COURSE CODE: 17P1WMA231   MAX. MARKS: 35

COURSE NAME: Advanced Linear Algebra

COURSE CREDITS: 3   MAX. TIME: 2 Hrs

Note: All questions are compulsory. Carrying of mobile phone during examinations will be treated as case of unfair means. Use of scientific calculator is allowed.

1. Let A be a normal matrix then prove the following results:

   (i) $\|Au\| = \|A^*u\|$; *for every* $u \in C^n$.

   (ii)   If $Au = \lambda u$, then $A^*u = \bar{\lambda}u$, where λ is a complex number.

   (iii)   Eigne vectors corresponding to distinct eigenvalues are orthogonal.

2. Define unitary similar matrices. Let $A = \begin{bmatrix} 2 & i \\ -i & 2 \end{bmatrix}$ and $U = \frac{1}{\sqrt{2}}\begin{bmatrix} 2 & i \\ -i & 2 \end{bmatrix}$. Show that A is a normal matrix and the columns of U are orthonormal eigenvectors of A.

3. What is a matrix norm? Explain the concept of sub-ordinate matrix norms and Forbenius norm and discuss various inequalities.

4. Find the eigenvalues of matrix $A = \begin{bmatrix} 10 & 3i \\ -3i & 2 \end{bmatrix}$ and the unitary matrix U such that $U^*AU = D$.

5. Find the least square solutions to the following over determined system of linear equations:

$$x + 3y = 80; \qquad 2x + 5y = 100$$
$$5x - 2y = 60; \qquad -x + 8y = 130$$
$$10x - y = 150.$$

6. What is a similarity transformation? Show that the similar matrices have the same characteristic polynomial and hence the same eignvalues with same multiplicities. Write the condition for A and B matrices share the same n independent eigenvectors.

7. Fit a second degree parabola to the following data:

X	1	3	5	6
Y	2	7	10	11

*****