. Player 1 starts by choosing any vertex. Each subsequent choice must be adjacent to the preceding choice (of the other player). Thus together they follow a path. The last player able to move wins.
Prove that the second player has a winning strategy if G has a perfect matching, and otherwise the first player has a winning strategy.
b. Find a way to place five queens on an eight-by-eight chessboard that attack all other squares.

3. [5 Marks]
a. Show how to use the Hungarian Algorithm to test for the existence of a perfect matching in a bipartite graph.
b. Determine $\kappa(G)$, $\kappa'(G)$, and $\delta(G)$ for graph shown in Figure A.

4. [5 Marks]
a. Draw the complement and line graph for the graph G shown in Figure B.
b. Prove that in a tree every vertex of degree greater than one is a cut-vertex.

5. [5 Marks]
Prove or disprove: Every circuit has an even number of edges in common with any cut-set.
b. A kitchen sink draws water from two tanks according to the network of pipes with capacities per unit time shown below (Figure C). Find the maximum flow. Prove that your answer is optimal by using the dual problem, and explain why this proves optimality.

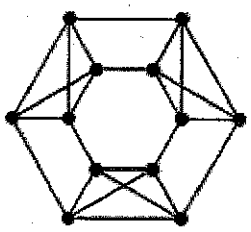

Figure A

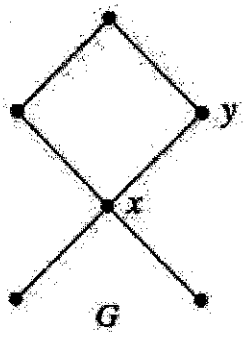


Figure B

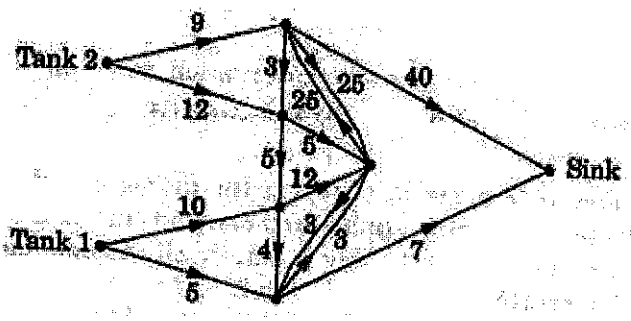


Figure C