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JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT
TEST -3 EXAMINATION- MAY 2019
PH. D. (MATHEMATICS): II SEMESTER

COURSE CODE: 13P1WMA232   MAX. MARKS: 35

COURSE NAME: **MATHEMATICAL ANALYSIS**

COURSE CREDITS: 3   MAX. TIME: **2 Hrs**

NOTE: *All questions are compulsory and carry equal marks. Carrying of mobile phone during examinations will be treated as case of unfair means.*

Q.1 Prove that the Cauchy product of two absolutely convergent series converges absolutely.

[4]

Q.2 What are the necessary and sufficient conditions for differentiability? Show that the function $f(z) = \sqrt{|xy|}$, is not analytic at the origin even though Cauchy Riemann equation are satisfied thereof.

[4]

Q.3 Let N and N' be normed liner space and T be a liner transformation of N into N'. Then T^{-1} exists and is continuous on its domain, if and only if, there exist a constant $k > 0$, such that:

$$k\|x\| \le \|T(x)\|, \quad \forall \, x \in N.$$

[5]

Q.4 Write the statements of following theorem with an examples:

     i.   **Hahn-Banach Theorem**

     ii.   **Open Mapping Theorem**

     iii.   **Closed Graph Theorem.**

[2+2+1=5]

Q.5 State and Prove "**Schwarz's Inequality**". Also discuss the case of equality.

[5]

Q.6 (a) State "**Bolzano Weierstrass Theorem**". Can we apply this theorem on set of integers?

(b) State "**Heine Borel Theorem**" with examples.

(c) Test the "**Uniform Convergence**" of sequences of function $< x e^{-nx} >$ on $[0, \infty]$.

[2+2+2=6]

Q.7 (a) State "**Liouville Theorem**". Use of Liouville theorem to prove fundamental theorem of algebra.

(b) Identify zeros of following functions and their order:

     i. $f(z) = z^2(1 - \cos z)$

     ii. $f(z) = z^2 - 1$

[3+3=6]

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