SIMPLIFICATION AND APPROXIMATION ANALYSIS OF DYNAMIC SYSTEMS FOR CONTROL DESIGN

Project report submitted in partial fulfillment of the requirement for the degree of Bachelor of Technology

In

ELECTRONICS AND COMMUNICATION ENGINEERING

By

SHREYA GOEL (141077)
AAKANSHA SAROHA (141079)

Under the supervision of

Mr. MOHIT GARG
(Assistant professor**)**

to

Jaypee University of Information Technology, Waknaghat, Solan-173234, Himachal Pradesh
May, 2018

DECLARATION BY THE SCHOLAR

We hereby declare that the work reported in the B.tech thesis entitled **"SimplificationAnd Approximation Analysis Of Dynamic Systems For Control Design"** submitted at **Jaypee University Of Information Technology, Waknaghat India**, in an authentic record of my work carried out under the supervision of **Mr. Mohit Garg** (Assistant Professor, Department Electronics and Communication Engineering).We have not submitted this work elsewhere for any other degree or diploma.

(Student Signature)
Shreya Goel
141077

(Student Signature)
Aakansha Saroha
141079

Department of Electronics and Communication
Jaypee University of Information Technology, Waknaghat , India
Date :

CERTIFICATE

This is to certify that the work reported in the B.Tech project report entitled "**Simplification And Approximation Analysis Of Dynamic Systems For Control design** "which is being submitted by **Shreya Goel (141077**) and **Aakansha Saroha (141079)** in fulfillment for the award of Bachelor of Technology in Electronics and Communication Engineering by the Jaypee University of Information Technology, is the record of candidate's own work carried out by him/her under my supervision. This work is original and has not been submitted partially or fully anywhere else for any other degree or diploma.

Mr. Mohit Garg

Assistant Professor

Department of Electronics & Communication Engineering

Jaypee University of Information Technology, Waknaghat

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LIST OF ABBREVIATIONS AND ACRONYMS

S.NO	ABBREVIATION	FULL FORM
1.	MOR	MODEL ORDER REDUCTION
2.	PSO	PARTICLE SWARM OPTIMIZATTION
3.	LTI	LINEAR TIME INVARIANT
4.	ISE	INTEGRAL SQUARE ERROR
5.	MSE	MEAN SQUARE ERROR
6.	SISO	SINGLE INPUT SINGLE OUTPUT
7.	MIMO	MULTIPLE INPUT MULTIPLE OUTPUT
8.	GA	GENETIC ALGORITHM
9.	SVC	STATIC VAR COMPENSATOR
10.	BB-BC	BIGBANG-BIG CRUNCH
11.	TDS	TIME DELAY SYSTEM
12.	ODE	ORDINARY DIFFERENTIAL EQUATION
13.	TF	TRANSFER FUNCTION
14.	DE	DIFFERENTIAL EQUATION
15.	MPC	MODIFIED POLE CLUSTERING TECHNIQUE

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ABSTRACT

Model order reduction is useful for the simulation and synthesis of faulty mechanism due to the associated computational costs. Large scale LTI dynamic systems which are described by the differential equations can be calculated using MOR methods. In MOR large order systems undergo certain techniques which results in its order reduction. This reduced order have numerous advantages over the higher order. Few advantages are listed below:

- Lower computational cost
- Simpler to implement
- Less run time
- Easy to understand

The reduced order preserves the essential properties of the original system.

Numerous approaches for MOR are there which includes both classical and evolutionary techniques. Few of them are stated and considered here:

- Modified pole clustering technique.
- Coefficient matching.
- Particle swarm optimization.
- Pade approximation.
- Big-Bang Big Crunch theory

The techniques are performed and then the responses of both original and reduced models are compared.

KEYWORDS

Model order reduction, Particle swarm optimization , Modified pole clustering technique, Coefficient matching.

CHAPTER 1
INTRODUCTION

1.1 Introduction

In engineering and sciences one of the unavoidable topics is the modeling of higher order system. These kind of models are complex to be used in real life problems. This shows that mathematical model made it easier to design and analyze such types of physical systems. The methods based on mathematical models and physical considerations are essential to achieve simplified models than the original ones.

Mathematical model of a physical system may consist of transfer function with much high order. When available techniques are applied to a higher order system then designing and analysis of that system becomes difficult. At this point in order to reduce process time and effort, we need to use model reduction methods at a large scale. In order to obtain lower order models from higher order models, it is of high urgency that we derive reduced order models which are stable from the already stable original model by also keeping in consideration that few quantities of the original order matches the quantities of reduced order model and this defined as Model order reduction(MOR).

In international literature various methods are available that deliver the main motive of large scale systems modeling. There are a number of physical laws such as Newton's and kirchoff's law that are used to execute the modeling of physical systems which are represented by the respective differential equations. These equations that are acquired can be regenerated into state space form or transfer function for control system design and analysis. The frequency and time domains that represent Linear Time Invariant or LTI are analyzed using the transfer function. This function when used is of a high order which makes the analysis of such systems time consuming and even difficult. For second order systems, there are various analytical methods obtainable in control theory. These same methods can be used to analyze and hence control reduced order system, by first simplifying the original model to a lower order..

There are multiple definitions available on (MOR) model order reduction , and depending on the context it is decided which definition is most suitable for use. Initially model order reduction was introduced in the field of systems and control theory. MOR is based on the study of properties of dynamic systems to reduce the complexity , while restoring their input-output behaviour. Model order reduction is a technique used in the field of research, both in control theory and numerical analysis and in systems.MOR as an whole experience a greater effect, indulging together distinct point of view and distinct techniques, thus providing valuable insights and helping progress the field of MOR.

Therefore the question is what is MOR ? Dynamical models contains various equations along with variables with higher power. These models need to deal with certain type of simplification . This type of simplification is required to perform simulations within a limited storage as well as using a short amount of time as well as speed but with enough reliable and verifiable output.

MOR tries to immediately store the important characteristic features of structure presented. What this depicts is that in the starting stages of the process the very basic traits of the original model should be there in minor approximations. Reduction process is stopped at a certain instant. At that instant all the important and essential properties of the original model should be stored along with enough accuracy. This entire process is done automatically.

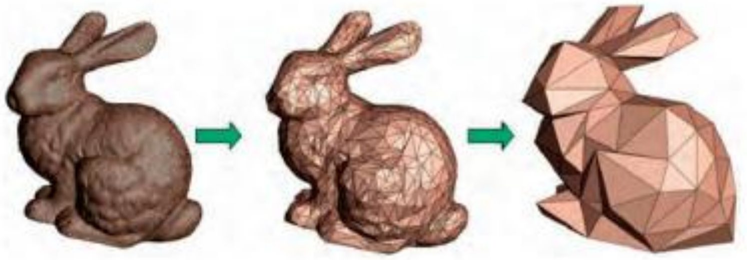


FIGURE 1.1 Graphical illustration of Model order reduction

Figure 1 represents the concept of model order reduction in easy-to-understand graphical manner, illustrating that sometimes minute amount of information is essential for describing a model. This figure containing the pictures of the Stanford Bunny demonstrates that even with very less features, the rabbit can yet be acknowledged as such. This example is used just to explain the basics of model order reduction and doe snot contain any reference regarding its mathematical functioning.

In most of the methods of MOR the integral square error which is found between time domain responses of reduced model and the original model is decreased. Thus the work conferred here too focuses on different performance specifications like steady state error, settling time, peak overshoot and rise time. Both the system responses should have similar steady state values.

Various techniques like classical algorithm and evolutionary algorithm are used further for model order reduction.

1.2 Approaches of model order reduction

In general, two ways are there to perform model order reduction:

Time Domain Approach: In this, initially the linear differential equation of first order(i.e. state space form) is used to convert the system dynamics.

Now consider a linear time invariant system of kth order as represented in eqns.1.1&1.2:

$$a(t)=Ar(t)+Bv(t) \qquad (1.1)$$

$$b(t)=Cr(t)+Dv(t) \qquad (1.2)$$

Where, a(t)- j dimensional state vector

v(t)-k dimensional input vector

b(t)-l dimensional output vector

A,B,C and D are matrices, whose size are (j X j),(j X k),(l X j) and (l X k) respectively.

The problem of model order reduction corresponds to searching of an approximate order r<j, given by:

$$ar(t)=Ara(t) + Brv(t) \qquad (1.3)$$

$$br(t)=Bra(t) + Drv(t) \qquad (1.4)$$

The eqn. (1.3) and eqn.(1.4) demonstrates the reduced model of the system which is close enough to the original system.

Frequency Domain Approach: Another approach used for MOR is frequency domain approach. In this Laplace transform of differential equation is taken, and the transfer function of the given system along with all the initial conditions fixed to zero is calculated. Suppose the transfer function of the system is given by:

$$G(s) = \frac{b_1 S^{n-1} + \ldots\ldots\ldots\ldots + b_n}{a_0 S^n + \ldots\ldots\ldots\ldots + a_n}$$

Now let us consider that reduced order model R(s) with order r <n that is close enough to the system G(s is given by:

$$R(s) = \frac{b_1 S^{r-1} + \ldots\ldots\ldots\ldots + b_r}{a_0 S^r + \ldots\ldots\ldots\ldots + a_r}$$

the coefficients of reduced order model are measured that greatly approximates the original system.

1.3 Motivation

In past two decades the problem of MOR of linear and non-linear dynamical systems has been broadly studied and this is still an active topic. Due to ever-enhancing ability of computers and methods to precisely model real-world systems, computational science or say simulation has been verified as a trustworthy method for predicting, identifying and analyzing the system’s behaviour. It is to such an extent that now a days simulation is viewed as an essential component of current technological world. Simulation is now acceptable as a third discipline, besides the classical discipline of experiments and theory. Now all the processes like physical, chemical or other processes makes the use of computer simulations. For a variety of problems, computer aided design and virtual environments are set up to make the work of engineers and designers easier. This helps in faster designing of new products that too reliably and without having to form expensive prototypes.

Simplifying the model helps in speeding up the computation time either in terms of complexity or size. Model order reduction(reducing the order of model) involves the reduction in the size of the mathematical model but keeping in mind that its essential properties are preserved.

1.4 Objectives

- Ample difference between the size of original system and reduced order model.
- Minute approximation error.
- System properties are preserved like steady state error, rise time, settling time.
- Numerically stable.
- Computational complexity of the model is reduced for system analysis.
- Computational cost is reduced.
- Controller size is decreased.
- System understanding is simplified for design purpose.

1.5 Main requirements of MOR

There are number of requirements that should be fulfilled while extracting the model from the physical description of the original system. These are few most essential requirements that assure the feasibleness of the resulting models.

- **Accuracy:** An adequately accurate model for the original system is acquired using reduction technique. The terminal behaviour of the original system is followed nearly by the reduced order model.
- **Compactness**: The number of states or variables should be reduced as compared to the original system.
- **System properties preservation:** The main physical properties(stability and passivity) of original system should be restored by the reduced order model.

- **Computationally efficient:** A model which is comparatively less expensive to simulate and gets easily stored in the computer's memory should be extracted by the model order reduction algorithm. The computational cost should be quite lower for simulation of reduced order model in comparison to original model.
- **Inexpensive algorithm:** The algorithm of model order reduction should be comparatively inexpensive to apply. In any phase of modeling, optimization or verification the extraction procedure needs to be practically repeatable with a quite reasonable cost by the designers.

1.6 Applications of MOR

- Micro hotplate device.
- Radio frequency microelectromechanical systems(RF MEMS).
- Micro accelerometer
- Time delay systems
- Structural health monitoring

Chapter 2
LITERATURE SURVEY

2.1 Order reduction of linear systems with an improved pole clustering.

For the reduction of LTI system, a new technique for MOR has been provided in the following research paper. In this technique the reduced denominator polynomial is calculated using an improved clustering algorithm and the corresponding reduced order model is calculated using simple mathematical calculation. The stability of the reduced algorithm is guaranteed using improved clustering algorithm and this algorithm also helps in conserving the essential properties in the given original model. The provided technique is also used to extend to the high order multivariable system which is represented by its matrix transfer function. The improved clustering method used in this paper is not same as the existing clustering technique. It is very different from the already present clustering process as it takes in consideration the distance between the poles of the system and the first pole in the group clustering process. The proposed scheme helps in obtaining better approximations in the reduction process. After we get the results from this process we compare them to the results in research papers which have used techniques like particle optimization and genetic algorithms often used for evolutionary analysis. Some numerical illustrations are shown to explain the model order reduction of single input single output and multiple input multiple output(SISO and MIMO) systems. ISE, ITAE as quality parameters are used to find the closeness between the reduced order model and the original model for the given test inputs like step, ramp and impulse inputs. If the provided higher order system is stable then the stability of the reduced model is also assured. On the basis of low values of ISE, IAE and other evolutionary methods like genetic algorithm and particle swarm optimization method can be used to modify the obtained reduced order numerator in order to improve the proposed method

2.2 A Comparative Study of GA, PSO and Big Bang-Big Crunch Optimization Techniques for Optimal Placement of SVC.

There is probability of line outage and continuous instability issues related to voltages at heavily loaded conditions in a power system it is essential to solve the problems related to reducing power losses and improving the voltage profile in an optimum way. In order to achieve this optimal solution , existing facilities can be utilized along with installation of FAACT devices. In this paper many optimization techniques are comparatively studied like particle swarm optimization, GA for optimum arrangement of static Var compensator. This is done in order to improve the stability of voltage and also for considering the cost function. In IEEE-14 bus test system the effectualness of the algorithm have been tested. This also concluded that this algorithm can also be applied to larger devices. The proposed algorithm doesn’t suffer with computation related difficulties. In comparison to PSO and GA, big bang big crunch is numerically less complex with comparatively less parameters. BB-BC provides better results and also consumes less storage memory. Maintaining the diverseness in search helps BB-BC in avoiding the traps related to local optimum. Using this it is also concluded that PSO method generates good quality outputs in quite shorter duration of time. Stable convergence traits are also obtained as compared to genetic algorithm but the problem is the dependency of initial parameters and coefficients. Any optimization issue which is represented using chromosome encoding can be solved using GA. But the GA doesn’t provide entire information related to the components of solution, but on the other hand it searches through multiple points instead of single point at one time in solution space. This paper demonstrates that BB-BC algorithm is comparatively more efficient than the PSO and GA in several aspects and PSO is comparatively better than GA. This paper showed the comparative study among these techniques .IEEE-68 &IEEE-30 bus systems will be used to test the effectiveness of the proposed techniques. IEEE-68 and IEEE-30 doesn’t suffer from difficulties related to computations.

2.3 Model Order Reduction of Higher Order Discrete Time Systems using Modified Pole Clustering Technique.

In this paper for larger order discrete time systems, a new scheme is proposed to find a reduced model. This proposed method consists of two techniques i.e. Pade approximations and modified pole clustering techniques which are computer oriented and simple. Modified pole clustering technique is used to acquire the denominator of reduced order model and Pade approximation is used to acquire the numerator coefficients of the reduced model. If the original order is stable then this method produces stable reduced order model. In this linear transformation is done to convert discrete time system to continuous time system and then the reduced model is acquired for continuous time system by using the above mentioned techniques. Then the reduced model is transformed to discrete time system using the inverse transformation method. Numerical illustrations are used to demonstrate the working of proposed scheme.

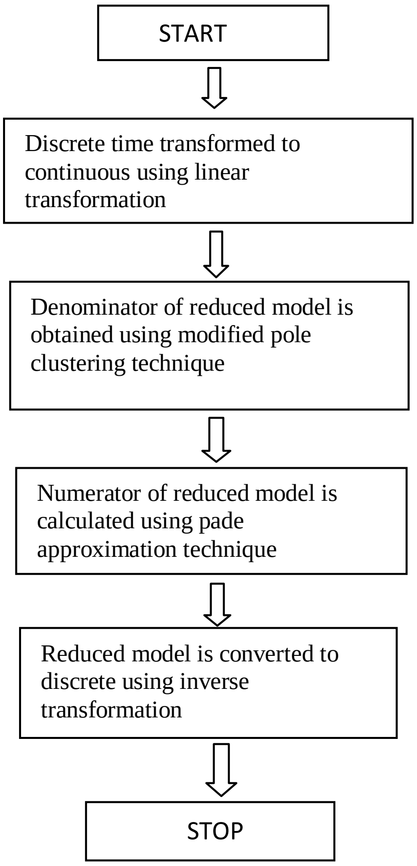

START
Discrete time transformed to continuous using linear transformation
Denominator of reduced model is obtained using modified pole clustering technique
Numerator of reduced model is calculated using pade approximation technique
Reduced model is converted to discrete using inverse transformation
STOP

FIGURE 2.2.1 Flowchart for the proposed scheme

2.4 Relative Mapping Errors of Linear Time Invariant Systems Caused By Particle Swarm Optimized Reduced Order Model.

An optimization technique is presented for MOR and its applications are presented for determining the relative mappping errors of LTI dynamic systems with the help of easy models. Relative ISE criterions are used to present the relative mapping error. These errors are observed for both impulse and unit step inputs. The proposed scheme for order reduction is based on the reduction of ISE by using particle swarm optimization algorithm referring to a unit step input. PSO algorithm is seen as a trustworthy algorithm for solving the optimization problems. It is a population based random optimization method. It is inspired by the movement of fish schooling or flocking of birds. PSO is in many ways similar to genetic algorithm. It involves the initialization of group of random solutions and search for optimum solution by updating generations. PSO unlike GA doesn't have evolution operator like mutation or crossover. The algorithm used is computer based, rugged and easy. It is demonstrated that the algorithm has several advantages, e.g. the stability and passivity of the original model are preserved in reduced model. Two numerical examples are shown to prove the advantages of proposed algorithm over the other existing methods. The algorithm used is represented in Matlab 7.0.1 on a Pentium-IV processor. This algorithm assures the similarities between unit step and impulse response. The relative step mapping error and impulse mapping error between the low order model and original model are also discovered and plotted with respect to time. This paper also demonstrated the comparative study of mapping errors for the proposed reduction method and the other existing reduction methods. The tables presented clears that the method proposed here compares well with the other existing techniques of order reduction.

2.5 The Particle Swarm—Explosion, Stability, and Convergence in a Multidimensional Complex Space.

The particle swarm optimization is an algorithm in which through the interaction among the individuals in a group of particles, optimal solutions of complex search areas could be found. Even if it is concluded that PSO works on the basis of social interactions and performs well, still researches have not adequately explained its working. The conventional version of this algorithm contain few undesirable properties i.e. the velocity of particles need to be limited in order control the particles' trajectories.
In this , particle's trajectory is analyzed during its movement in discrete time and then it further progresses to the view in continuous time. System is represented entirely with the help of five dimension depiction. These analyses help in obtaining a generalized model of the proposed algorithm, which contains a number of coefficients in order to control the system's convergence tendencies. The current analysis begins with a very simple version of PSO so as to explain how it searches for the problem space and then further continues on to analyze the entire stochastic system. A generalized model is developed that contain techniques for controlling the convergence properties of the given particle system. Lastly, few empirical results are obtained that demonstrates the performance of several implementations of the technique on a set of test functions. This paper represents the working of PSO algorithm on the basis of individual particle's point of view. The 5-D depiction summarizes the working of particle entirely and allows the evolution of methods for Controlling the explosions that are obtained due to the system's random behaviour.

In order to assure convergence, coefficients can be applied to several parts of the formula, while promoting exploration. Various coefficient adjustments are shown in present paper.

2.6 A new optimization method: Big Bang–Big Crunch

Nature is considered to be the principle source for suggesting new optimization techniques such as simulated annealing and genetic algorithm methods. All the traditional techniques are experimental population-based search methods which integrate arbitrary variations and selection. The major contribution of this is that it suggests a novel optimization technique that depends on the theories of evolution of the universe. One of this mainly the BB-BC Theory. In the Big Bang phase, a randomness and disorder is produced through the energy dissipation which is the major characteristics of this phase. In the Big Crunch phase is the arbitrarily distributed particles are collected into an order. Encouraged by this theory, a new optimization algorithm is developed, that is known as the BB-BC method also denoted by BB-BC. In the BB phase random points are generated while those points are compressed to a single representative point in the Big Crunch phase via the minimal cost or centre of mass approach. Is has been found that the presentation of the new BB-BC technique defines superiority over the advanced and modified genetic search algorithm.

While finding the compound problem’s global optimum with many local optima, main fight is among reliability, accuracy and computational time. In the customary optimization technique fails to give effectively dependable results, in this case evolutionary algorithm may consist of the interesting alternative. The conventional Genetic Algorithm approach may experience extra slow convergence. They are usually inactive in approaching the global optimum precisely and consistently in a small time period.

The major limitation of classical GA can be overcome by the new BB-BC technique. It has been found in the benchmark test that the speed convergence has been upgraded. From the analysis some other conclusions can be made that up gradation also depends on the traits of the function that is to be minimized. Also as the function becomes more and more selective its speed convergence reduces.

The BB-BC method gives the accurate global optimum point for the sphere and step functions with the largest numbers of iteration allowed.

2.7 Reduced order modeling of linear time invariant systems using big bang big crunch optimization and time moment matching method

A new approach is designed to estimate the high-order LTI system to its reduced model. The designed model is a combined method approach of order reduction containing the time-moment matching method and the latest developed BB-BC optimization algorithm. The designed method is applied to SISO and MIMO systems and also to the time delayed LTI systems. The given approach is verified with multiple numerical examples of high and low-order systems. Also the results are compared with the present approaches of simplified order modelling that shows unique improvisation in the integral square error (ISE) and also some other time domain specifications.

MOR is usually performed in the area control engineering where the attributes of dynamic systems that are examined to simplify the difficulties and restore the input output behaviour as much as can be done. MOR simplifies the systems and reduces the calculative burden in the simulations. Unlike the design of compound H_∞ and μ synthesis based control schemes, it allows control practioners to make easy control laws and making controller cost effective and calculative. In multiple engineering process advanced robust control concepts such as μ synthesis and H_∞ control are largely used. Various MOR approaches have been designed using the multiple concepts to cope with the above mentioned challenges faced while working with big scale dynamical systems.

In many years of research in MOR in linear time invariant system, the designed method conceptualize attributes like the singular value decomposition, dominant pole reduction and Hankel norm based approximation, Krylov subspace method.. One of the most important thing in the study is optimization of integral error criterion between the reduced model and actual plant

The described scheme mixes the concept of BB-BC optimization along with time moment matching in order to obtain the reduced model. In denominator part of the original system the time moment matching scheme is applied that helps in restoring the main characteristics of the original systems to the reduced model. The big bang big crunch is applied to the numerator to obtain the reduced model. In the last, simulation studies finalises the proposed technique in both frequency and time domain analysis.

2.8 Multivariable Systems Model Reduction Based on the Dominant Modes and GA

The aim here is to construct a model order reduction algorithmic program that is supported by the MIMO MOR. It depends entirely on the entire order model leading modes preservation. It essentially involves the conversion of all the knowledge that's contained within the original complete order system to the reduced order approximant which allows the development of our approximant denominator. The approximated numerator is calculated by sq. error criterion and genetic rule tools.

The optimum approximant of lower order comes as a result. To boost some vital of its attributes, to check the performances, and to conclude to its effectiveness, a relative study is administrated. Some mathematical examples area unit delineated , where the original model is compared to reduced order models calculated from two latest and vital techniques based on modified pole clustering and on GA tools, especially the stability equation.

Optimization technique essentially deals with finding the simplest answer amongst the multiple possible solutions out there with minimum price . the thought is taken additional to approximate the system theory and to regulate, wherever heaps of labor is to be administrated. One among the optimization tools is genetic rule denoted by GA. Search actions are applied which are supported by the survival of the fittest and on the mechanics of natural process

On the basis of retaining the most significant key characteristics of the entire original order system a new simplified order optimal system is built in a low-order system that gives us permission to simulate and study the original system behaviour with simplicity, accuracy, less computation effort, and minimum cost. The lower-order denominator is constructed in this way. The reduced order approximated denominator can be derived from a combination of square error criterion and GA tools. Our approximant’s quality is compared to those that are stated by stability equation that is the modified pole clustering technique.

For linear continuous time MIMO systems an optimal reduced order approximant was given. It was basically created on the basis of holding the fast modes of the original order system in the reduced order denominator . By the means of minimizing the ISE criterion along with GA tools and the numerator with a simplified order was calculated. To find out the efficacy of the defined model, it was compared to approximants delivered from the improvised pole clustering methods and stability equation. From different measures and different frequency and time responses, this model tells the interesting advantages that simple it is to implement, its ease of use, and the substantial benefit is execution time, which does not require any calculation effort comparing to the other lower order systems. Here, in this method stability is always found along with the best behaviour fitting the original order system.

CHAPTER 3
DYNAMIC SYSTEMS-STATE SPACE VS TRANSFER FUNCTION

3.1 Dynamical Systems

For placing MOR in a mathematical framework, it is essential to understand that various systems consists of a system of ODE's developed in computational science and are also supplemented with boundary conditions. For example, Maxwell's equations in computational fluid dynamics. When the behaviour is described using partial DE, then one realizes that the autonomous variables involved are space and time. Thus an ODE's system is acquired in time after the semi-discrediting in the space. Therefore the discussion is not taken further and is limited to ODE's. Now we take in consideration the following finite dimensional explicit dynamical system:

$$\frac{dx}{dt} = \left(h(x,v)\right)$$

$$y=s(x,v)$$

Here, v is the input, y is the output and x is the state variable. Dynamical systems are thus shown as input-output system in the Figure3.1. The number of state variables define the complexity of the system i.e the dimension j of the state space vector x. The like dynamical systems can also be characterized in form of differential algebraic equations.

In such situation, the set of equations in eq.1 is replaced by H(dx/dt,x,v)=0. MOR may also be considered as the process of reduction in the dimension of state space vector and also conserving the essential features of input-output relations.

In order words we need to discover a dynamical system of such form:

$$\frac{dx}{dt} = \dot{h}(x,v)$$

$$y=\dot{s}(x,v)$$

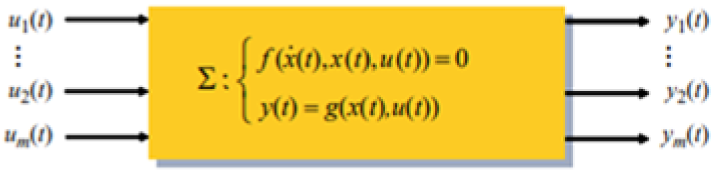

$u_1(t)$
$u_2(t)$
$u_m(t)$
$\Sigma : \begin{cases} f(\dot{x}(t), x(t), u(t)) = 0 \\ y(t) = g(x(t), u(t)) \end{cases}$
$y_1(t)$
$y_2(t)$
$y_m(t)$

FIGURE.3.1 Input-Output system

Where the dimension of ^x is much lesser than j. A number of conditions need to be justified in order to get better approximation of the original input-output system.
Minimum approximation error.
Preservation of essential properties like stability and passivity.
The reduction procedure to be followed should be computationally efficient.

3.2 Modeling of Dynamical Systems

.

Modeling and analyzing of dynamic systems can be done during the study of control dynamics. A set of equations that presents the systems dynamic's accurately or atleast close enough then it is defines as mathematical model of dynamic systems. It is not so that mathematical systems are unique for a given system. There are quite many ways to represent a dynamic system and therefore the system can have varying SUCH model depending on the person's perspective. The dynamics of various systems, call it be electrical, mechanical, thermal, biological can be described in form of differential equations. These differential equations can be formed with the help of physical laws used for that particular system, for example kirchhof's law for electrical systems. The deriving of mathematical model is considered as the most essential part in the process of analysis.

Mathematical models. They may have numerous different forms. Due to particular system and related circumstances the suited mathematical model for the given system is selected for the process. For example, in frequency or transient response analysis, SISO systems, linear systems, the transfer function approach is considered more convenient than any other approach. On the other side, in optimal control problems, it is beneficial to make use of state space formulation.

3.3 Transfer Function

In control theory, the functions are used to describe the input-output relations of systems or components that can be defined as LTI, differential equations is called transfer function. The transfer function of a LTI differential equation system can be called as ratio of Laplace transform of the output function to that of the input function considering that all the initial conditions are zero.

The transfer function can be given as:

Transfer function = G(s) = $\frac{L[output]}{L[input]}$ (with all initial conditions to be zero)

$$= \frac{X(s)}{Y(S)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_2 s^2 + b_1 s + b_0}$$

With the help of transfer function, system dynamics can be represented by algebraic equations in s-domain. If the largest of s represented in the denominator of the TF, is find equal to n then the system is said to be the nth order system.

The concept of the transfer function is applicable for the TF that are LTI differential equation systems. This approach is use for the design and analysis of such type of systems. Some important things concerned with the transfer function are:

- The TF of the system can be described as a mathematical model in which it is an operational method of representing the DE which forms a relation between the output and the input variable.
- TF is the characteristic of the system which is independent of the nature and magnitude of the input or driving function.
- TF includes the units which are necessary to form a relation between the input and the output. It does not provide any information related to the physical structure of the system.

- If the TF is known the input and output responses are known for the multiple forms of inputs with the view of understanding the system’s nature.
- If the TF is not known, it can established by introducing the inputs and analyzing the output of the system. If it is established once, the TF provides the full information of the dynamic characteristics of the system that are distinct from its physical description.

3.4 State Space

In state space analysis, we are interested in three types of variables which are needed in modeling a dynamic system. These variables are input variables, output variables and state variables. The state space representation of a given system is not different, except that the total number of state variables is same for any dissimilar state-space representation of same system.

The dynamic system should consist of elements that remember the values of the input. The output of such type of integrators can be reflected as the variables which decribes the internal state of the dynamic system, since the integrators in a continuous-time control system provides as the memory devices. Therefore the output of the integrators acts as state variables. These variables totally describes the dynamics of the system which is same as the number of the integrators that are a part of the system.

Let us consider the MIMO system that involves n integrators. Suppose that r number of inputs $u_1(t)$, $u_2(t)$, ..., $u_r(t)$and m number of outputs $y_1(t)$, $y_2(t)$, ...,$y_m(t)$are present. Define the n number of outputs of the integrators as the state variables: $x_1(t)$, $x_2(t)$,...., $x_n(t)$. So the system can be defined

$$
\begin{aligned}
\dot{x}_1(t) &= f_1(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\
\dot{x}_2(t) &= f_2(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\
&\;\cdot \\
&\;\cdot \\
&\;\cdot \\
\dot{x}_n(t) &= f_n(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t)
\end{aligned}
$$

The system outputs can be given as:

$$
\begin{aligned}
y_1(t) &= g_1(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t)\\
y_2(t) &= g_2(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t)\\
&\cdot\\
&\cdot\\
&\cdot\\
y_m(t) &= g_m(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t)
\end{aligned}
$$

Also if we define

$$
\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \cdot \\ \cdot \\ \cdot \\ x_n(t) \end{bmatrix}, \qquad
\mathbf{f}(\mathbf{x}, \mathbf{u}, t) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\ f_2(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\ \cdot \\ \cdot \\ \cdot \\ f_n(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \end{bmatrix}
$$

$$
\mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \cdot \\ \cdot \\ \cdot \\ y_m(t) \end{bmatrix}, \qquad
\mathbf{g}(\mathbf{x}, \mathbf{u}, t) = \begin{bmatrix} g_1(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\ g_2(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \\ \cdot \\ \cdot \\ \cdot \\ g_m(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_r; t) \end{bmatrix}, \qquad
\mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \cdot \\ \cdot \\ \cdot \\ u_r(t) \end{bmatrix}
$$

The equation becomes

$$
\begin{aligned}
\dot{x}(t) &= f(x,u,t)\\
y(t) &= g(x,u,t)
\end{aligned}
$$

where the above given equations are approximately same. If the vector functions f or g includes the time t, then system is called a time-varying system.

If the above equations are linearized, then the following are the linearized state and the output equation

Here ,we have A(t) as the state matrix, B(t) as the input matrix, C(t) as the output matrix and D(t) as the direct transition matrix.

$$\dot{x}(t)=A(t)x(t)+B(t)u(t)$$
$$y(t)=C(t)x(t)+D(t)u(t)$$

The block diagram of the above described equation in shown in fig. 3.2

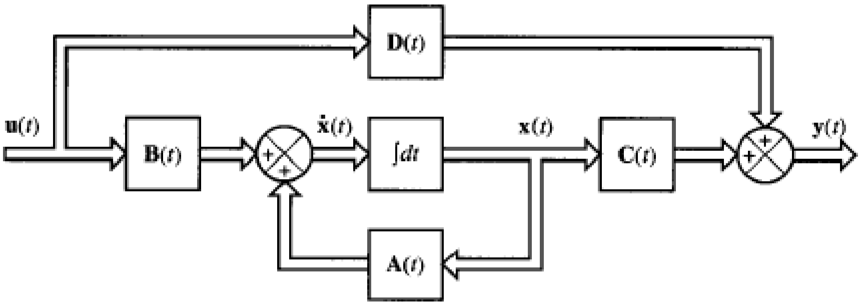


FIGURE 3.2 Block diagram of the linear continuous-time control system represented in the state space

The system is known as the time-varying system, if the vector functions f and g do not include the time t. so the above equations can be simplified as

$$x(t)=Ax(t)+Bu(t)$$
$$y(t)=Cx(t)+Du(t)$$

CHAPTER 4

MODEL ORDER REDUCTION TECHNIQUES

Since the early times, a large number of classical and evolutionary techniques has been proposed and worked upon to analyse a given system. There are multiple methods present in literature for reduction of higher order linear systems.

4.1 Classical approach- Modified Pole Clustering and Coefficient matching

Here, a mixed system order reduction technique is selected in which numerator polynomial coefficients are retrieved by coefficient matching method and denominator polynomial order is simplified by modified pole clustering method. The work represented directs attention to the specifications like rise time, settling time, peak overshoot and steady-state error. It is necessary that both the reduced order system specifications and original system values should have the same steady state values. In this method of reducing order, the numerator polynomials are calculated using coefficient matching method and denominator polynomial are calculated using MPC technique in which clustering of poles is done.

4.1.1 Description of proposed method

A TF of the mth order of a SISO LTI system is formulated and can be given as

$$G_0(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_2 s^2 + b_1 s + b_0}$$

$$= \frac{\sum_{i=0}^{n} a_i s^i}{\sum_{j=0}^{m} b_j s^j} = \frac{N(s)}{D(s)}$$

Where n ≤m and u_i , v_j are the numerator and denominator polynomial coefficients of high order transfer function. At the point when u_0 = v_0 , the steady state value of the framework will progress toward becoming unity. The above condition can be composed in calculated frame as

$$G_0(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0}{(s+\lambda_1)(s+\lambda_2)\ldots\ldots\ldots(s+\lambda_m)}$$

Where λ_1, λ_2........λ_m are the original transfer function poles.

To discover the reduced order transfer function from the above given higher order transfer is the fundamental goal. Suppose the diminished pth order transfer function which is spoken to by

$$G_k(s) = \frac{\alpha_l s^l + \alpha_{l-1} s^{l-1} + \cdots + \alpha_2 s^2 + \alpha_1 s + \alpha_0}{\beta_p s^p + \beta_{p-1} s^{p-1} + \cdots + \beta_2 s^2 + \beta_1 s + \beta_0}$$

$$= \frac{\sum_{i=0}^{l} \alpha_i s^i}{\sum_{j=0}^{p} \beta_j s^j} = \frac{Q(s)}{N(s)}$$

Where $l \leq p$, and α_i ,β_j are considered to be the coefficients of numerator polynomial and denominator polynomial for the reduced order transfer function.

4.1.2 Proposed methodology

To find the coefficients of reduced order numerator and denominator the method can be described as follows:

Step 1: In denominator polynomial P(s) coefficients are calculated utilizing the modified pole clustering method.

Let us consider that the transfer function consist of m number of poles in the original system transfer function. The pole clustering centres should be equal to the order (p) of reduced model. The limitation for the system poles grouped per cluster centre should be at minimum one and maximum number of poles per cluster has zero limitation. In original transfer function the poles which are at origin (s=0) are grouped separately in cluster centre so as to be present in reduced order transfer function also.

In pole clustering algorithm further steps are involved:

(i) Let us consider there are x number of real poles such that $|\lambda_1|<|\lambda_2|......|\lambda_x|$ are grouped together so as to form a pole cluster.

(ii) For j=1, calculate the modified pole cluster centre using $v_j=[\sum_{t=1}^{x}(-1/|\lambda_t|) \div x]^{-1}$.

(iii) check whether j=x. if j=x, terminate the process and pole cluster centres are retrieved as $\lambda_{ek}=v_j$.

(iv) calculate the modified pole cluster centres using $v_j=[(-1/|\lambda_1|+(-1/|v_{j-1}|))\div 2]^{-1}$ for j=j+1.

(v) again check for j=r. If we find that the value of j=r, terminate the procedure and pole cluster centres can be retrieved as $\lambda_{ek}= v_j$.

(vi) until the number of pole cluster centres are same as the order of reduced order TF, repeat the above steps.

After getting the updated pole cluster centres, the reduced order denominator polynomial can be formed as

(a) If there exist all the real poles,

The reduced order system denominator is formed by

$$P(s) = (s-\lambda_{c1})(s-\lambda_{c2})\ldots\ldots(s-\lambda_{cx})$$

Where $\lambda_{c1},\lambda_{c2}\ldots..\lambda_{cx}$ are the modified pole cluster values.

(b) if there exist complex conjugate pairs,

The clustered pole values can be given by applying the described technique for the real and imaginary functions exclusively and the modified cluster centres can be calculated by

$\varphi_{cj} = A_{c\ j} \pm B_{cj} i$ where A_{cj}, B_{cj} are the values or modified pole cluster for real and imaginary functions respectively.

The reduced order system denominator can be formed by

$$P(s) = (s-|\varphi_{c1}|)(s-|\varphi_{c2}|)\ldots..(s-|\varphi_{ck}|)$$

(c) if there exist poles which consist of some real and some complex conjugate,

The MPC centres can be calculated using the using the technique individually for real poles and complex poles as mentioned in (a) and (b).

Step 2: further, the numerator Q(s) coefficients can be determined by using coefficient matching technique. In coefficient matching algorithm further steps are involved:

(i) The reduced order model is equated to the original order system TF

$$\frac{N(s)}{D(s)} = \frac{Q(s)}{P(s)}$$

(ii) Put the values of reduced order denominator polynomial P(s) in calculated in step 1 in the general transfer function form of reduced model.

(iii) After substituting the values cross multiply the both sides of equation (4) and the powers of L.H.S and R.H.S are to be compared.

(iv) The steps described in (i)-(iii) gives the numerator polynomial. The numerator polynomial P(s) can be attained by equating the lowest and highest powers of s on L.H.S and R.H.S

.

4.2 Evolutionary technique – Particle Swarm Optimization

With the developing need of demonstrating exceptionally compound physical models, more estimation quality is required in the zone of simulation. Despite the fact that equipment calculation forces of the hardware can be expanded however this procedure will end at a specific farthest point as characterized by Moore'slaw. Optimization algorithm is another technique to take care of the issue and rearrange the compound models and work with its improved form with bearable error.

The lessening strategy to take care of the issue of model order reduction for higher order TF is a PSO based arrangement. The fundamental benefits of this scheme are that it is generic, with relatively lesser time and sensible precision. Likewise, it moderates the frequency and time response of the first order TF. PSO algorithm was created by Russell Eberhart and James Kennedy in 1995, for the displaying and reenactment of schooling pattern of fish or the exercises of flocking of birds. The fundamental thought process is to find out numerous arrangements, and displaying them as particles or birds, and to give every one of the particles a chance to locate their own specific manner relying upon its prior choice, and furthermore to locate the global variable which is the decision of all flying creatures.

It was initially produced for computer software simulation of flocking birds around the food sources and later it was acknowledged how productively the calculation can be utilized for optimization problems. To comprehend the calculation, let us assume the situation of a gathering of birds which are self-assertively looking for food in territory. The birds do think about where the food is yet in each cycle they become more acquainted with how far the food is. In this way, the best system to discover the food is to take after the flying creature which is nearest to the food. In this way, the bird who is closest to the nourishment twitters the loudest a similar way every one of the flying creatures swings around. Additionally, if some other bird comes closer to the objective that the first, it twitters louder and others turns over towards it. This example goes ahead till one of the fledgling finds the food. This calculation is simple and easy to actualize.

PSO learns from the above described scenario and it is used to solve the optimization problems. In PSO, every single solution is a considered as a bird in search space which is called as a particle. There is a fitness value associated with every particle which is analysed by the fitness function which is to be optimized. They have velocities that directs the flying of the particles.

PSO initiates with a gathering of arbitrary particles otherwise called arrangements. It at that point looks through the optima by refreshing the ages. Each molecule is refreshed by the two best qualities in every emphasis. The first is known as the Pbest which is the best arrangement that every molecule has achieved till now by any molecule in the populace. Another best esteem is the global best or the Gbest which is followed by the particle swarm optimizer. It is the best esteem accomplished till now by any molecule in the populace. Each molecule contains the information communicating the conceivable arrangement, an individual best otherwise called Pbest esteem speaking to how closest the information of the molecule has ever gone to the objective and the speed esteem speaking to how much the information can be changed.

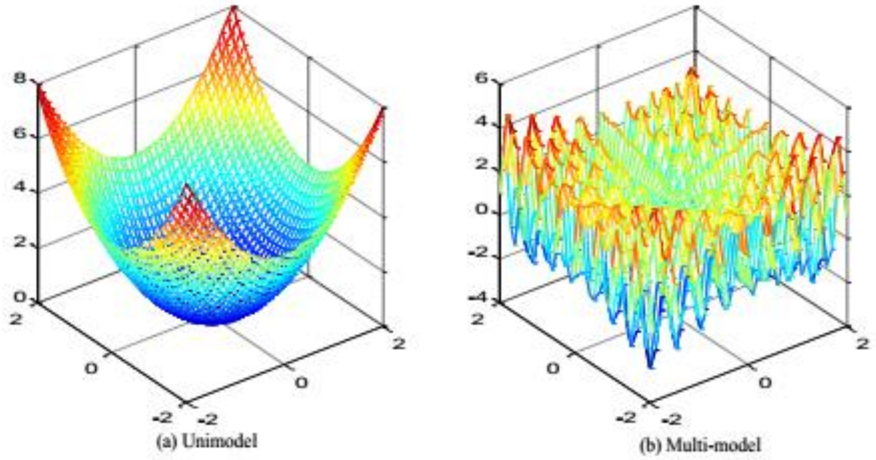
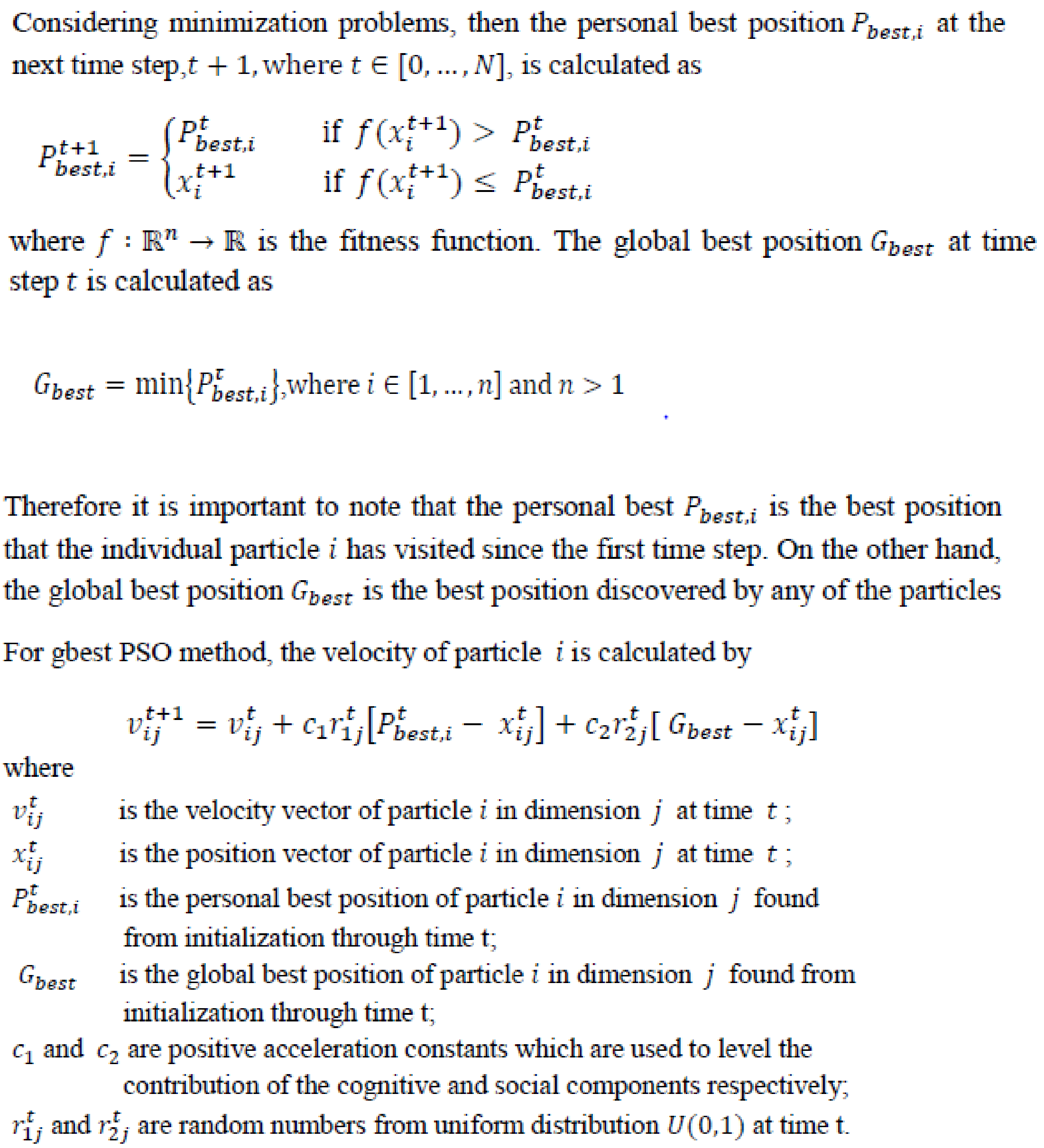


FIGURE 4.1 PSO model

The information of the molecule could be anything. In the above case of flocking birds, the information could be the X,Y,Z coordinated of a solitary bird. The particular directions of each bird would attempt run closer to the directions of the food which is closer to the coordinates of the bird which is the Gbest esteem. On the off chance that the information is a grouping or an example then each and every bit of information would be changed till the example coordinates the objective pattern.

The estimation of the velocity is figured subsequent to taking care of how far the information of the individual is from the objective. The more remote the molecule's information is, more noteworthy is the estimation of the speed. In the illustration portrayed over, the person which is most distant from the food would try to remain with others and would do it by flying speedier towards the Gbest bird. On the off chance that the information is an arrangement or an example, at that point how extraordinary the example is from the objective and the amount it must be changed to coordinate the objective would be depicted by the speed. The molecule's Pbest esteem shows how shut the information has ever gone to the objective since the calculation was instated.

Algorithm for finding the global best value:



Considering minimization problems, then the personal best position $P_{best,i}$ at the next time step,$t+1$, where $t \in [0, \dots, N]$, is calculated as

$$P_{best,i}^{t+1} = \begin{cases} P_{best,i}^{t} & \text{if } f(x_i^{t+1}) > P_{best,i}^{t} \\ x_i^{t+1} & \text{if } f(x_i^{t+1}) \leq P_{best,i}^{t} \end{cases}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is the fitness function. The global best position G_{best} at time step t is calculated as

$$G_{best} = \min\{P_{best,i}^{t}\}, \text{where } i \in [1, \dots, n] \text{ and } n > 1$$

Therefore it is important to note that the personal best $P_{best,i}$ is the best position that the individual particle i has visited since the first time step. On the other hand, the global best position G_{best} is the best position discovered by any of the particles

For gbest PSO method, the velocity of particle i is calculated by

$$v_{ij}^{t+1} = v_{ij}^{t} + c_1 r_{1j}^{t}[P_{best,i}^{t} - x_{ij}^{t}] + c_2 r_{2j}^{t}[\, G_{best} - x_{ij}^{t}]$$

where

v_{ij}^{t} is the velocity vector of particle i in dimension j at time t ;

x_{ij}^{t} is the position vector of particle i in dimension j at time t ;

$P_{best,i}^{t}$ is the personal best position of particle i in dimension j found from initialization through time t;

G_{best} is the global best position of particle i in dimension j found from initialization through time t;

c_1 and c_2 are positive acceleration constants which are used to level the contribution of the cognitive and social components respectively;

r_{1j}^{t} and r_{2j}^{t} are random numbers from uniform distribution $U(0,1)$ at time t.

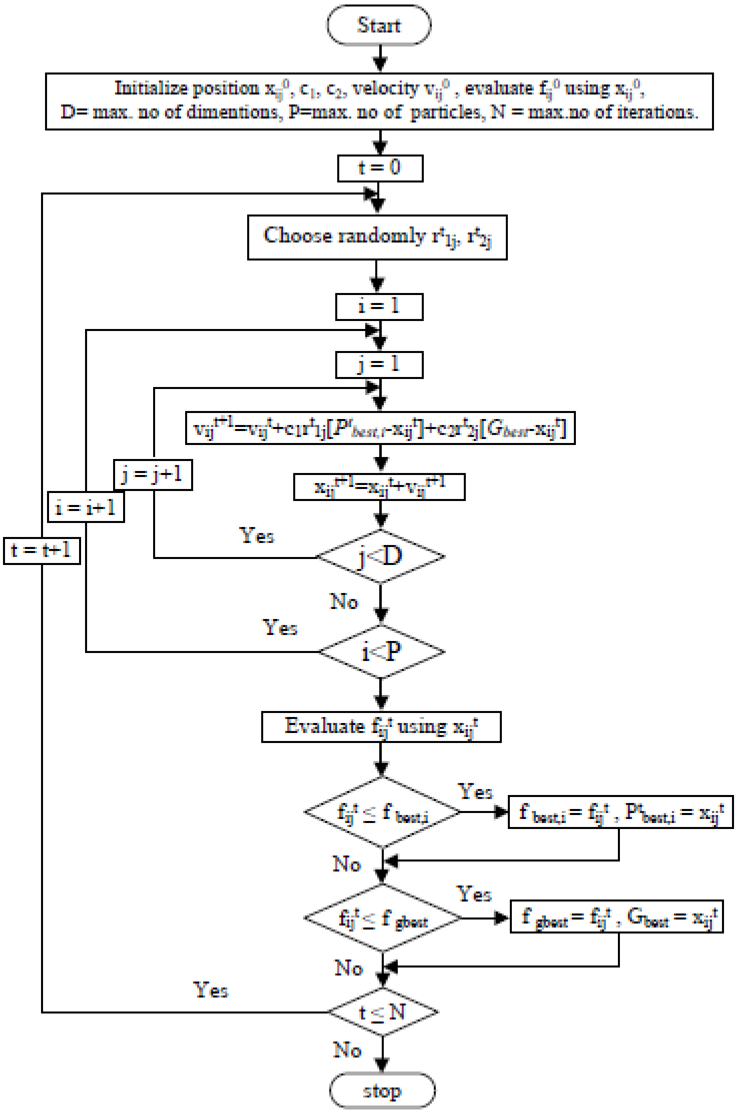

Start
Initialize position x_{ij}^0, c_1, c_2, velocity v_{ij}^0 , evaluate f_j^0 using x_{ij}^0,
D= max. no of dimentions, P=max. no of particles, N = max.no of iterations.
t = 0
Choose randomly r^t_{1j}, r^t_{2j}
i = 1
j = 1
$v_{ij}^{t+1}=v_{ij}^t+c_1r^t_{1j}[P^t_{best,i}-x_{ij}^t]+c_2r^t_{2j}[G_{best}-x_{ij}^t]$
$x_{ij}^{t+1}=x_{ij}^t+v_{ij}^{t+1}$
j = j+1
i = i+1
t = t+1
Yes
j<D
No
Yes
i<P
Evaluate f_{ij}^t using x_{ij}^t
$f_{ij}^t \leq f_{best,i}$
Yes
$f_{best,i} = f_{ij}^t$, $P^t_{best,i} = x_{ij}^t$
No
$f_{ij}^t \leq f_{gbest}$
Yes
$f_{gbest} = f_{ij}^t$, $G_{best} = x_{ij}^t$
No
Yes
$t \leq N$
No
stop

FIGURE.4.2 Flowchart for PSO Gbest algorithm

After the main introduction of the best qualities, the molecule adjusts the position and velocity at every cycle by the beneath portrayed condition:

*vel[i]=wgt*vel[i]+e2*lran()*(Pbest[i]-present[i])+e1*rand()*(Gbest[i]-prsnt[i] (1)*

Present[i+1]=present[i] + v [i]

Here,

vel[i]=current particle’s velocity.

prsnt[i]=value of the current particle.

Wgt=weight factor at inertia.

e1,e2 =social and congnitive accelerations.

ran() = random numbers that are evenly distributed in the range (0, 1).

.

Pbest= = the best value that is attained by the current particle till now.

Gbest= Global best of all the group.

Flow Chart of PSO algorithm is demonstrated in figure below:

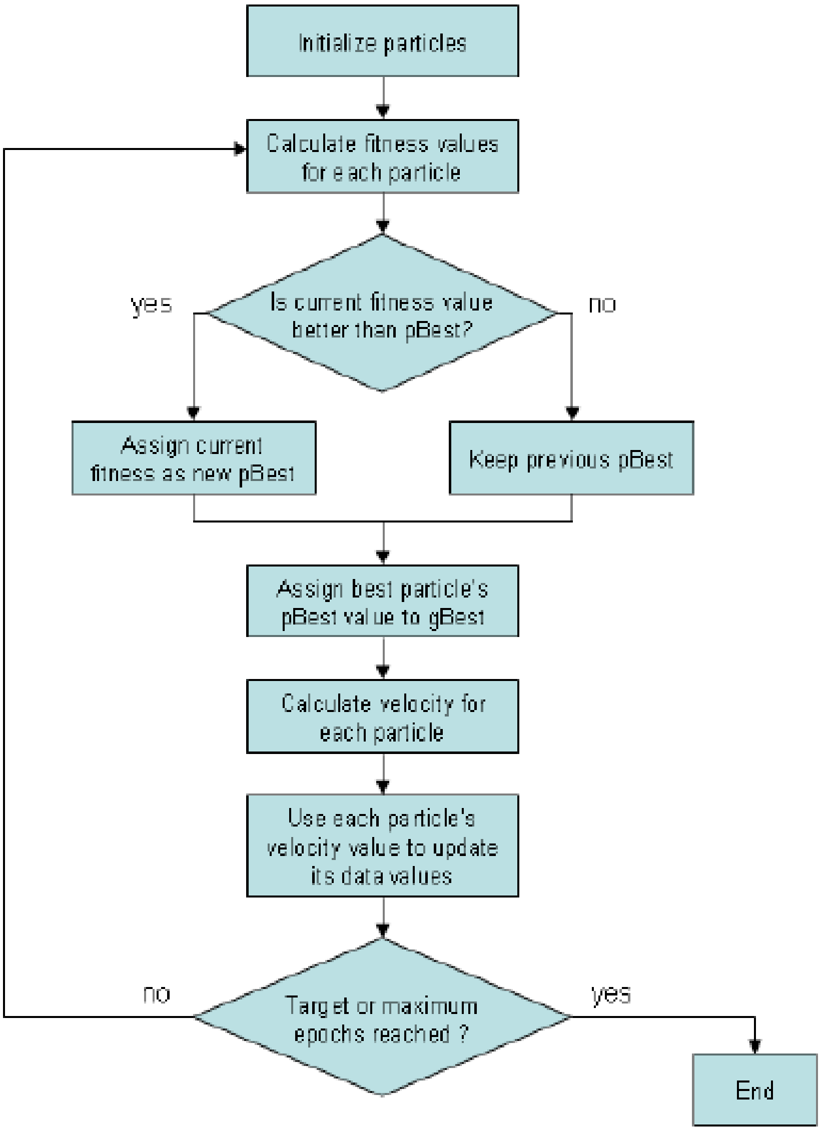

Initialize particles
Calculate fitness values for each particle
yes
Is current fitness value better than pBest?
no
Assign current fitness as new pBest
Keep previous pBest
Assign best particle's pBest value to gBest
Calculate velocity for each particle
Use each particle's velocity value to update its data values
no
Target or maximum epochs reached ?
yes
End

FIGURE 4.3 PSO algorithm

The motive of this algorithm is to simplify the fitness function i.e. MSE between the full order and reduced order system.

The major limitation of this algorithm is the parameter tuning which is different for every new TF. To make the most benefit from the algorithm it wants an experienced user. The process can be automated by parameter tuning by looping all possible combinations of parameters so as to find the best values that causes the minimum error of them all. The main problem here is the time that is needed for the looping, which is comparatively high as compared to the other algorithms.

The PSO algorithm works with the following steps:

Step 1: To make the order order system parameters.

In this the algorithm is furnished with the first non-reduced model. It is done as such as to look at the lessened order model created by the calculation with the first non-reduced model and choose when to end.

Step 2: To set true level step inputs for the system.

To discover the match between the reduced order and original models, we give same information is given to them to look at their responses. Here, advance info is utilized to the models and their responses are looked at.

Step 3: To choose an appropriate order of the lessened order system.

The suitable order of reduced model must be selected so that the algorithm can select the data width of every particle and also to find the fitness function and simulate the model of this reduced order.

Step 4: To fix the PSO parameter.

Certain parameters such as the numbers of particles, the maximum number of iterations, the social and cognitive acceleration factors, e_1 and e_2 , the randomization range of the particles velocity and data needs to be set.

Step 5: To define a fitness function.

It is a connection between the physical issue close by and the optimization algorithm. Here, the fitness function is portrayed by the mean squared error MSE of the distinction between the step response of the full order and decreased order model.

Supposing, x_1 = step response of the original model and

x_2 = step response of the reduced model, so the MSE can be defined as:

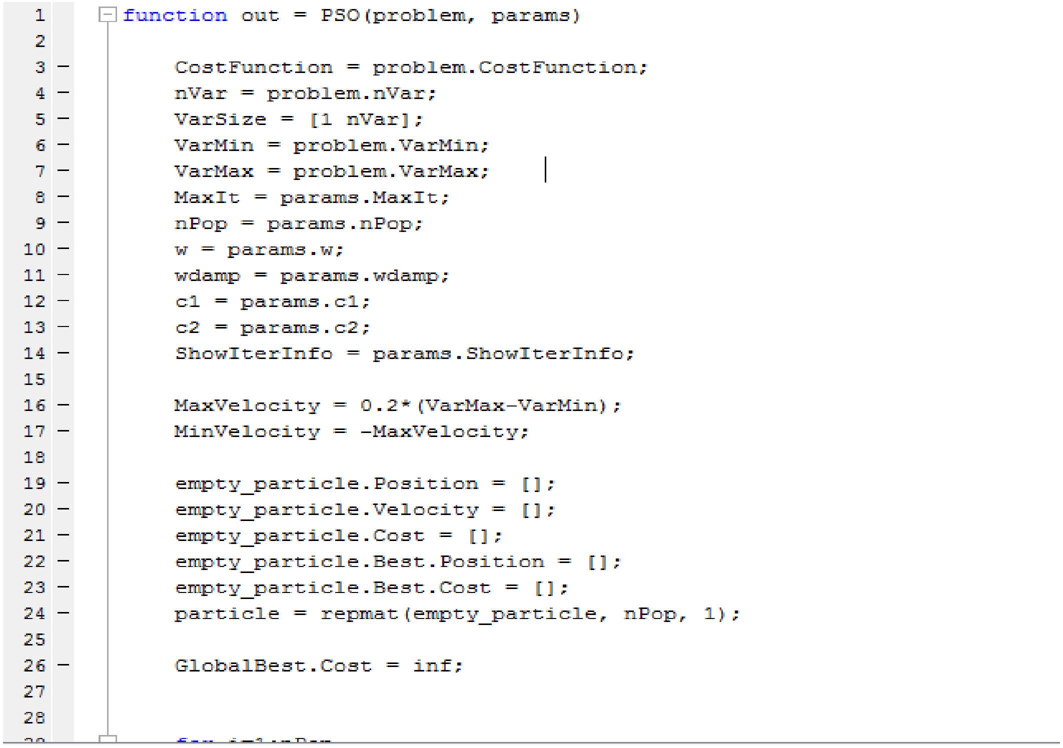
$$MSE = mean\ (x_2\text{-}x_1)^2$$

The motive of this algorithm is to simplify the fitness function i.e. the mean square error between the full order and reduced order models.

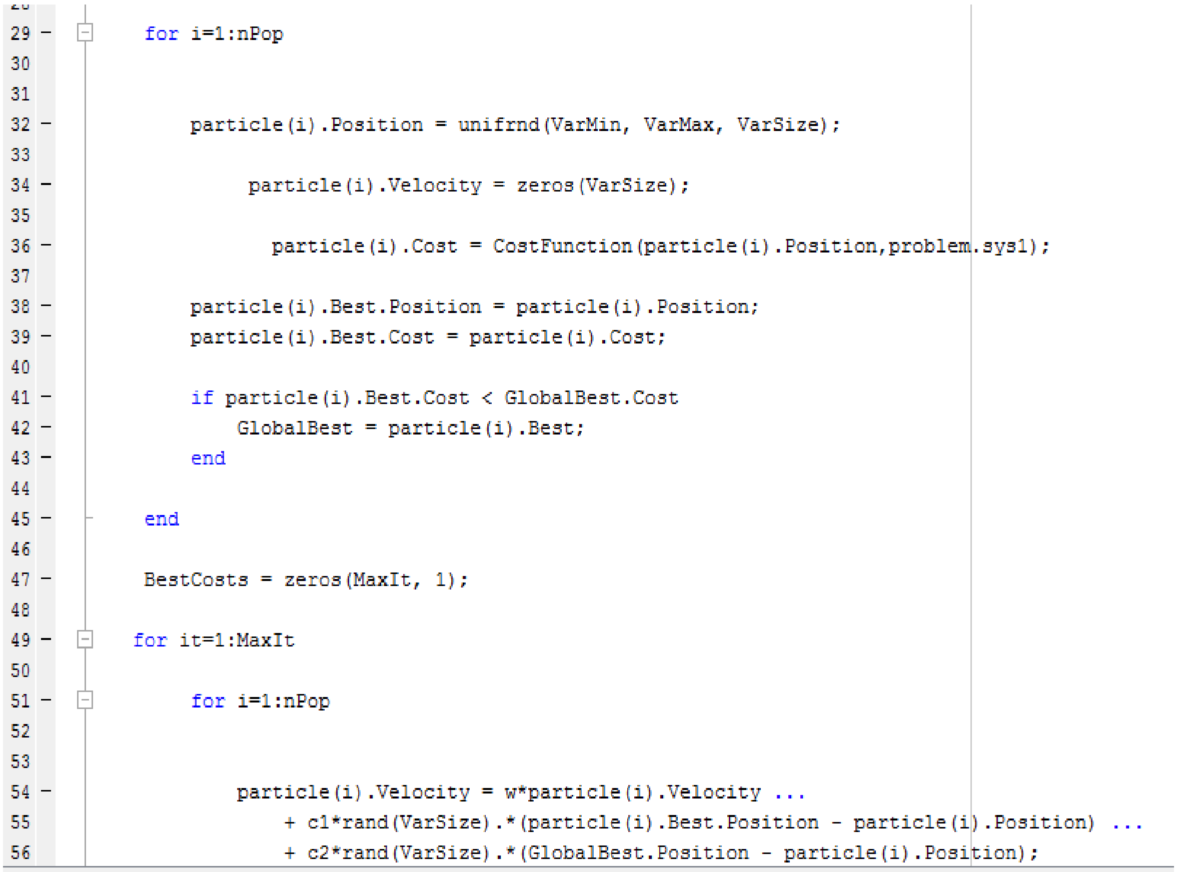
The significant disadvantage of this calculation is the parameter tuning which is distinctive for each new transfer function. To make the most advantage from the calculation it needs an accomplished client. The procedure can be robotized by parameter tuning by circling every single conceivable blend of parameters to locate the best qualities that causes the minimum error of all. The primary issue here is the time that is required for the circling, which is similarly high when contrasted with alternate calculations.

4.3 Codes for PSO in MOR.

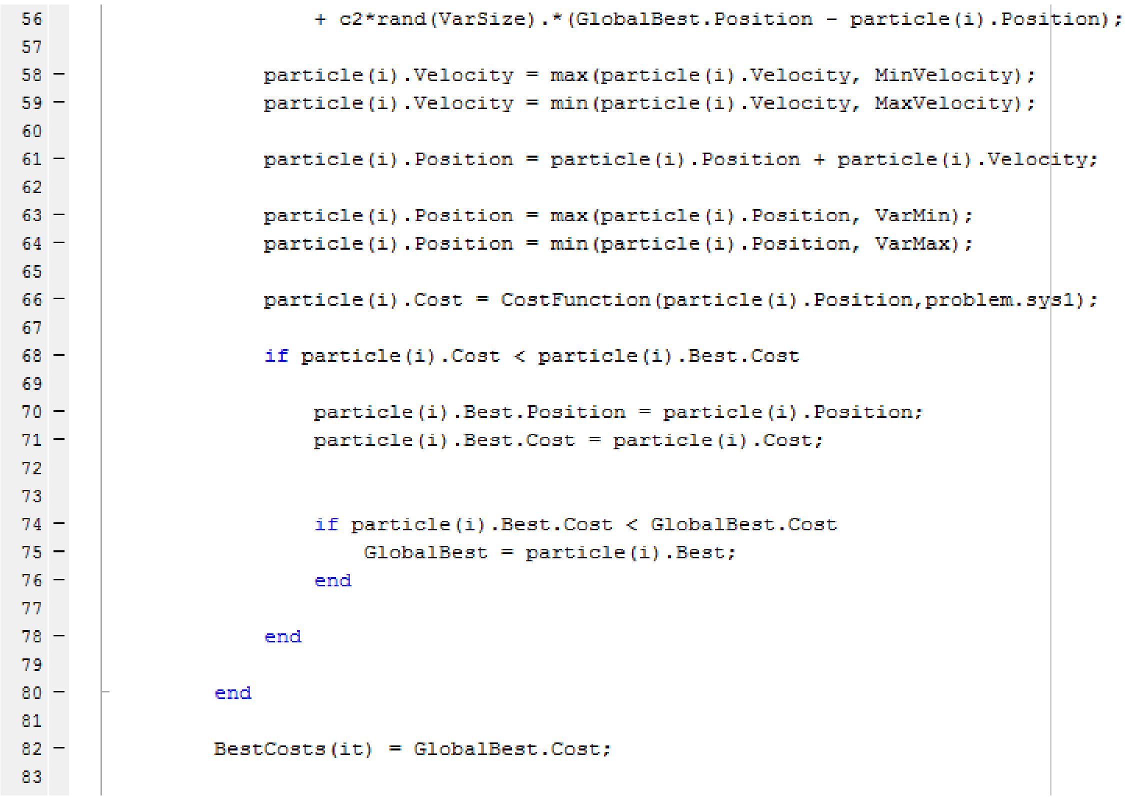
CODE FOR PSO



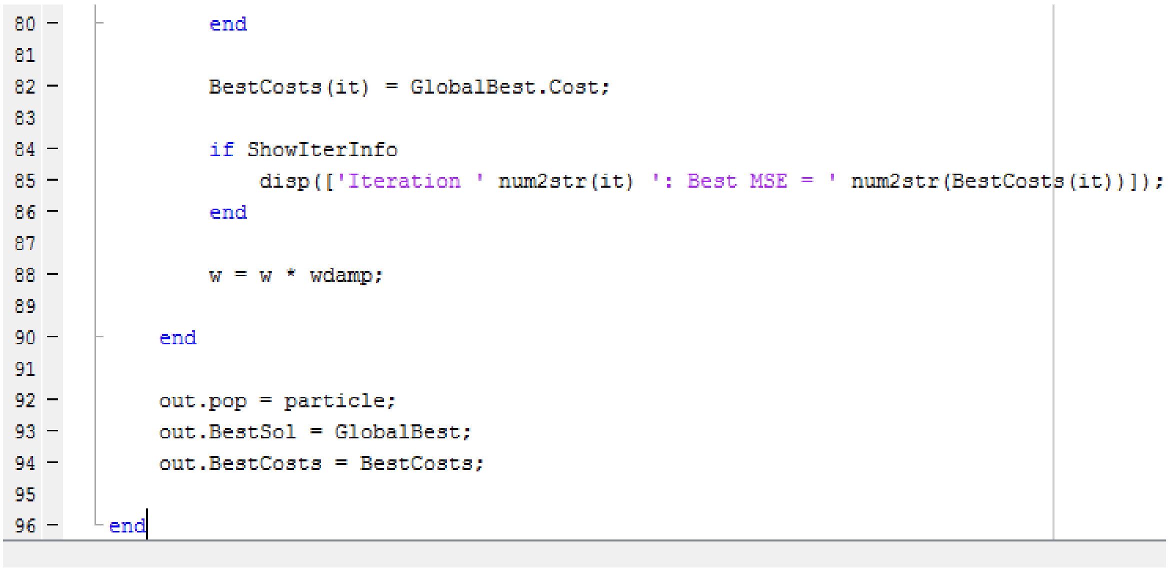
```
function out = PSO(problem, params)

    CostFunction = problem.CostFunction;
    nVar = problem.nVar;
    VarSize = [1 nVar];
    VarMin = problem.VarMin;
    VarMax = problem.VarMax;
    MaxIt = params.MaxIt;
    nPop = params.nPop;
    w = params.w;
    wdamp = params.wdamp;
    c1 = params.c1;
    c2 = params.c2;
    ShowIterInfo = params.ShowIterInfo;

    MaxVelocity = 0.2*(VarMax-VarMin);
    MinVelocity = -MaxVelocity;

    empty_particle.Position = [];
    empty_particle.Velocity = [];
    empty_particle.Cost = [];
    empty_particle.Best.Position = [];
    empty_particle.Best.Cost = [];
    particle = repmat(empty_particle, nPop, 1);

    GlobalBest.Cost = inf;


```



```
 for i=1:nPop


     particle(i).Position = unifrnd(VarMin, VarMax, VarSize);

          particle(i).Velocity = zeros(VarSize);

            particle(i).Cost = CostFunction(particle(i).Position,problem.sys1);

     particle(i).Best.Position = particle(i).Position;
     particle(i).Best.Cost = particle(i).Cost;

     if particle(i).Best.Cost < GlobalBest.Cost
         GlobalBest = particle(i).Best;
     end

 end

 BestCosts = zeros(MaxIt, 1);

for it=1:MaxIt

     for i=1:nPop


         particle(i).Velocity = w*particle(i).Velocity ...
             + c1*rand(VarSize).*(particle(i).Best.Position - particle(i).Position) ...
             + c2*rand(VarSize).*(GlobalBest.Position - particle(i).Position);
```

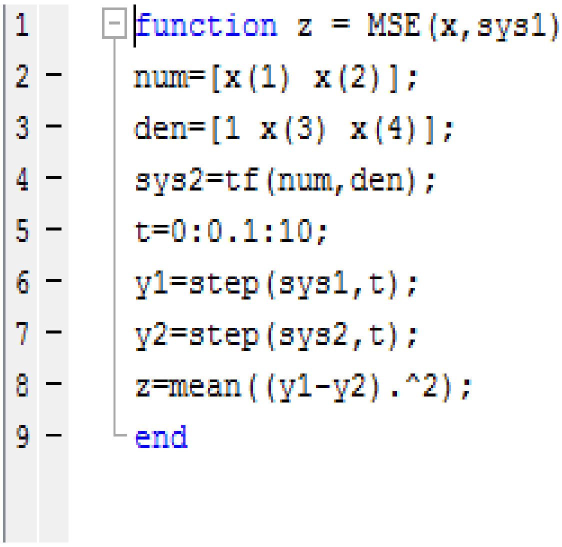


```
             + c2*rand(VarSize).*(GlobalBest.Position - particle(i).Position);

         particle(i).Velocity = max(particle(i).Velocity, MinVelocity);
         particle(i).Velocity = min(particle(i).Velocity, MaxVelocity);

         particle(i).Position = particle(i).Position + particle(i).Velocity;

         particle(i).Position = max(particle(i).Position, VarMin);
         particle(i).Position = min(particle(i).Position, VarMax);

         particle(i).Cost = CostFunction(particle(i).Position,problem.sys1);

         if particle(i).Cost < particle(i).Best.Cost

             particle(i).Best.Position = particle(i).Position;
             particle(i).Best.Cost = particle(i).Cost;


             if particle(i).Best.Cost < GlobalBest.Cost
                 GlobalBest = particle(i).Best;
             end

         end

     end

     BestCosts(it) = GlobalBest.Cost;
```



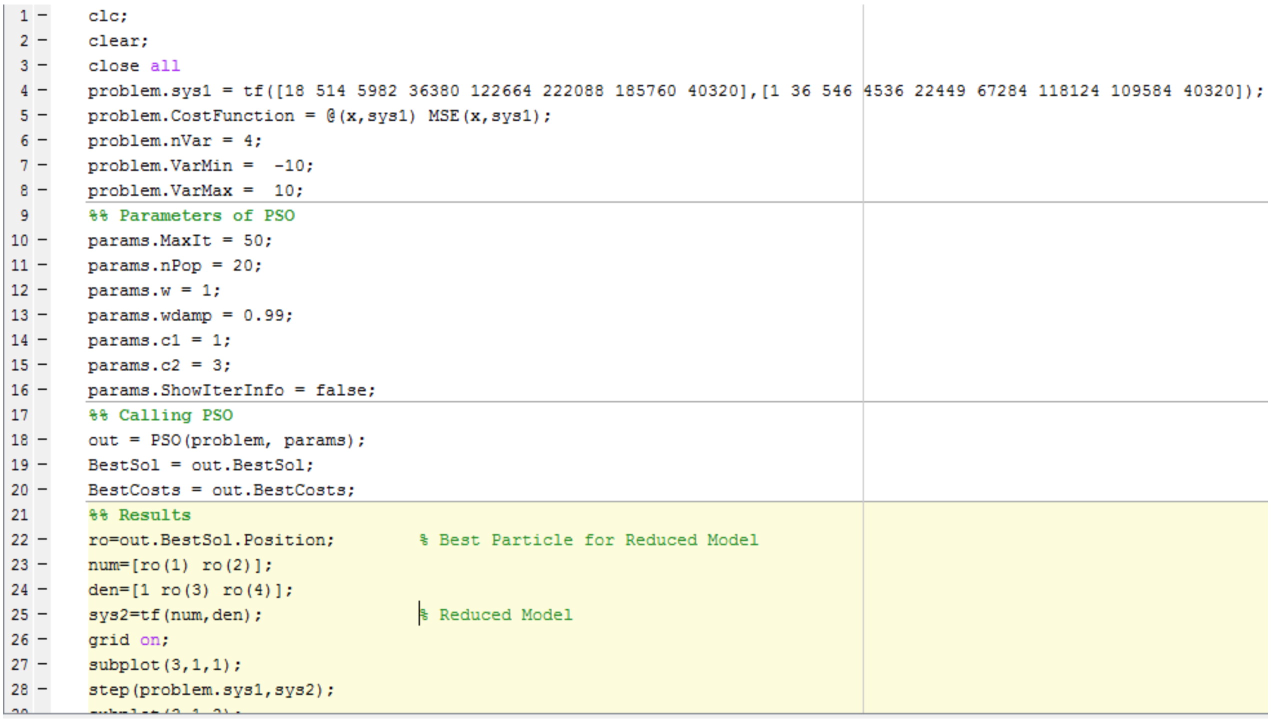
```
        end

        BestCosts(it) = GlobalBest.Cost;

        if ShowIterInfo
            disp(['Iteration ' num2str(it) ': Best MSE = ' num2str(BestCosts(it))]);
        end

        w = w * wdamp;

    end

    out.pop = particle;
    out.BestSol = GlobalBest;
    out.BestCosts = BestCosts;

end
```

CODE FOR MSE :

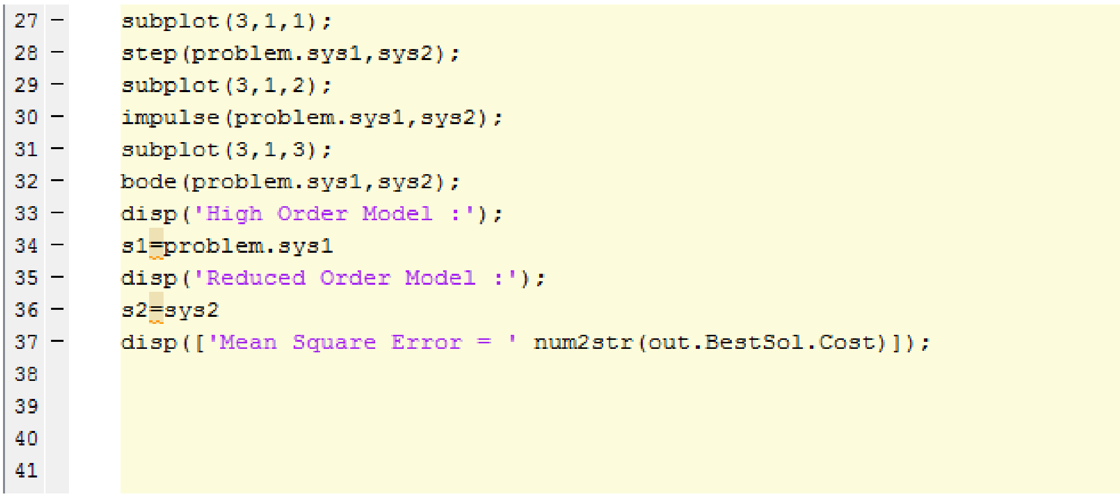


```
function z = MSE(x,sys1)
num=[x(1)  x(2)];
den=[1  x(3)  x(4)];
sys2=tf(num,den);
t=0:0.1:10;
y1=step(sys1,t);
y2=step(sys2,t);
z=mean((y1-y2).^2);
end
```

CODE FOR MOR.



```
clc;
clear;
close all
problem.sys1 = tf([18 514 5982 36380 122664 222088 185760 40320],[1 36 546 4536 22449 67284 118124 109584 40320]);
problem.CostFunction = @(x,sys1) MSE(x,sys1);
problem.nVar = 4;
problem.VarMin =  -10;
problem.VarMax =  10;
%% Parameters of PSO
params.MaxIt = 50;
params.nPop = 20;
params.w = 1;
params.wdamp = 0.99;
params.c1 = 1;
params.c2 = 3;
params.ShowIterInfo = false;
%% Calling PSO
out = PSO(problem, params);
BestSol = out.BestSol;
BestCosts = out.BestCosts;
%% Results
ro=out.BestSol.Position;        % Best Particle for Reduced Model
num=[ro(1) ro(2)];
den=[1 ro(3) ro(4)];
sys2=tf(num,den);               % Reduced Model
grid on;
subplot(3,1,1);
step(problem.sys1,sys2);
```



```
subplot(3,1,1);
step(problem.sys1,sys2);
subplot(3,1,2);
impulse(problem.sys1,sys2);
subplot(3,1,3);
bode(problem.sys1,sys2);
disp('High Order Model :');
s1=problem.sys1
disp('Reduced Order Model :');
s2=sys2
disp(['Mean Square Error = ' num2str(out.BestSol.Cost)]);
```

CHAPTER 5

SIMULATION RESULTS AND DISCUSSIONS

5.1 MODEL ORDER REDUCTION

In fig 5.1, the time domain responses of the unit step input of the reduced order system along with the original order system is shown. Within the figure the response of the original system along with reduced order system is conferred on the basis of MPC technique. Each the reduced and the original order systems contains similar steady-state values, and it shows an in depth approximation to their transient behaviour. In figure 5.2 comparisons of the time-responses of reduced order systems is obtained by using proposed method and different notable schemes are explained. In fig 5.3 the comparison of the reduced order and therefore the original order system is demonstrated on the basis of MPC technique and coefficient matching method,. The performance parameters of the planned scheme along with few different existing schemes are compared within the given Table 5.1 that clearly demonstrates, that the planned scheme helps to resolve the response to its steady state value quicker than the other available methodology.

Let us take into account the transfer function:

$$G(s) = \frac{s^3+7s^2+24s+24}{s^4+10s^3+35s^2+50s+24}$$

Reduced order model is calculated using the MPC technique.

$$G_r(s) = \frac{0.8s+2.0004}{s^2+3.0004s+2.0004}$$

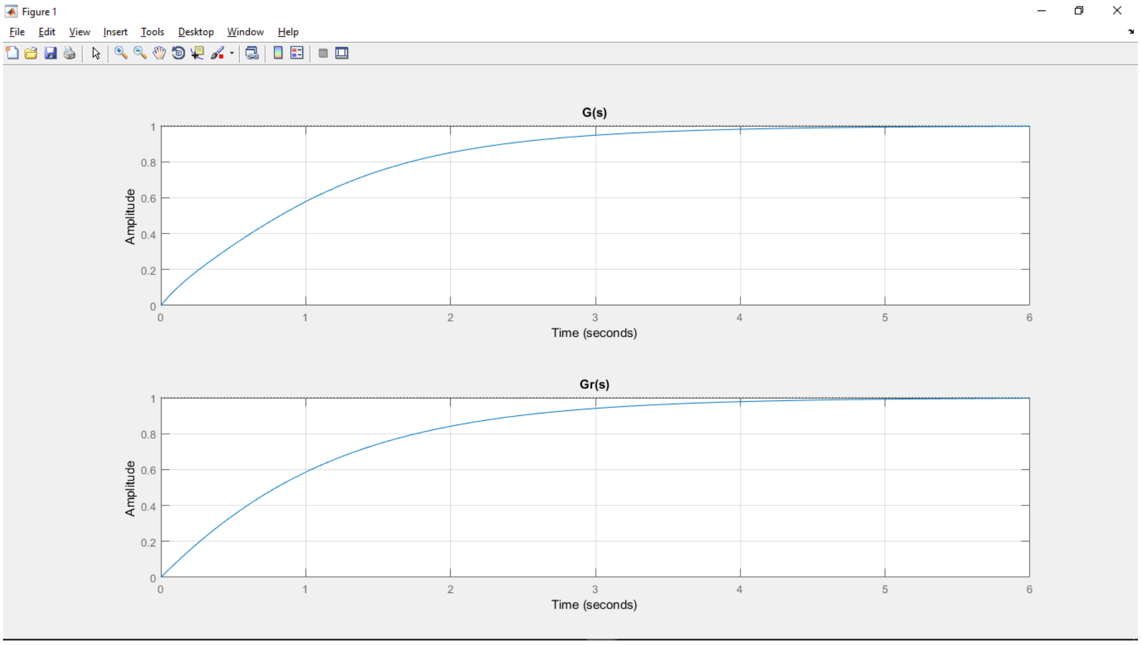


Fig. 5.1 Step Response

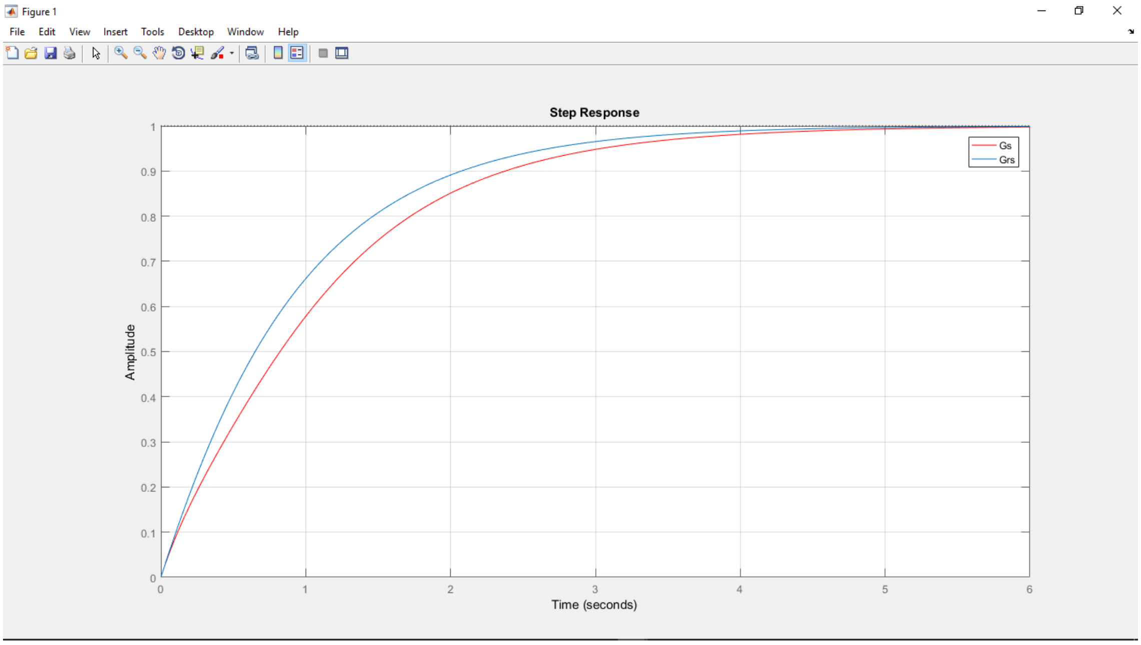


Fig. 5.2 Comparison of the Step Responses

In the same way,

By using the modified pole clustering and the coefficient matching method of MOR, the reduced order model TF for the original TF can be given by,

$$G_r(s) = \frac{s + 3.654}{s^2 + 4.342s + 3.654}$$

In fig 4.3, Comparison of the step responses of the TFs is given. It is clearly visible that the difference between the two responses is almost negligible.

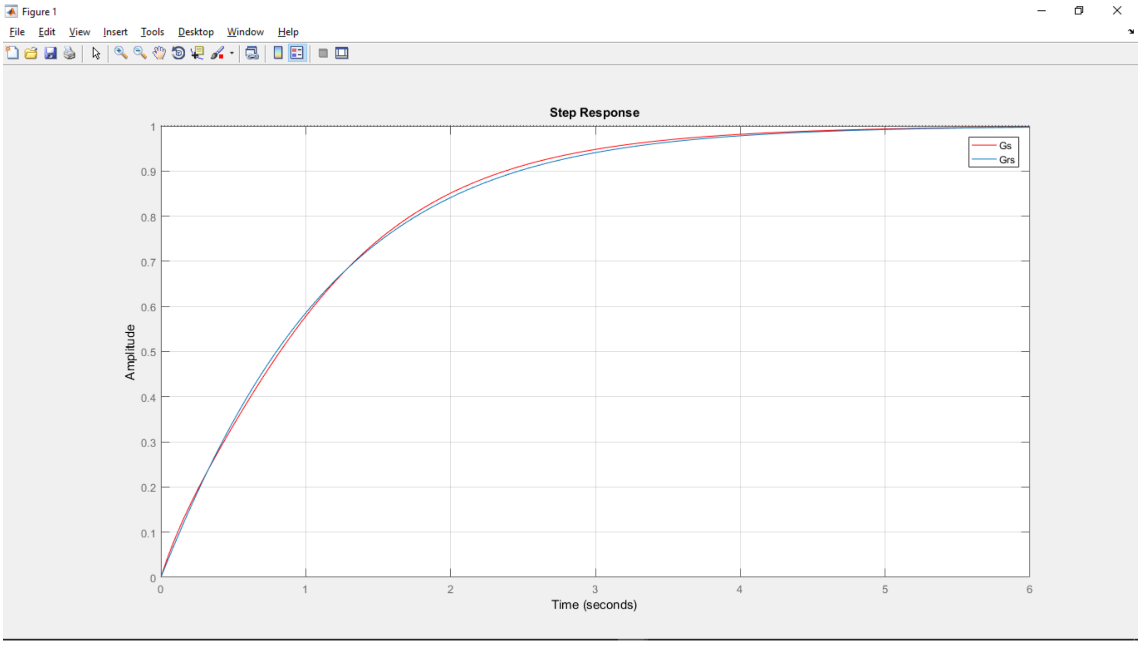
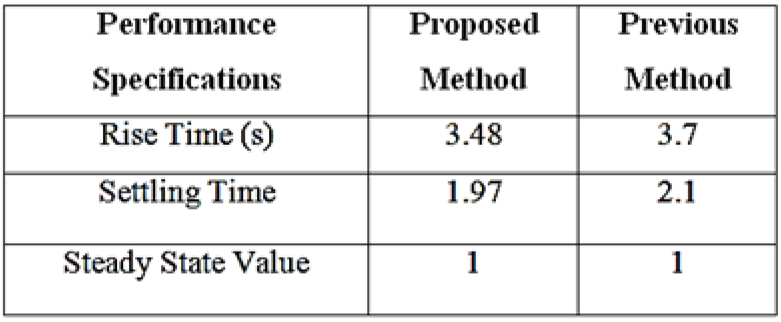


Fig 5.3: Comparison between two step responses

Comparison of the performance specifications for the proposed method and other MOR techniques is given in the table below:



Performance Specifications	Proposed Method	Previous Method
Rise Time (s)	3.48	3.7
Settling Time	1.97	2.1
Steady State Value	1	1

TABLE 5.1 Comparison of performance specifications

5.2 PARTICLE SWARM OPTIMIZATION

Model order reduction is transformation of higher order model to lower order model along with the best cost (i.e. computer and time resources) and least error .Work presented here includes the SISO TF of the higher order system and our aim is to find other TF of lesser order which represents the same system but with minimum error. These results illustrate the use of PSO in MOR. Here the result obtained by PSO is the coefficients of the reduced order TF. So for example, we have considered 8th order transfer function.

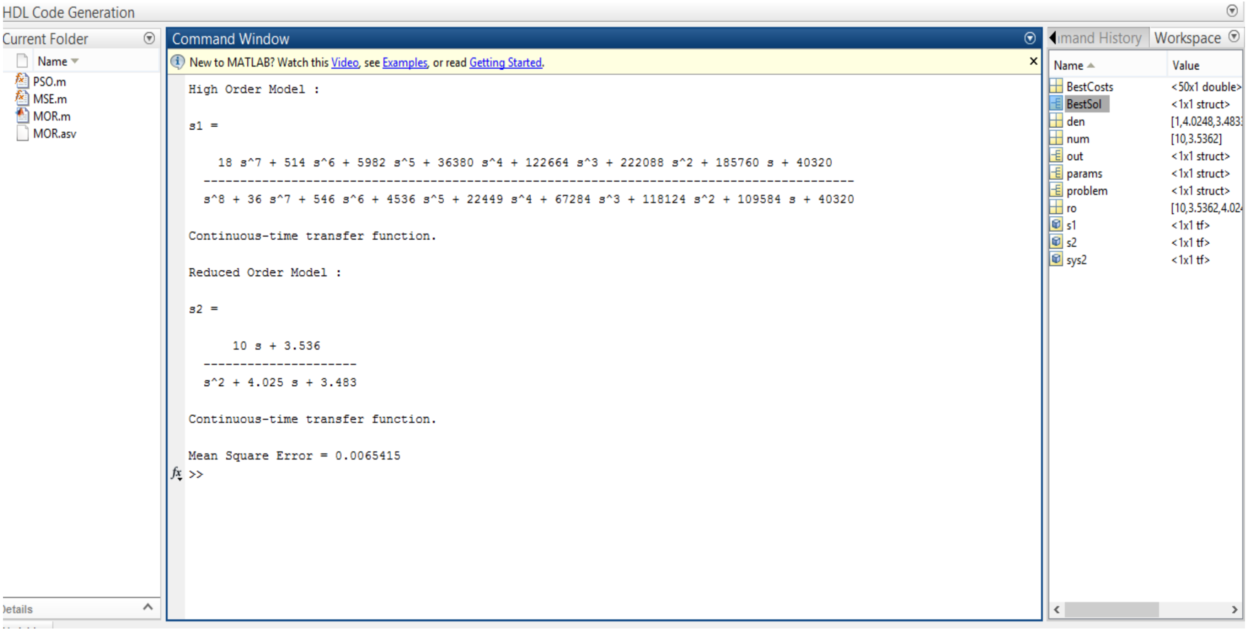
$$G(s)=\frac{18s^7+514s^6+5982s^5+36380s^4+122664s^3+222088s^2+185760s+40320}{s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3+118124s^2+109584s+40320}$$

Now, our aim is to find another TF of 2nd order. So the particles data will be represented by the coefficients of the new transfer function that the method will iterate to find ; for example , the new system is taken as:

$$G_r(s)=\frac{x_1s+x_2}{s^2+x_3s+x_4}$$

Each and every particle presented here will be a vector of four elements i.e. x1,x2,x3 and x4 . The algorithm used here will try to find the coefficients which are presented by each particle’s local best and the global best of all the particles involved.

The PSO algorithm is applied a transfer function of higher order. Then this model is implemented using MATLAB R2013. The figures below show the results of reduction. Fig 5.4 shows the order reduction of higher order model to reduced order model using PSO algorithm in model order reduction and Fig 5.5 shows the comparison among the step response, impulse response and frequency response of both higher order model and reduced order model.



```
High Order Model :

s1 =

    18 s^7 + 514 s^6 + 5982 s^5 + 36380 s^4 + 122664 s^3 + 222088 s^2 + 185760 s + 40320
  -----------------------------------------------------------------------------------------
  s^8 + 36 s^7 + 546 s^6 + 4536 s^5 + 22449 s^4 + 67284 s^3 + 118124 s^2 + 109584 s + 40320

Continuous-time transfer function.

Reduced Order Model :

s2 =

      10 s + 3.536
  ---------------------
  s^2 + 4.025 s + 3.483

Continuous-time transfer function.

Mean Square Error = 0.0065415
fx >>
```

Name	Value
BestCosts	<50x1 double>
BestSol	<1x1 struct>
den	[1,4.0248,3.483
num	[10,3.5362]
out	<1x1 struct>
params	<1x1 struct>
problem	<1x1 struct>
ro	[10,3.5362,4.02
s1	<1x1 tf>
s2	<1x1 tf>
sys2	<1x1 tf>

FIGURE 5.4 Reduced Model

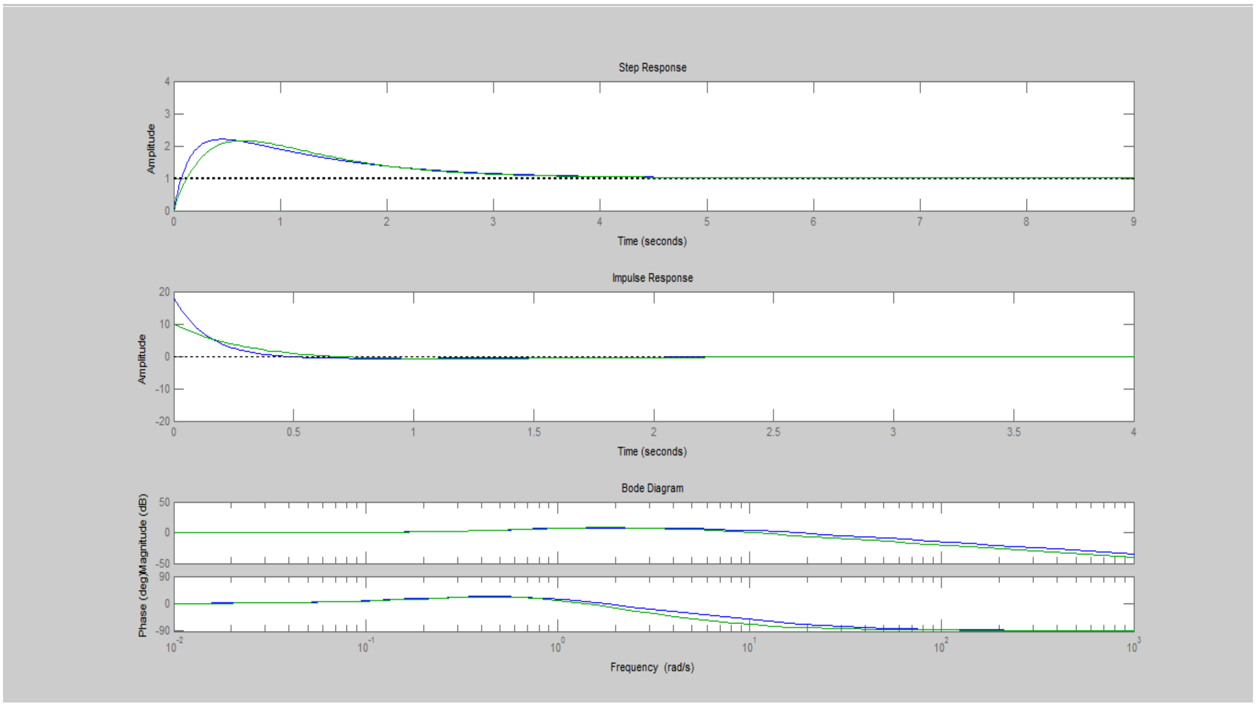


FIGURE 5.5 Step response, Impulse response and Frequency response.

Original function	Numerator = [18,514,5982,36380,122664,222088,185760,40320]
	Denominator =[1,36,546,4536,22449,67284,118124,109584,40320]

Reduced function	Numerator=[10,3.536]
	Denominator=[1,4.025,3.483]

Original order	8
Reduced order	2
MSE	0.0065415

TABLE 5.2

CHAPTER 6

CONCLUSION

The MOR approach supported TF framework is projected. The projected technique is valuable for study and management of higher order dynamic systems. The MPC technique is employed in order to obtain the poles and coefficient matching technique is employed to obtain the zeros of reduced order system. An example of fourth order dynamic system is taken into account for MOR. The step responses of original fourth order system and reduced second order system are conferred. The transient and steady state behavior of reduced order system demonstrates a detailed estimation to the original system. Both the proposed and alternative order reduction techniques are compared. The proposed scheme is beneficial when used in dynamic systems wherever speed of the response is additionally a vital issue which need to considered and it will be explored within the design of control system.

A new generic approach based on PSO is used. The aim is to solve the issues related to MOR for high order TFs. The main benefits of this approach is that it's generic, with comparatively little time and proper accuracy. Though the algorithm functions in time domain, it conserves each time and frequency responses of the original TF.

REFERENCES

Books and Research Papers

[1]. Ramesh Komarasamy et.al "Order Reduction of linear systems with an improved pole clustering" , Journal of Vibration and Control, June 2011

[2]. Dr. HK Verma et.al "A Comparative study of GA, PSO and Big Bang-Big Crunch Optimization Techniques for Optimal Placement of SCV", International Journal of Electronics , Communication and Computer Engineering,Volume 3,Issue 3 , 2012.

[3]. N. Sai Dinesh et. al "Model Order Reduction of Higher Order Discrete Time System.", International Journal of Engineering Research and Technology, Vol.2 Issue 11, November 2013.

[4]. G. Parmar et. al "Relative mapping errors of Linear Time Invariant System caused by particle Swarm Optimized Reduced Order Model" , International Journal of Electrical , Computer ,Energetic, Electronics and Communication Engineering,Vol.1,No.4, 2007

[5]. Maurice clerc et.al "The particle swarm-Explosion, stability, and convergence in a multidimensional complex space.", ", IEEE transactions on Evolutionary Computation,Vol.6, No.1, February 2002.

[6]. Osman K. Erol et.al "A new optimization method: Big Bang-Big Crunch", Advances in Engineering Software 37, 2006

[7]. Shivanagouda Biradar et.al "Reduced order modelling of linear time invariant systems using big bang big crunch optimization and time moment matching method" , Applied Mathematical Modelling, 2016

[8]. Amel Baha Houda Adamou-Mitiche et.al "Multivariable Systems Model Reduction Based on the Dominant Modes and Genetic Algorithm", IEEE transactions on industrial electronics,VOL 64, No.2, February 2017

Websites:

[9]. https://www.mathworks.com/examples/robust/category/model-order-reduction

[10]. https://www.hindawi.com/journals/mse/2009/540895/

[11]. https://www.sciencedirect.com/science/article/pii/S0307904X15004400

[12]. https://en.wikipedia.org/wiki/Particle_swarm_optimization