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Multi-criteria Decision Making with Triangular Intuitionistic Fuzzy Number based on Distance Measure & Parametric Entropy Approach

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Abstract

The present communication aims to study the Triangular Intuitionistic Fuzzy Multiple Criteria Decision Making (TIF-MCDM) problem for finding the best option where the phonetic factors for the criteria are pre-characterized. In perspective of this, a new parametric entropy under α -cut/ (α, β) - cut based distance measures has been proposed and implemented for various conceivable estimations of parameters. A survey structure based on a questionnaire for the purchase of a car has also been studied and devised the ranking procedure for the evaluation criteria. Further, a ranking algorithm for TIF-MCDM issue for the accessible options by processing the various distances between the perfect option and all the accessible options has been proposed. An illustrative example to rank the alternatives in view of different opinions has also been included.

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Keywords: Triangular Intuitionistic Fuzzy Number; Multiple Criteria Decision Making; Ranking; Closeness Coefficient; Distance Measures.

1. Introduction

Keeping in mind the end goal to pick a thing, the piece of human practices affected by some interrelating components is a critical factor, in a customer basic decision making process. The outside attributes, for example, price, brand, capability and so forth are additionally emphasized in settling on a decision. Fuzzy set hypothesis, created by Lotfi A. Zadeh [1] has capacity to portray the dubious circumstances, uncertainty, inaccuracy, ambiguity and perception. As an augmentation of the fuzzy set, the idea of intuitionistic Fuzzy Set (IFS), was presented by Atanassov [2, 3] in 1986, can express and process of uncertainty much better than fuzzy sets because of having an additional hesitation parameter inherited. Intuitionistic Fuzzy Set is portrayed by two functions- the degree of membership function (be-

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longingness or enrollment)/degree of non membership function (nonbelongingness or non-enrollment) of an element of the sets along with hesitation margin and is observed to be more useful in catching the ambiguous, deficient or indeterminate data.

2. Literature Survey and Basic Definitions

Gau and Buehrer [4] in 1993, presented the idea of vague set. Grattan-Guinness, K.U. Jahn and R. Sambuc [5, 6, 7] presented the theory of interval valued fuzzy set, which is well known generalization of ordinary fuzzy set. Among various extensions of fuzzy sets such as IFS, Vague Set, Interval-valued Fuzzy Set, Triangular Intuitionistic fuzzy set etc. , IFS are found to be more consistent with human behavior. The theory of uncertainty measure makes IFS useful in many scientific disciplines, such as knowledge discovery, analyzing data, data mining, pattern recognition, logic programming, image segmentation, medical diagnosis, image edge detection and modeling various real life activities like evaluation, negotiation and multi criteria decision making.

The idea of multiple criteria decision making (MCDM) provides a systematic quantitative approach for decision making problems and includes a gathering of decision makers, surveying multiple criteria and action where judgement of human beings plays a major role. In the course of recent decades, numerous MCDM techniques have been produced to fathom, numerous real life decision circumstances in the field of open organization, administration science, operation examine, designing, society, financial matters, military research and proficient journals [8, 9, 10]. During the MCDM process, decision makers usually use qualitative measure (market reputation, relationship closeness etc.) and/or quantitative measure (economical) for the assessment and choice of most suitable alternative with respect to each criterion and consider the relative significance of every criterion with regard to overall goal. The priority from a decision maker for a particular criterion has been expressed through phonetic judgment, for example, - ‘excellent’, ‘good’, ‘average’, ‘poor’ and ‘extremely poor’. The idea of Linguistic factors is exceptionally valuable in managing practical circumstance which are excessively mind boggling or not all around characterized, to be reasonably described in conventional quantitative expressions. Shu and Cheng [11] described Triangular Intuitionistic Fuzzy Numbers (TIFNs) which have a more noticeable ability to manage more adequate and versatile information than triangular fuzzy numbers.

Next, we present some of the basics IFSs and TIFNs in reference with some arithmetic operations and distance measures, which are well known in literature.

Definition 2.1. *Atanassov’s [2, 3] intuitionistic fuzzy set (IFS) over a finite non empty fixed set X , is a set $\tilde{A} = \{ \langle x, \mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x) \rangle \mid x \in X \}$ which assigns to each element $x \in X$ to the set \tilde{A} , which is a subset of X having the degree of membership $\mu_{\tilde{A}}(x) : X \rightarrow [0, 1]$ and degree of non-membership $\gamma_{\tilde{A}}(x) : X \rightarrow [0, 1]$, satisfying $0 \leq \mu_{\tilde{A}}(x) + \gamma_{\tilde{A}}(x) \leq 1$, for all $x \in X$ ”. “For each IFS in X , a hesitation margin $\pi_{\tilde{A}}(x)$, which is the intuitionistic fuzzy index of element x in the IFS \tilde{A} , defined by $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \gamma_{\tilde{A}}(x)$, denotes a measure of non-determinancy. We denote $IFS(X)$, the set of all the IFSs on X .*

Definition 2.2. *Triangular intuitionistic fuzzy number $\tilde{\chi} = \langle (t, t, \bar{t}); u_{\tilde{\chi}}, w_{\tilde{\chi}} \rangle$ (TIFN) is a special intuitionistic fuzzy set, whose membership function and non-membership function are defined by Atanassov’s [12] as follows:*

$$\mu_{\tilde{\chi}}(x) = \begin{cases} u_{\tilde{\chi}}(x - \underline{t}) / (t - \underline{t}) & \text{if } \underline{t} \leq x < t \\ u_{\tilde{\chi}} & \text{if } x = t \\ u_{\tilde{\chi}}(\bar{t} - x) / (\bar{t} - t) & \text{if } t < x \leq \bar{t} \\ 0 & \text{if } x < \underline{t} \text{ or } x > \bar{t} \end{cases}$$

and

$$\nu_{\tilde{\chi}}(x) = \begin{cases} [t - x + w_{\tilde{\chi}}(x - \underline{t})] / (t - \underline{t}) & \text{if } \underline{t} \leq x < t \\ w_{\tilde{\chi}} & \text{if } x = t \\ [x - t + w_{\tilde{\chi}}(\bar{t} - x)] / (\bar{t} - t) & \text{if } t < x \leq \bar{t} \\ 1 & \text{if } x < \underline{t} \text{ or } x > \bar{t} \end{cases}$$

respectively, where the values $u_{\tilde{\chi}}$ and $w_{\tilde{\chi}}$ represent the maximum degree of membership and the minimum degree of non-membership, respectively, such that they satisfy $0 \leq u_{\tilde{\chi}} \leq 1$, $0 \leq w_{\tilde{\chi}} \leq 1$, $0 \leq u_{\tilde{\chi}} + w_{\tilde{\chi}} \leq 1$.

Let $\pi_{\tilde{\chi}}(x) = 1 - \mu_{\tilde{\chi}}(x) - \nu_{\tilde{\chi}}(x)$, which is called as intuitionistic fuzzy index of an element x in $\tilde{\chi}$. It is the degree of indeterminacy membership of the element x in $\tilde{\chi}$. The TIFN $\tilde{\chi} = \langle (t, t, \bar{t}); u_{\tilde{\chi}}, w_{\tilde{\chi}} \rangle$ is called as a positive TIFN,

denoted by $\tilde{\chi} > 0$, if $t \geq 0$ and one of the three values \underline{t} , t and \bar{t} is not equal to zero. Similarly, if $\bar{t} \leq 0$ and one of the three values \underline{t} , t and \bar{t} is not equal to zero, then the TIFN $\tilde{\chi} = \langle (\underline{t}, t, \bar{t}); u_{\tilde{\chi}}, w_{\tilde{\chi}} \rangle$ is called as a negative TIFN.

Definition 2.3. Let $\tilde{\chi} = \langle (\underline{t}, t, \bar{t}); u_{\tilde{\chi}}, w_{\tilde{\chi}} \rangle$ and $\tilde{\lambda} = \langle (\underline{s}, s, \bar{s}); u_{\tilde{\lambda}}, w_{\tilde{\lambda}} \rangle$ be two TIFNs and δ is a real number. Some arithmetical operations (addition, multiplication etc.) are well defined in literature. [Ref. Wang and Zhang [13]]

Definition 2.4. (i) Triangular intuitionistic fuzzy positive ideal solution is $\tilde{I}^+ = \langle (\underline{t}^+, t^+, \bar{t}^+); u^+, w^+ \rangle = \langle (1, 1, 1); 1, 0 \rangle$. (ii) Triangular intuitionistic fuzzy negative ideal solution is $\tilde{I}^- = \langle (\underline{t}^-, t^-, \bar{t}^-); u^-, w^- \rangle = \langle (0, 0, 0); 0, 1 \rangle$.

Definition 2.5. An (α, β) -cut set of $\tilde{\chi} = \langle (\underline{t}, t, \bar{t}); u_{\tilde{\chi}}, w_{\tilde{\chi}} \rangle$ is a crisp subset of \mathbb{R} , which is defined in literature (Ref. [14, 15]) as $\tilde{\chi}_\beta^\alpha = \{x | \mu_{\tilde{\chi}}(x) \geq \alpha, \nu_{\tilde{\chi}}(x) \leq \beta\}$; where $0 \leq \alpha \leq u_{\tilde{\chi}}, w_{\tilde{\chi}} \leq \beta \leq 1$ and $0 \leq \alpha + \beta \leq 1$.

Definition 2.6. A α -cut set of $\tilde{\chi} = \langle (\underline{t}, t, \bar{t}); u_{\tilde{\chi}}, w_{\tilde{\chi}} \rangle$ is a crisp subset of \mathbb{R} , which is defined as $\tilde{\chi}^\alpha = \{x | \mu_{\tilde{\chi}}(x) \geq \alpha\}$.

From definitions 2.2 and 2.6, it follows that $\tilde{\chi}^\alpha$ is a closed interval, denoted by $\tilde{\chi}^\alpha = [L^\alpha(\tilde{\chi}), R^\alpha(\tilde{\chi})]$, which can be calculated as $[L^\alpha(\tilde{\chi}), R^\alpha(\tilde{\chi})] = \left[\underline{t} + \frac{\alpha(t-\underline{t})}{u_{\tilde{\chi}}}, \bar{t} - \frac{\alpha(\bar{t}-t)}{w_{\tilde{\chi}}} \right]$.

Definition 2.7. A β -cut set of $\tilde{\chi} = \langle (\underline{t}, t, \bar{t}); u_{\tilde{\chi}}, w_{\tilde{\chi}} \rangle$ is a crisp subset of \mathbb{R} , which is defined as $\tilde{\chi}_\beta = \{x | \nu_{\tilde{\chi}}(x) \leq \beta\}$.

Using definitions 2.2 and 2.7, it follows that $\tilde{\chi}_\beta$ is a closed interval, denoted by $\tilde{\chi}_\beta = [L_\beta(\tilde{\chi}), R_\beta(\tilde{\chi})]$, which can be calculated as $[L_\beta(\tilde{\chi}), R_\beta(\tilde{\chi})] = \left[\frac{[(1-\beta)t + (\beta-w_{\tilde{\chi}})\underline{t}]}{1-w_{\tilde{\chi}}}, \frac{[(1-\beta)t + (\beta-w_{\tilde{\chi}})\bar{t}]}{1-w_{\tilde{\chi}}} \right]$.

Definition 2.8. Let $\tilde{\chi} = \langle \tilde{t}; u_{\tilde{\chi}}, w_{\tilde{\chi}} \rangle$ and $\tilde{\lambda} = \langle \tilde{s}; u_{\tilde{\lambda}}, w_{\tilde{\lambda}} \rangle$ be two arbitrary triangular intuitionistic fuzzy numbers where \tilde{t} and \tilde{s} are two triangular fuzzy numbers with α -cut representations, $\tilde{t}_\alpha = [t^L(\alpha), t^R(\alpha)]$ and $\tilde{s}_\alpha = [s^L(\alpha), s^R(\alpha)]$. The distance between $\tilde{\chi}$ and $\tilde{\lambda}$ is defined by Li and Chen [16] is as follows:

$$d(\tilde{\chi}, \tilde{\lambda}) = \sqrt{\int_0^1 [(t^L(\alpha) - s^L(\alpha))^2 + (t^R(\alpha) - s^R(\alpha))^2] d\alpha} + \sqrt{\frac{1}{2} [(u_{\tilde{\chi}} - u_{\tilde{\lambda}})^2 + (w_{\tilde{\chi}} - w_{\tilde{\lambda}})^2 + (u_{\tilde{\chi}} + w_{\tilde{\chi}} - u_{\tilde{\lambda}} - w_{\tilde{\lambda}})^2]}$$

Considering the decision maker’s subjective assessments to the various criteria for the positioning of the accessible alternatives, the importance/weight of every standard has been ascertained with the assistance of parametric entropy under α -cut/ (α, β) - cut based distance measures for various conceivable estimations of parameters. The same has been shown in section 3. In section 4, a survey structure based on a questionnaire for the purchase of a car has been presented with the ranking of the evaluation criteria. In section 5, a ranking algorithm for Triangular Intuitionistic Fuzzy Multi-criteria Decision Making (TIF-MCDM) problem has been proposed. An illustrative example to rank the alternatives in view of different opinions has additionally been given in section 6.

3. Evaluating Weights of Criteria

In this section, we reveal the technique to use the entropy system for surveying weights of attributes with triangular intuitionistic fuzzy numbers. We have consider a multiple criteria decision making problem where a discrete arrangement of m conceivable alternatives $AL = \{AL_1, AL_2, \dots, AL_m\}$, which depends on a discrete arrangement of n assessment criterions $CR = \{CR_1, CR_2, \dots, CR_n\}$. We can represent the decision matrix of triangular intuitionistic fuzzy multiple criteria as $\tilde{D} = [\tilde{s}_{ij}]_{m \times n} = \{[f_{ij}, f_{ij}, \bar{f}_{ij}]; u_{ij}, w_{ij}\}_{m \times n}$; where \tilde{s}_{ij} is the rating of the i^{th} alternative ($i = 1, 2, \dots, m$) meeting the j^{th} criteria ($j = 1, 2, \dots, n$) which is together given by decision makers. It may be seen that weight measure is a basic idea in various multiple criteria decision making issues and have a quick relationship with the distance measure between two fuzzy numbers. To oversee choice data information with triangular intuitionistic fuzzy numbers, we use the distance between triangular intuitionistic fuzzy numbers as given in definitions 2.8.

Let \tilde{W}_j speaks to the weight vector of j^{th} criteria, where the weights of the criteria have been given by the decision maker’s subjective opinion. For the purpose of plan of the qualitative assessment, we instinctively characterize the accompanying Table 1:

Table 1: Linguistic Variables in terms of TIFNs

Sr. No.	Linguistic Variables	TIFNs
1	Extremely Poor (EP)	$\langle (0.2308, 0.3, 0.4286); 0.8, 0.1 \rangle$
2	Poor (PO)	$\langle (0.3, 0.4286, 0.75); 0.8, 0.1 \rangle$
3	Average (AV)	$\langle (0.55, 0.7, 0.85); 0.6443, 0.252 \rangle$
4	Good (GO)	$\langle (0.7, 0.8667, 0.9667); 0.7846, 0.1587 \rangle$
5	Excellent (EX)	$\langle (0.8, 1, 1); 0.8413, 0.126 \rangle$

In the event, there are g individuals in a basic decision making committee, who subjectively describe the weights of the n criteria, at that point the successful weight of each criteria in the from of triangular intuitionistic fuzzy number can be evaluated as $\tilde{W}_j = \frac{1}{g} (\tilde{W}_j^1 + \tilde{W}_j^2 + \dots + \tilde{W}_j^g)$.

If $d(\tilde{W}_j, \tilde{I}^+)$ is distance between the weights \tilde{W}_j (TIFN) and the triangular intuitionistic fuzzy ideal solution \tilde{I}^+ , then the distance vector is given by $V = [d(\tilde{W}_1, \tilde{I}^+), d(\tilde{W}_2, \tilde{I}^+), \dots, d(\tilde{W}_n, \tilde{I}^+)]$.

Further, the normalized distance vector V' is given by $V' = [\varepsilon_j] = \left[\frac{d(\tilde{W}_j, \tilde{I}^+)}{\max d(\tilde{W}_j, \tilde{I}^+); j=1,2,3,\dots,n} \right], j = 1, 2, \dots, n$.

For a discrete random variable with probability distribution $P = (p_1, p_2, \dots, p_n)$ related with an experiment, Renyi [17] characterized the parametric probabilistic entropy measure as

$$e^\Omega = \frac{1}{1 - \Omega} \log \sum_{k=1}^n (p_k)^\Omega; \quad 0 < \Omega < 1.$$

The entropy measure of the j^{th} criteria (C_j) can be obtained from Renyi’s [17] entropy in an analogous way as follows:

$$e_j^\Omega = \frac{1}{1 - \Omega} \log \sum_{k=j} \left(\frac{\varepsilon_k}{\sum_{k=1}^n \varepsilon_k} \right)^\Omega$$

At long last, the crisp value of weight for j^{th} criterion in perspective of positive perfect arrangement (\tilde{I}^+), which relies upon the above entropy measure, can be figured as:

$$W_j^+ = \frac{1 - e_j^\Omega}{n - \sum_{k=1}^n e_k^\Omega}; \quad j = 1, 2, \dots, n.$$

Correspondingly, the crisp value of the weight for j^{th} criterion in context of negative perfect arrangement (\tilde{I}^-) can be ascertained. From there on, both the figured weights are being utilized as a part of the proposed algorithm in section 5. From that point onwards, both the figured weights are being used as a piece of the proposed calculation.

4. Survey Structure

In order to apply the methodology of evaluating the weights of different criteria in a multi-criteria decision making shown in section 3, a small survey has been conducted among a certain domain of intellectual peoples. The survey comprises of a short priority sheet for finding the ranking of various evaluation criteria under different category while purchasing a car. The priority from a customer (decision maker) for a particular criterion has been taken in terms of linguistic variables -Excellent(EX), Good(GO), Average(AV), Poor(PO) and Extremely Poor(EP) are used in this research paper to determine the satisfaction level of the customer. For a particular criterion, the customers have been asked to indicate the degree of priority level on discrete scale of 1(EP) to 5 (EX). The broad categories of the priority sheet have been chosen to be performance, style, comfort, safety, specifications and after sale services. Further, in the priority sheet, these categories have been sub-divided into twenty different evaluation criteria from top to bottom in the framework presented in Fig 1.

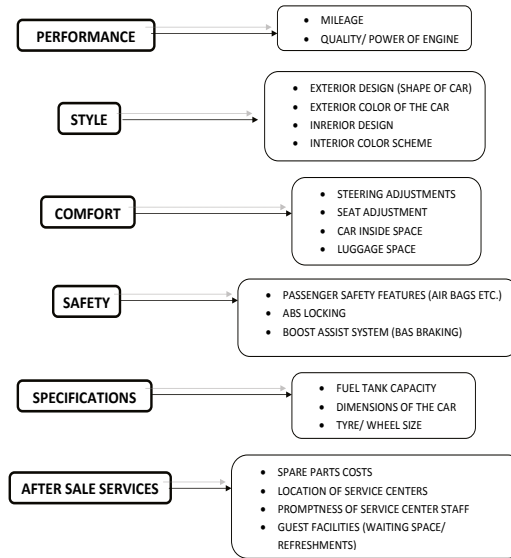


Fig. 1: Framework of Evaluation Criteria

On compiling all the data obtained through the brief survey conducted and applying the methodology of evaluating the weights of criteria as discussed in section 3, we display the accompanying table demonstrating the positioning of the assessment criteria:

Table 2: Ranking of the Evaluation Criteria

Evaluation Criteria	Weights ($\Omega = 0.1$)	Ranking	Weights ($\Omega = 0.5$)	Ranking
C ₁	0.05012	6	0.05035	6
C ₂	0.05019	4	0.05058	4
C ₃	0.05022	2	0.05065	2
C ₄	0.04977	18	0.04931	18
C ₅	0.05016	5	0.05048	5
C ₆	0.04985	16	0.04953	16
C ₇	0.04997	11	0.04993	11
C ₈	0.04995	13	0.04985	13
C ₉	0.05008	9	0.05023	9
C ₁₀	0.05011	7	0.05033	7
C ₁₁	0.05011	8	0.05033	8
C ₁₂	0.04986	14	0.04959	14
C ₁₃	0.04975	19	0.04926	19
C ₁₄	0.04984	17	0.04952	17
C ₁₅	0.05002	10	0.05006	10
C ₁₆	0.04986	15	0.04959	15
C ₁₇	0.04996	12	0.04989	12
C ₁₈	0.05044	1	0.05133	1
C ₁₉	0.05021	3	0.05062	3
C ₂₀	0.04952	20	0.04856	20

From Table 2, we observe that the criterion ‘location of service centers’ scores the highest rank and the criterion ‘interior color scheme’ was ranked as the least.

5. Ranking Algorithm for Triangular Intuitionistic Fuzzy MCDM

The procedure of ranking for a discrete game plan of m conceivable alternatives stuck on a course of action of n evaluation criteria if there should be an occurrence of triangular intuitionistic fuzzy multiple criteria decision making (TIF-MCDM) problem is given underneath:

Input An arrangement of m possible alternatives $AL = \{AL_1, AL_2, \dots, AL_m\}$, a discrete arrangement of n evaluation criterions $CR = \{CR_1, CR_2, \dots, CR_n\}$ and processed weights of criteria stuck on subjective opinions of leaders/decision makers.

Step 1 : In the event that there are g people in a basic decision making committee, then develop the decision matrix DM by computing the rating of every alternative meeting the criteria as $\tilde{s}_{ij} = \frac{1}{g} (\tilde{s}_{ij}^1 + \tilde{s}_{ij}^2 + \dots + \tilde{s}_{ij}^g)$.

Step 2: As the information about the weights of attributes is obscure, we find the attributes weights utilizing the entropy strategy as analyzed in section 3.

Step 3: Make utilization of definition 2.8 and the acquired weight vector to compute the distances $d(AL_i, \tilde{I}^+)$ and $d(AL_i, \tilde{I}^-)$ for every i as under: $d(AL_i, \tilde{I}^+) = \sum_{j=1}^n W_j^+ d(\tilde{I}^+, \tilde{s}_{ij})$ and $d(AL_i, \tilde{I}^-) = \sum_{j=1}^n W_j^- d(\tilde{I}^-, \tilde{s}_{ij})$.

Step 4: Ascertain the closeness coefficient, CC_i ($i = 1, 2, \dots, m$) of all alternatives and rank all alternatives, as indicated by the closeness coefficient as $CC_i = \frac{d(AL_i, \tilde{I}^-)}{d(AL_i, \tilde{I}^+) + d(AL_i, \tilde{I}^-)}$.

Step 5: Last and final step is to rank the alternatives. The basic thought of positioning the alternatives used is - higher the estimation of CC_i better the execution.

6. Illustrative Example

The relevance, pertinence and ampleness of triangular intuitionistic fuzzy multiple criteria decision making (TIF-MCDM) model is addressed by a numerical layout. Consider an instance of an association whose objective is to recruit a Director (DIR). Suppose there are three short-listed candidates DIR_1, DIR_2 and DIR_3 after preliminary screening. A gathering of three chiefs is surrounded, which will survey the three rivals in perspective of the criteria including stability (CR_1), economical (CR_2), working (CR_3), identity (CR_4) and self-assurance (CR_5). Leaders/decision makers utilize the phonetic factors, for instance, extremely poor, poor, average, good, excellent to depict the weights of criteria and rating of alternatives subjectively. Weights of criteria as far as phonetic factors have been given in Table 3.

Table 3: Linguistic Variables for Weight of Criteria

Criteria/Decisions	D_1	D_2	D_3
CR_1	EX	AV	AV
CR_2	GO	PO	GO
CR_3	EX	GO	AV
CR_4	GO	GO	EX
CR_5	AV	EP	EP

The rating of the decisions as for different paradigm as given by the leaders have been dealt with in Table 4.

Table 4: Rating of Alternatives by Decision Makers in Different Criterion

Criteria	Alternative	D_1	D_2	D_3
CR_1	DIR_1	GO	EX	EX
	DIR_2	AV	AV	GO
	DIR_3	GO	PO	AV
CR_2	DIR_1	PO	GO	GO
	DIR_2	GO	GO	AV
	DIR_3	EX	AV	EX
CR_3	DIR_1	GO	AV	GO
	DIR_2	AV	GO	GO
	DIR_3	AV	AV	GO
CR_4	DIR_1	GO	AV	AV
	DIR_2	EX	GO	EX
	DIR_3	AV	GO	AV
CR_5	DIR_1	PO	AV	PO
	DIR_2	GO	AV	GO
	DIR_3	GO	EX	AV

With a particular ultimate objective to deal with the issue, we at first assess the weights of each paradigm with the help of pre-portrayed etymological factors as TIFNs and organize them in the accompanying Table 5.

Table 5: Linguistic Variables equivalent to Summated Weight (TIFN)

Criteria	Weight
CR ₁	< (0.63, 0.80, 0.90); 0.327, 0.003 >
CR ₂	< (0.57, 0.72, 0.89); 0.330, 0.001 >
CR ₃	< (0.68, 0.86, 0.94); 0.329, 0.002 >
CR ₄	< (0.73, 0.91, 0.98); 0.331, 0.001 >
CR ₅	< (0.34, 0.43, 0.57); 0.329, 0.001 >

Based on the normalized decision matrix given in Table 6, the criteria weights can be calculated by using the entropy method with TIFNs:

Table 6: Decision Matrix of Triangular Intuitionistic Fuzzy Numbers

	DIR ₁	DIR ₂	DIR ₃
CR ₁	< (0.77, 0.96, 0.99); 0.33, 0.001 >	< (0.60, 0.76, 0.89); 0.32, 0.003 >	< (0.52, 0.67, 0.86); 0.32, 0.001 >
CR ₂	< (0.57, 0.72, 0.89); 0.33, 0.001 >	< (0.65, 0.81, 0.93); 0.33, 0.002 >	< (0.72, 0.90, 0.95); 0.33, 0.001 >
CR ₃	< (0.65, 0.81, 0.93); 0.33, 0.002 >	< (0.65, 0.81, 0.93); 0.33, 0.002 >	< (0.60, 0.76, 0.89); 0.32, 0.003 >
CR ₄	< (0.60, 0.76, 0.89); 0.32, 0.003 >	< (0.77, 0.96, 0.99); 0.33, 0.001 >	< (0.60, 0.76, 0.89); 0.32, 0.003 >
CR ₅	< (0.38, 0.52, 0.78); 0.32, 0.0001 >	< (0.65, 0.81, 0.93); 0.33, 0.002 >	< (0.68, 0.86, 0.94); 0.32, 0.002 >

The estimated weights with Triangular Intuitionistic Fuzzy positive & negative ideal solution and distance measure given by Li and Chen [16] are tabulated as follows in Table 7:

Table 7: Tabulation of Estimated Weights

	W ₁ ⁺	W ₁ ⁻	W ₂ ⁺	W ₂ ⁻	W ₃ ⁺	W ₃ ⁻	W ₄ ⁺	W ₄ ⁻	W ₅ ⁺	W ₅ ⁻
Ω = 0.1	.2013	.1991	.1992	.2004	.2029	.1982	.2038	.1976	.1927	.2051
Ω = 0.5	.2054	.1959	.1968	.2013	.2118	.1924	.2154	.1900	.1705	.2204
Ω = 0.9	.2082	.1938	.1952	.2020	.2178	.1885	.2233	.1849	.1555	.2309

By using the weight vector and distance measure defined by Li and Chen [16], we get the closeness coefficient and ranking order of choosing/selecting a Director for different values of Ω are shown in Table 8:

Table 8: Ranking Results

For Ω=0.1	<i>d</i> (DIR _{<i>i</i>} , \bar{I}^+)	<i>d</i> (DIR _{<i>i</i>} , \bar{I}^-)	CC _{<i>i</i>}	Ranking	
	DIR ₁	1.09421	1.97896	0.6439	3
	DIR ₂	1.08705	2.09335	0.6582	2
	DIR ₃	1.02884	2.03923	0.6647	1
For Ω=0.5	<i>d</i> (DIR _{<i>i</i>} , \bar{I}^+)	<i>d</i> (DIR _{<i>i</i>} , \bar{I}^-)	CC _{<i>i</i>}	Ranking	
	DIR ₁	1.0838	1.9709	0.6452	3
	DIR ₂	1.0917	2.0906	0.6569	2
	DIR ₃	1.0328	2.0412	0.6640	1
For Ω=0.9	<i>d</i> (DIR _{<i>i</i>} , \bar{I}^+)	<i>d</i> (DIR _{<i>i</i>} , \bar{I}^-)	CC _{<i>i</i>}	Ranking	
	DIR ₁	1.0769	1.9664	0.6461	3
	DIR ₂	1.0952	2.0895	0.6561	2
	DIR ₃	1.0357	2.0437	0.6637	1

It has been further observed that the values of closeness coefficients given by Grzegorzewski’s distance measure [19] in Gandotra et al. [18], is more distinguishable than Li and Chen’s distance measure [16]. Thus, we conclude that results obtained by Grzegorzewski’s distance measure are better than the results obtained by Li and Chen distance measure.

7. Conclusions

Manufacturing environment, product design, creation framework, practical perspectives at workstation and cost required in, are a portion of the major impacting parameter that seem directly or indirectly influence the decision-making process. In some real-life circumstances, there exist issues that the data about the weight of criteria is unknown, and accordingly, to create techniques to deal with this issue is an imperative research heading. Here in this paper we have given the methodology of evaluating the weights of criteria, in a multi-criteria decision making. We have applied the same methodology on a survey structure based on questionnaire for the purchase of a car. Further, a ranking algorithm for Triangular Intuitionistic Fuzzy Multi-criteria Decision Making (TIF-MCDM) issue for the accessible options by processing the various distances between the perfect option and all the accessible options has been proposed. In this manner, we finish up that outcomes obtained using Grzegorzewski's distance measure are better than the outcomes obtained by Li and Chen distance measure.

References

- [1] Zadeh LA. Fuzzy Sets. *Information and Control* 1965; 8: 338–353.
- [2] Atanassov K. Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems* 1986; 20: 87–96.
- [3] Atanassov K. More on Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems* 1989; 33: 37–46.
- [4] Gau WL, Buehrer DJ. Vague Sets. *IEEE Transactions on Systems, Man and Cybernetics* 1993; 23(2): 610–614.
- [5] Grattan-Guinness I. Fuzzy membership mapped onto intervals and many-valued quantities. *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 1976; 22(2): 149–160.
- [6] Jahn KU. *Interval-Wertige Mengen*. *Mathematische Nachrichten* 1975; 68:115–132.
- [7] Sambuc R. *Fonctions phi-floues Application a l'Aide au diagnostic en pathologie thyroïdienne*, Ph.D. thesis, University of Marseille, Marseille, France, 1975.
- [8] Yang ZL, BONSALL S and WANG J. Use of Hybrid Multiple Uncertain Attribute Decision Making Techniques in Safety Management. *Expert Systems with Applications* 2009; 36(2): 1569–1586.
- [9] Wang YM, Elhag TM. Fuzzy TOPSIS Method Based on Alpha level Sets with an Application to Bridge Risk Assessment. *Expert Systems with Applications* 2006; 36(2): 309–319.
- [10] Cavallaro F. Fuzzy Topsis Approach For Assessing Thermal Energy Storage in Concentrated Solar Power (CSP) Systems. *Applied Energy* 2010; 87(2): 496–503.
- [11] Shu MH, Cheng CH, Chang JR. Using Intuitionistic Fuzzy Sets for Fault-Tree Analysis on Printed Circuit Board Assembly. *Microelectronics Reliability* 2006; 46: 2139–2148.
- [12] Atanassov K. *Intuitionistic Fuzzy Sets : Theory and Applications*. Springer Physica-Verlag, Berlin 1999.
- [13] Wang JQ, Zhang Z. Aggregation Operators on Intuitionistic Trapezoidal Fuzzy Number and its Application to Multi-Criteria Decision Making Problems. *Journal of Systems Engineering and Electronics* 2009; 20(2): 321–326.
- [14] Li DF, Nan JX, Zhang MJ. A Ranking Method of Triangular Intuitionistic Fuzzy Numbers and Application to Decision Making. *International Journal of Computational Intelligence Systems* 2010; 3(5): 522–530.
- [15] Guha D, Chakraborty D. A Theoretical Development of Distance Measure for Intuitionistic Fuzzy Numbers. *International Journal of Mathematics and Mathematical Sciences* 2010; Article ID 949143: 1–25.
- [16] Chen Y, Li B. Dynamic Multi-Attribute Decision Making Model Based on Triangular Intuitionistic Fuzzy Numbers. *Scientia Iranica B* 2011; 18(2): 268–274.
- [17] Renyi A. On Measures of Entropy and Information. *Proceedings of Fourth Berkeley Symp on Mathematical Statistics and Probability* 1961; 1: 547–561.
- [18] Gandotra N, Bajaj RK, Mathew J. On Ranking in Triangular Intuitionistic Fuzzy MCDM under (α, β) -Cut with 'Useful' Parametric Entropy. *IEEE proceedings of 5th International Conference on Advances in Computing & Communications* 2015; 69–74.
- [19] Grzegorzewski P. Distances and Orderings in a Family of Intuitionistic Fuzzy Numbers. *Proceedings of the 3rd Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT 03)*, Zittau, Germany, (2003); 223–227.