Signal Generation Employing Chebyshev Polynomial For Pulse Compression With Small Relative Side-Lobe Level

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***Abstract*—The theme of this paper is to present the improvement in the peak side-lobe levels (PSL) and time-bandwidth product with Chebyshev polynomial. This PSL behavior is observed by the matched filter (MF) response, which contains main-lobe width as well as side-lobes. Here to get a better reduction in the side-lobes, Chebyshev polynomials are modified by using zero-crossing there by getting the positive and negative pulse. Here two cases have been considered, in first ordinary Chebyshev polynomial are analyzed, second is a modification in the cycles of Chebyshev polynomial is incorporated. After this the smallest duration of the pulse has been used in determining the optimal duration which has the smallest mean square error (MSE) between the number of pulses incorporated and original signal. This is giving a much larger signal with less PSL by reducing the search domain considerably. This new method tries to implement a side lobe level reduction technique. All of the mentioned procedure is carried out by mathematical equations and simulation verification.**

Keywords—Pulse compression; peak side-lobe level; matched filter; Chebyshev polynomial.

I. Introduction

Radar is generally used for the detection and to find the location of the target. For this purpose a narrow pulse is transmitted into the space and then the reflected echo signal from the target is captured [1-3] and this captured signal is processed at the receiver to find the characteristics of the target. To detect echo signal from the target, transmitting signal should have more strength. So, the transmitted pulse should be more efficient in terms of energy otherwise the reflected pulse may get attenuated [1] during the course of transmission in the environment. Range resolution, $R = \frac{c\tau}{2}$ is the capability of the system to separate two or more nearby targets in terms of their range. Here τ is the transmitted pulse duration. The distance of the target can be measured by calculating the round trip time [1, 2] that is the time taken by a wave from radar to target and target to radar. For the measurement of range, the delay between the transmitted pulse and reflected pulse from the target has to be measured.
Usually short duration pulses are considered to get better range resolution, which is desired property in radar system [2].

Practically, the duration of the transmitting pulse cannot be decreased too much because for long distance, transmission short duration pulse requires high peak power to get adequate energy. To handle high peak power overall radar system becomes heavier by which overall cost of the system increases, thereby the pulse having less peak power and longer duration is required at the transmitting end for long range detection [2].

For the determination of velocity, Doppler frequency shift is used, that is shift in the frequency of the transmitted signal due to the relative motion between target and radar. This shift in the frequency is proportional to the velocity of the target relative to the radar [2]; if target is in motion then it can be called as radial velocity. So, for good detection many radar systems transmit long duration pulses to achieve high signal strength at the receiver side. But for good range resolution radar uses short duration pulses. By using angle modulation long duration pulse bandwidth can be made larger and this larger bandwidth shows narrow effective pulse duration that can be used for range resolution. This signal processing technique used in radar system is known as pulse compression [3].

It means a short duration pulse for range measurement and long duration pulses are required for better detection. To determine both range and velocity simultaneously pulse Doppler radars are used which uses this pulse compression technique.

II. Related Work

Many techniques have been developed to handle this problem in a radar system, started with linear frequency modulation (LFM) [3], non-linear frequency modulation (NLFM), the classic Barker codes [4] have been developed. In [5], the authors generate the orthogonal phase-coded waveforms to random coded length for an uninformed number of transmitters to suppress the side-lobes. In [6], a method for an active electronic protection system against some electronic intelligence devices has been produced.

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Vizitiu [7] has produced a technique to overcome the problems of LFM signal that is widening of the mainlobe width which interrupt the range resolution by using nonlinear method. In [8], the authors have proposed a periodic cross-correlation technique for the non-coherent pulse compression using Legendre sequences of any prime length. A hyper chaotic coding scheme and corresponding finest selection method are proposed by authors [9, 10] to obtain the phase code signal, which exhibits excessive performance for side-lobes reduction.

Weighting in frequency and time domain can generally be applied to reduce the side-lobes [11]. A new radar pulse compression procedure based on approximation theoretic principles is proposed by the authors [12]. The determining power of the proposed algorithm is improved due to the complete elimination of the side-lobes, which earlier existed in the auto correlation function (ACF). In [13], the authors have developed a model in which a mismatched filter, comprised of a MF is cascaded with a parameterized multiplicative limited-duration impulse response filter.

In [14, 15], the authors have present a sidelobe reduction technique whose results are good. In this sidelobe reduction technique, Woo filter and modified forms of Woo filter are investigated which works on the polyphase codes. In [16], the authors have proposed a technique in which ACF for modified two and tri-stage NLFM signal are examined and -19dB sidelobe suppression is achieved without disturbing the relative main-lobe width. Baghel and Panda [17], have proposed a hybrid model in which MF output is modulated by the output of the radial function for different Barker codes.

In the literature, there is little analysis of the pulse compression with Classical Orthogonal polynomials and this paper addresses the detail analysis on PSL of MF, bandwidth and AF with modified Chebyshev polynomials. The rest of the paper is organized as follows. The section III gives the problem formulation while section IV illustrates the simulations and results. Section V gives the conclusion based on the detailed study.

III. Problem Formulation

The aim of this paper is to analyze the sideline behavior for the Chebyshev polynomial based signal. To avail the benefits of short duration pulse for good range measurement and long duration pulse for better detection, pulse compression technique has been developed in radar system. In radar system, transmitted signal is represented as [2]

$$x(t) = \alpha(t)\cos(2\pi ft + \varphi(t)) \tag{1}$$

where $\alpha(t)$ represents the amplitude modulation whose amplitude varies with respect to the time and $\varphi(t)$ is the angle modulation. The received or echo signal is represented as

$$r(t) = \cos(2\pi ft + \Omega(t) - T_d) \tag{2}$$

where $\Omega(t)$ can be taken as the phase differences coming from relative velocity between the target and the radar system, while T_d can be taken as the round trip time delay.

This received signal is passed through the MF in order to improve the Signal to Noise Ratio (SNR) because probability of detection is depend on the SNR rather than exact shape of the signal [14]. So, it is required to increase the SNR rather than maintaining the shape of the signal, for this purpose MF is used. Generally, for signal synthesis or to observe the sidelobe behaviour ACF is used [1, 2] which gives output of MF of expected signal. For a general complex-valued sequence a code of length N, the ACF is given by

$$R(k) = \sum_{n=0}^{N-k-1} a_n a^*_{n+k} \tag{3}$$

where $k = 0,1,2...,N-1$. The output of the MF is used to find the main response, but it also contains the side-lobes. These side-lobes are undesired because they may responsible for false target detection.

In summary, during the process of pulse compression side-lobes are undesired. In two ways these side-lobes can be suppressed, in one matched filter's output can be passed through some weighting filters [1, 2], doing this SNR of the system is reduced to some extent. In the second method, the search for the transmitting waveform which has low side-lobes in the MF response can be used during angle modulation. In this paper second case has been considered to reduce the side-lobes, so the search has to be conducted for the best possible function $\varphi(t)$ as represented in (1). There are many functions possible for $\varphi(t)$, but here this function is confined to the Chebyshev polynomial due to the wide applications of this polynomial in engineering domain [18]. This paper presents an in-depth analysis on side-lobes reduction in the pulse compression with respect to modified Chebyshev polynomial.

IV. Simulations and Results

First, Chebyshev polynomial is taken and its MF response is analyzed, after that modification in the cycles of the polynomial is incorporated, full and half cycles of the polynomials are modified. Full cycle modification means each full cycle of the polynomial and half cycle modification means each half cycle of the Chebyshev polynomial has to be modified. For nth order, $\frac{n-1}{2}$ and $n-1$ cycles exist for full cycle and half cycle modification respectively. Here emphasis is given on half cycle modified polynomial because this modification is possible for both even and odd ordered polynomial and it also gives better PSL in MF response. To generate full cycle modification, only odd number ordered polynomial has to be considered, if even number order polynomials are incorporated then some part of the signal is truncated which alter the desired results. Once this half cycle modification in the polynomial is done, then analyze the performance measuring parameters viz. bandwidth. After this calculate the least duration of the cycles; divide this duration by total length of the polynomial termed as optimal duration (T_{opt}).

This T_{opt} is calculated only for that modified polynomial which has better PSL in the MF response. After this divide all cycle duration with this T_{opt} and rounding these durations to its nearest integer value. Now all the cycle duration is an integer multiple to the optimal one. MSE is calculated between original and rounded duration of the cycles after division with the least duration pulse. Then only that duration of the modified half cycle polynomial is taken which has least MSE and occupy maximum duration of the optimal one. Chebyshev polynomial of different order can be calculated [18] as

$$T_{n+1} = 2xT_n - T_{n-1} \qquad (4)$$

Steps to generate modification in the Chebyshev polynomial cycles

Step 1: Take a Chebyshev polynomial.

Step 2: Modify cycles of the polynomial in the following manner.

To find the zero crossings of the polynomial. Count the number of positive and negative pulses in the region -1 to 1.

Generate all the possible binary sequences according to the number of pulses obtained above.

Multiply this binary sequence with Chebyshev polynomial.

Out of these possibilities, the smallest PSL is stored and designate this as best PSL.

Step 3: Bandwidth and AF analysis are carried out.

Step 4: Find the optimal duration (T_{opt}) for the modified half cycle polynomial.

Calculate the least duration of the pulse of zero-crossing and divide by it with overall length of the polynomial.

Get the fractional duration of all the pulses.

Round-off this duration to its nearest integer value.

Step 5: Calculate mean square between rounded and actual duration.

Take the duration which has least MSE.

Calculate number of cycles that can be added to the modified half cycle polynomial.

For the above obtained durations, expand the each pulse of an integer number of T_{opt} duration.

Step 6: Analyzed their MF response.

A. PSL Observation

For the representation of polynomials in time-domain, duration of the pulse has been taken from -1 to +1 and 4k Hz sampling frequency is taken. To observe the PSL behavior of polynomial, its MF is analyzed which tells about the main lobe-width as well as side-lobes. Here in Fig. 1, order of the polynomial is taken as fourteen because it gives best PSL. To suppress these side-lobes some modification in the cycles of the polynomial is incorporated.

Chebyshev polynomial cycles are multiplied with some polarities i.e. +1 or -1 to modify the cycles; duration of this polarity is generated by zero-crossing of Chebyshev polynomial cycles. After the generation of this polarity based multiplying sequence (shown by red color) has to be multiply with above polynomial cycles. First row corresponds to the Chebyshev polynomial and its MF response which contains high side-lobes. Second row corresponds to the modified half cycle polynomials whose MF response has relatively less side lobes.

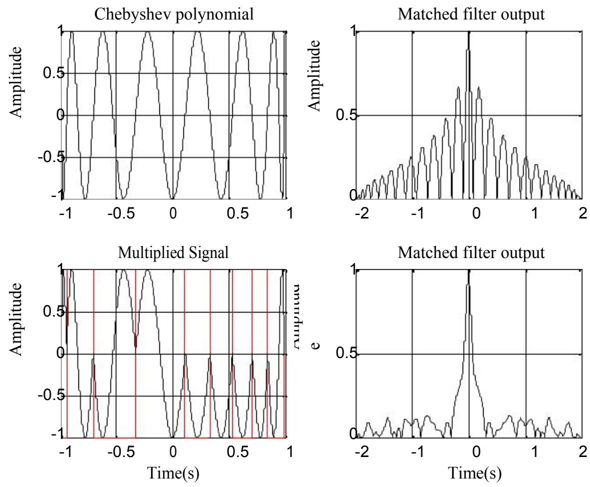

Fig. 1. Time-domain representation and MF output of Chebyshev polynomial for n=14

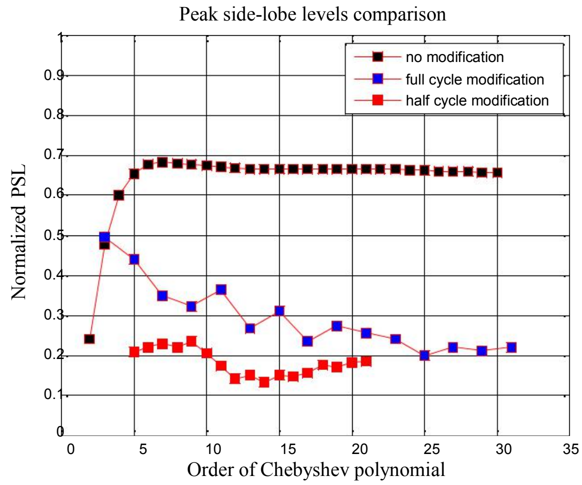

Fig. 2. Relative side-lobe levels for different order Chebyshev polynomial

Depending upon the order of the polynomials there are many possibilities to change the cycle of polynomials, here out of these possibilities best case (gives better PSL) has been considered. The PSL of Chebyshev polynomial, full and half cycle modified polynomial are observed and represented in Fig.2. This figure represents the best PSL level out of all the possibility and it is observed that half cycle modified polynomial gives better PSL than other two cases.

It is observed that full cycle modified polynomials are better than Chebyshev polynomials but worse than half cycle modified polynomials for the reduction of PSL. So, in this article half cycle modification has been considered because it is giving better PSL. The multiplied cycle's polarity is generated for half cycle modification which gives best PSL is mentioned in Table 1.

B. Bandwidth Analysis

As time-bandwidth product is the main parameter in pulse compression. It should be more for good detection as well as for good resolutions. If the bandwidth of the transmitting signal is high, then the pulse duration is small and this gives a good range resolution.

TABLE 1. MULTIPLIED CYCLES POLARITIES

Order(*n*)	**Polarity Format**
11	-1 -1 -1 -1 -1 +1 +1 -1 -1 +1
12	+1 +1 +1 +1 +1 +1 -1 -1 +1 +1 +1
13	-1 +1 +1 -1 -1 +1 +1 +1 +1 +1 +1 +1
14	+1 +1 -1 -1 +1 +1 -1 -1 -1 -1 -1 -1 +1
15	-1 -1 +1 +1 -1 -1 +1 +1 +1 -1 +1 -1 -1 +1
16	+1 +1 +1 -1 -1 +1 +1 +1 +1 +1 -1 +1 -1 -1 +1

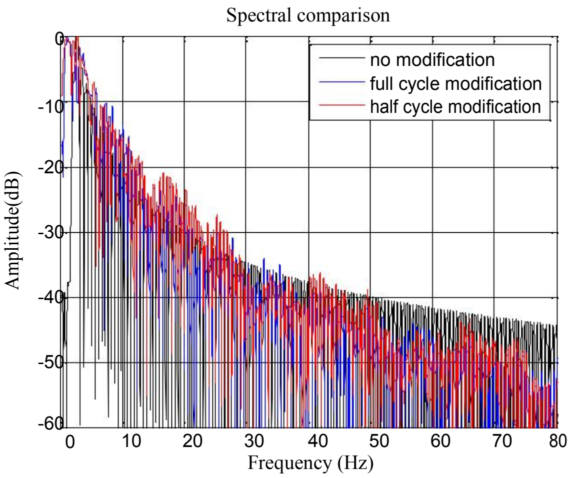

Fig. 3. Spectral comparison for n=15

Usually, duration of the pulse cannot be reduced too much because Fourier theory says that a signal having bandwidth B have always duration greater than 1/B i.e. its time-bandwidth product is always greater than unity. So, in Fig.3 spectral contents of Chebyshev polynomials for order fifteen are compared because both modifications are possible for odd order only. It is observed that the modified half cycle polynomial has considerably wider frequency response than the other two. For better range resolution, the bandwidth of the pulse has to be very large which indicates a shorter pulse. This shorter pulse makes difficulty in the decision on the target. Hence the bandwidth of the pulse has to be increased as much as possible while maintaining the duration of the pulse fixed.

C. MSE Calculation

MSE is calculated between original and rounded duration of the cycles after division with the least duration pulse. All cycles in the polynomial are not of equal duration as observed in the Fig. 1. It is clear that after dividing by T_{opt}, cycles duration is in the form of fraction, this fractional duration is rounded to its nearest integer value. It means number of pulses which are integer multiple to the optimal duration can be added into the modified half cycle polynomial, each added cycle have duration equal to the optimal one.

During rounding the fractional duration to its nearest integer number, some part of the signal may be discarded that alter the performance of the system. So, only those modified polynomials are considered which have MMSE. $e(t)$ is the error deviation [18] in the actual $x(t)$ and the rounded duration ($x(\hat{t})$).

$$e(t) = x(t) - x(\hat{t}) \tag{5}$$

MSE can be calculated as

$$MSE = \frac{1}{T}\int_0^T e(t)^2\, dt \tag{6}$$

T is the overall duration of the polynomial. Here Fig. 4 represents the polarities of the multiplied signal and percentage of error present in rounded duration with respect to the original duration after division.

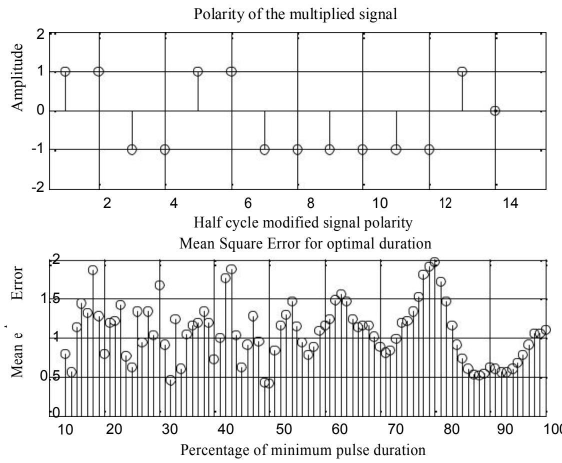

Fig. 4. Multiplied signal polarity and optimal duration MSE for n=14

D. Cycle Addition And ACF Analysis

Only those modified polynomial sequences whose MSE is small and contain the maximum duration of the optimal one, are considered here. First two best cases have been analyzed for order fourteen and fifteen. From Fig. 4, it is observed that low MSE at 93% (first optimal), and at 88% (second optimal) of the minimum duration pulse, depending on these particular points, modification in the cycles is incorporated by adding more number of cycles. For first optimal (7,13,19,23,27,29,30,29,27,23,19,13,7) cycles can be added, it means seven cycles in the first duration having polarity +1, thirteen cycles in the second duration having polarity +1, nineteen cycles in the third duration having polarity -1 and so on. Each added cycle has a duration equal to the optimal one. Here polarities of these added cycles are same as that of the

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modified polynomial. Now modified polynomial contains total 266 half cycles and their ACF are observed as shown in Fig. 5 and it has less PSL. Similarly, for the second optimal (6,11,16,20,23,24,25,24,23,20,16,11,6) cycles can be added to their respective cycle duration.

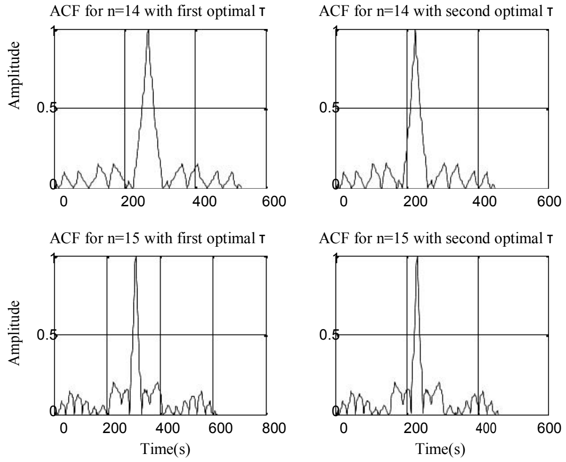

Fig. 5. ACF for n= 14 and 15 having different τ_{opt}

In the same way for order fifteen, first two optimal duration, having least MSE are considered and numbers of cycles are calculated which can be added into the modified half cycle polynomial. First best exist at 96 % of the optimal duration, have number of cycles that can be added are (7,13,19,24,28,31,32,32,31,28,24,19,13,7) and second optimal exist at 91% of the optimal duration, have number of cycles that can be added are (5,10,14,18,21,23,24,24,23,21,18,14,10, 5). From the simulation, it can be concluded that by appending cycles in to half cycle modified polynomial, PSL levels are suppressed and total numbers of cycles are increased, this corresponds to the higher order polynomial.

So, using low order polynomials, the PSL behavior of higher orders can be analyzed by which simulation time can be saved to run higher order polynomials. Usually, in pulse compression if side-lobes are suppressed, then its main-lobe width is increased slightly as observed in Fig. 5. So, these half cycles modified polynomial can be used in the pulse compression for the reduction of PSL in place of $\varphi(t)$ in (1).

V. Conclusion

The detailed analysis is carried out on the derived signal with respect to the PSL and time band-width product. It is observed that half cycle modified polynomials gives better results than full cycle modifications. Along with this, there is improvement in the bandwidth. By adding more number of cycles which have a duration equal to the optimal one, correspond to the least MSE into the modified half cycle polynomial, gives better results for PSL. It is also concluded that by using low order polynomial, ACF for higher order polynomial can be observed which saves the simulation time. In summary, the method has been demonstrated to be real sidelobe reduction technique having applicability inside the pulse compression radar systems.

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