Meta Heuristic Algorithms for Travelling Salesman Problem

Project report submitted in partial fulfillment of the requirement for the degree of Bachelor of Technology

In

Computer Science and Engineering

By

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Under the supervision of

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To

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Candidate's Declaration

We hereby declare that the work presented in this report entitled **"Meta Heuristic Algorithms for Travelling Salesman Problem"** in partial fulfillment of the requirements for the award of the degree of **Bachelor of Technology** in **Computer Science and Engineering/Information Technology** submitted in the department of Computer Science & Engineering and Information Technology**,** Jaypee University of Information Technology Waknaghat is an authentic record of my own work carried out over a period from August 2017 to December 2017 under the supervision of **Dr. Yugal Kumar**.

The matter embodied in the report has not been submitted for the award of any other degree or diploma.

Aditi Sirkek (141320)

This is to certify that the above statement made by the candidate is true to the best of my knowledge.

Dr. Yugal Kumar

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Dated:

Acknowledgement

I take this opportunity to express my profound gratitude and deep regards to my guide Dr. Yugal Kumar for his exemplary guidance, monitoring and constant encouragement throughout the course of this project. The blessing, help and guidance given by his time to time shall carry us a long way in the journey of life on which we are about to embark.

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List of Abbreviations

S.NO.	ABBREVIAION	DESCRIPTION
1	TSP	Travelling Salesman Problem
2	ACO	Ant colony Optimization

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Abstract

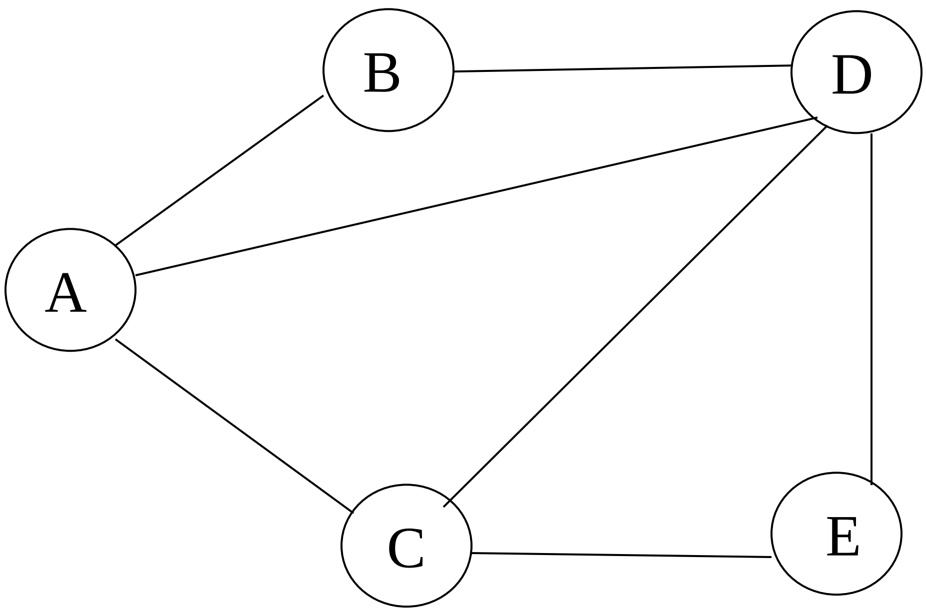
In this work, an efficient technique is proposed to solve the travelling salesman problem (TSP) using Ant Colony Optimization. The main aim of this problem is to search the shortest tour for a salesman to visit all cities exactly once and finally return to the starting city. Ant Colony Optimization algorithm is a heuristic algorithm which has been proven a successful technique and applied to a number of combinatorial optimization problems and is taken as one of the high performance computing methods for Traveling salesman problem (TSP). The Traveling Salesman Problem (TSP) is one of the standard test problems used in performance analysis of discrete optimization algorithms. Till date, large numbers of algorithms are adopted to solve the TSP problem. In this project, a new meta-heuristic algorithm is proposed to solve the TSP problem. The performance of proposed hybrid algorithm will be investigated on ten different benchmark problems taken from literature and compared to the performance of some well-known algorithms.

Chapter-1 INTRODUCTION

1.1 Introduction

Travelling Salesman Problem is a classical NP hard problem. Travelling Salesman Problem is a problem in which a salesman intends to visit a large number of cities exactly once and returning to starting point while minimizing the total distance travelled or the overall cost of the trip. It is a NP-hard problem in combinatorial optimization and also has its importance in operations research and theoretical computer science. TSP is also used as a benchmark for many optimization methods. Although the problem is computationally difficult, many heuristics and algorithms are known which can be used to solve the problem with thousands of cities and even problems with millions of cities can be approximated within a fraction of one percent. The TSP has many applications, in the manufacture of microchips, planning and logistics. TSP also appears as a sub-problem in many areas, such as DNA sequencing. In these applications, the concept city represents DNA fragments, customers, or soldering points and the distance represents travelling time or overall cost of the trip, or a similarity measure between DNA fragments.

TSP can be modeled as an undirected weighted graph, in which the vertices of the graph denote the cities and the paths are the graph's edges, and a path's distance is the edge's weight. It is a minimization problem starting and finishing at a specified vertex after having visited each other vertex exactly once. Often, the model is a complete graph with every vertices connected by an edge. If there is no path between two cities, then an additional arbitrary edge can be used to connect the cities without affecting the optimal path. The symmetric Traveling Salesman Problem can be represented as an undirected graph as the distance between two cities is the same in each opposite direction. This symmetry halves the number of possible solutions. In the asymmetric TSP, paths may not exist in both directions or the distances might be different, forming a directed graph. There can be many scenarios of symmetric TSP changing to asymmetric TSP like traffic collisions, one-way streets etc.



Graph 1

- **AB-1**
- **AC-1**
- **AD-3**
- **BD-2**
- **DE-3**
- **CE-1**
- **CD-2**

Adjacency Matrix:

	A	B	C	D	E
A	0	1	1	3	0
B	1	0	0	2	0
C	1	0	0	2	1
D	3	2	2	0	3
E	0	0	1	3	0

Table-1

In literature, different formulation of TSP problem is considered. Some famous variants of TSP problem are mentioned below:

1. Pickup and delivery TSP with multiple stacks

Pickup and delivery traveling salesman problem with multiple stacks is a variation of well-known traveling salesman problem [14]. In pickup and delivery traveling salesman problem a vehicle has to serve the customer's request of definite pair of pickup and delivery. Every item is picked up from source location and is unloaded at its destination location. The vehicle also contains fixed number of stacks. Further, each stack has a finite capacity and its loading and unloading sequence must follow the last-in-first-out (LIFO) policy that is for each stack just the last item loaded can be unloaded at its corresponding delivery location.

2. Symmetric quadratic TSP

The symmetric quadratic travelling salesman problem (SQTSP) is a variation of the well-known Traveling salesman problem [5]. In SQSP a weight is associated with every three vertices traversed in succession. The main goal of SQTSP is to find a least cost in an edge-weighted graph, in which every edge pair has a wait associated with it. If these weights correspond to the turning angles of the tour then the problem can be modeled into an angular- metric travelling salesman problem.

3. Colored TSP

A colored traveling salesman problem (CTSP) is an extension of the well-known multiple traveling salesman problem [11]. In CTSP, two set of cities are defined i.e. every group has cities of single color and salesman is also allocated a color. The salesman has to visit the cities in group with same color as the salesman. This allows having many salesmen and each of them visiting the cities in group with same color as the salesman a color is allocated to the salesman and every city including many all salesmen's colors depending on the problem types. In CTSP the salesman travels the city with the similar color exactly once.

4. Multi-stripe TSP

Multi-stripe TSP is an extension of the classical traveling salesman problem [8]. The main objective in traveling salesman problem to visit every city exactly once and summing the cost of visiting the city. The q-stripe TSP is a type of multi-stripe TSP in which q can take values anything greater than zero. The main objective of q-stripeaTSP is visit the cities while adding the cost of travelling from one city to each of next q cities in the tour. The resulting q-stripe TSP generalizes the TSP and forms a special case of the quadratic assignment problem.

5. Close enough TSP

Close enough TSP is a variation of the well-known traveling salesman problem [1]. In close enough TSP there are a set of target points in any Euclidean space. The main objective is to determine the minimum length of the tour which starts and ends at a depot while visiting each target point exactly once. In CETSP each target point is associatedawith a neighborhood. The neighborhood is a compact region of the space consisting of the target point. CETSP consist in finding the shortest tour that starts and ends at the depot and intersects each neighborhood once.

6. Consistent TSP

Consistent TSP is a variant of the classical Travelling Salesman Problem. In consistent TSP the objective is to identify the minimum cost route that a vehicle should take during multiple time periods. The route should enable the vehicle to serve all the given set of customers. Every customer may require service in one or multiple time periods and the requirement for consistent service applies at each customer location that requires service in more than one timeaperiod. This requirement corresponds to restricting the difference between the earliest and latest vehicle arrival times, across the multiple periods, to not exceed some given allowable limit.

7. TSP with time windows (TSPTW)

Travelling Salesman problem with Time window is an extension of the classical Travelling Salesman Problem (TSP) [17]. In TSPTW there are set of customers, service time and time window. A salesman has to travel and reach to every customer exactly once. There is due date before which the customer has to be visited. If the salesman reaches a customer before its ready time then, the salesman has to wait. Moreover, there are tours in which the due dates of customers are compromised and such a tour is called infeasible while the other tours in which the due dates of the customers are respected are called feasible tours. The main objective is to reduce the cost of the distance travelled.

8. Black-and-white TSP

The black-and-white travelling salesman problem (BWTSP) is a variation of the well-known Travellingasalesmanaproblem [19]. In BWTSP the vertices representing the cities are partitioned into black and white. Further, cardinality and length constraints are used to find the distance between two consecutive black and white vertices. BWTSP has many applications in telecommunication, logistics etc.

9. Equality generalized TSP

The equality generalized traveling salesman problem (EGTSP) is a variant of the traveling salesman problem (TSP) [21]. In EGTSP the cities are divided into set of clusters. The objective of EGTSP is that salesman has to visit every cluster exactly once and to minimize the overall cost of the tour. EGTSP can be modified into asymmetric TSP and can be solved using a regular TSP solver.

10. Multi objective TSP

The Multi objective Traveling Salesman Problem (moTSP) is an extension of the classic travelingasalesmanaproblem [12]. In moTSP more than one salesman can be used to find the optimal solution. The characteristics of moTSP is more applicable to real life scenarios like vehicle routing problems by adding some additional variables and constraints.

11. Physical TSP

The Physical Travelling Salesman Problem (PTSP) is a variant of the well-known Travelling Salesman Problem (TSP) [32]. PTSP has wide applications in converting TSP into single player real time game. In PTSP the player or the salesman controls a spaceship and has to visit a number of points which are dispersed in a maze in shortest time possible. PTSP is real time game and an action must be taken in every forty milliseconds.

12. TSP with multiple time windows and hotel Selection

Travelling Salesman Problem with Multiple Time Windows and Hotel Selection (TSP-MTWHS) is an extension of Travelling Salesman Problem with Time Windows (TSPTW) [29]. TSP-MTWHS is a recently added extension to the Travelling Salesman Problem with Hotel Selection. The TSP-MTWHS consists of finding the route for a salesman who has to visit different customers present at different locations with different time windows for each customer. The salesman has to travel for several days and need to stay at hotels. The main objective is to reduce the overall cost of the tour which includes the wages, hotel costs, travelling expenses and also a penalty amount for every missed customer.

13. Travelling salesman problem with draft limits

Travelling salesman problem with draft limits (TSPDL) is a recent extension of the well-known traveling salesman problem [4]. TSPDL arises in the context of maritime transportation. In TSPDL the main goal is to find the optimal path in the Hamiltonian cycle for a ship. The ship has to travel and visit a set of ports. The ship has to deliver products at given ports within the draft limit of the port.

14. Asymmetric Traveling Salesman Problem

The asymmetric traveling salesman problem (ATSP) is a well-known part of the classical travelling salesman problem [6]. In asymmetric traveling salesman problem the distance between pair of cities is different and the goal is to visit every vertex exactly once while minimizing the total distance travelled. The main objective is to reduce the overall cost of the trip while visiting each and every node. The asymmetric travelling salesman problem is one of the most studied and widely used variation of the classical travelling salesman problem.

15. Traveling Salesman Problem with Pickups, Deliveries, and Draft Limits

Traveling salesman problem with pickup, delivery and draft limits is a new generalization of the classical traveling salesman problem [10]. This new extension uproots from the applications in maritime logistics. In TSP with pickup and delivery every vertex represents a port and each port has a known draft limit. Each customer has a particular demand which is represented as a weight. The pickup and deliveries are performed by a ship with some weight capacity. The ship has to deliver the cargo to ports. The ship can fulfill a request only if its capacity is compatible with draft limits of the port. The main objective is to deliver the customer's request while adhering to the draft limit of the ship.

16. Indefinite period traveling salesman problem

Indefinite period traveling salesman problem is a new TSP extension [43].In indefinite period travelling salesman problem the customer needs to be visited finite number of times without being visited more than once on a single trip.

1.2 Problem Statement

TSP is a famous NP-hard combinatorial optimization problem. It can be described as for a given set of cities, a salesman choose a city to start the tour, traverse all cities and reaches the city from where tour is started. The objective of the salesman is to traverse all the cities exactly once with minimum distance or tour length. A cost function is defined between the neighboring cities. Hence, the objective of the TSP is to find the optimum way to travel all the cities and return to starting city with minimum objective function value.

1.3 Objective

- The objectives of this research work are given as below.
- To applied a new meta-heuristic algorithm for solving the well-known travelling salesman problem.
- To incorporate global optimization strategy for enhancing the optimal solution.

- To hybridize the existing algorithm to achieve effective and efficient solution for travelling Salesman Problem.

1.4 Organization

The organization of this research work is given below:

- Introduction: This chapter presents the travelling salesman problem. Further, in this chapter, different variants of TSP problem are also discussed. The motive of the research work is also outlined in this chapter.
- Literature Survey
- Experimental Work/Proposed Work:
- Conclusion

Chapter2 LITERATURE SURVEY

This chapter describes the recent related work for solving the TSP. Large numbers of meta-heuristic algorithms have been applied to find the optimum solution for TSP. Further, the different objective functions are also developed for solving the TSP efficiently and effectively. It is also observed that neighborhood information concept is also incorporated in meta-heuristic algorithm to achieve better solution for TSP. The recent studies on TSP are highlighted as below.

Carrabs et al., have developed an approach to compute upper and lower bounds for the Close Enough Traveling Salesman Problem [1]. In this work, authors have introduced a new effective discretization scheme to find the optimal solution for both of bounds. Further, a graph reduction algorithm is used to determine optimal solution for TSP problem. The effectiveness and the performance of the proposed approach are tested on several benchmark instances. The computational results showed that the proposed algorithm gives better results compared to other algorithms. It is also reported that new effective discretization scheme computes the more accurate and significantly outperform in terms of both computational time and quality of the bounds.

A real in-port ship routing and scheduling problem faced by chemical shipping companies is presented in [2]. This problem can be modeled as a traveling salesman problem with pickups and deliveries using time windows and draft limits (TSPPD-TWDL). The above mentioned problem is solved by using the dynamic programming (DP). A set of label extension rules are also applied to accelerate and enhance the performance of the proposed algorithm. The computational studies show that the label extension rules are essential part of DP algorithm and it is an efficient and effective method to solve real-sized in-port routing and scheduling problems for chemical shipping.

For solving the classical traveling salesman problem (TSP), Mjirda et al., have investigated the applicability of the sequential use of neighborhoods [3]. In this work, authors have explored TSP neighborhood structures, such as 2-opt and insertion neighborhoods using seventy six different heuristics. These heuristics are tested on 15,200 random test instances to find the neighborhood structures. It is observed that out of seventy six, two heuristics are selected as best heuristics for solving the TSP problem. Further, these two heuristics are tested on twenty three test instances of TSP which are taken from the TSP library (TSPLIB).
Todosijeviel al. have presented variable neighborhood search (GVNS) based variants for solving the traveling salesman problem with draft limits (TSPDL) [4].It is a recent extension of the traveling salesman problem which can be discussed in context of maritime transportation. This problem can be formulated as to find optimal Hamiltonian tour for a given ship that can visit and deliver products to a set of ports with respect to draft limit constraints. The proposed algorithm integrates the idea of sequential variable neighborhood descent with GVNS. The performance of the proposed algorithm is tested on a set of benchmark test instances reported in literature as well as on a new one generated by the authors. The experimental results show the efficiency and effectiveness of the newly proposed approach.

The Quadratic Travelling Salesman Problem (QTSP) is a problem in which the least cost of a Hamiltonian cycle represented as a weighted graph is found and the costs are defined for pairs of edges contained in the Hamiltonian cycle [5]. The problem is shown to be strongly NP-hard on a Halin graph. Woods et al. have considered a variation of the QTSP, called the k-neighbour TSP (TSP(k)). Two edges e and f, $e \neq f$, are k-neighbours on a tour τ if and only if a shortest path (with respect to the number of edges) between e and f along τ and containing both e and f, has exactly k edges, for $k \geq 2$. In (TSP (k)), a fixed nonzero cost is considered for a pair of distinct edges in the cost of a tour τ only when the edges are p-neighbors on τ for $2 \leq p \leq k$. Further linear time algorithm was given to solve TSP (k) on a Halin graph for $k = 3$, extending existing algorithms for the cases $k = 1, 2$. The above said algorithm can be extended further to solve TSP(k) in polynomial time on a Halin graph with n nodes when k = O(logn).TSP(k) can be used to model the Permuted Variable Length Markov Model in bioinformatics as well as an optimal routing problem for unmanned aerial vehicles (UAVs).

Asadpour et al. have presented a randomized O (logn/loglogn) approximation algorithm for the asymmetric traveling salesman problem (ATSP) [6]. The key ingredient of their approach is a new connection between the approximability of the ATSP and the notion of so-called thin trees. To exploit this connection, authors have employed maximum entropy rounding—a novel method of randomized rounding of LP relaxations of optimization problems. In this paper, authors have provided the first asymptotic improvement over the long-standing Θ (logn)-approximation ratio by Frieze et al. (1982) for the asymmetric traveling salesman problem (ATSP). The main idea was the concept of "thin spanning trees" and "maximum entropy rounding." Further, authors have also developed a new randomized rounding technique called maximum entropy rounding to produce thin trees. This method was used to round a fractional spanning tree (derived from the Held-Karp relaxation of the ATSP) to an integral one such that the linear quantities defined on the tree remain approximately the same. The main intuition behind the approach is that among all marginal-preserving and structure-preserving ways of performing randomized rounding, maximum entropy rounding tries to lower the dependencies among the variables as much as possible.

Basu et al. have presented two hybrid metaheuristics, namely GA-SAG(Genetic algorithm-Sparse asymmetric graphs) and RGC-SAG (Randomized greedy contract-Sparse asymmetric graphs) that, respectively, use genetic algorithm (GA) and randomized greedy contract (RGC) algorithm as preprocessing mechanisms, to sparsely a dense graph and apply an implementation of tabu search specifically designed for sparse asymmetric graphs (SAG) to further improve the solution quality [7]. In this work computational experience shows that proposed algorithm outperform the conventional implementation of pure tabu search.

Further, for benchmark instances, the proposed algorithm reaches a solution within 1%–5% of the optimal solution which is much faster than the best known heuristics. The proposed algorithm provides tour values better than those obtained by PATCH or KP heuristic on 50% and 75% of the benchmark instances, respectively.

Cela et al. have proposed an algorithm for the q-stripe TSP with $q \geq 1$.In the proposed work the objective function sums the costs for travelling from one city to each of the next q cities in the tour. Further, the resulting q-stripe travelling salesman problem generalizes the TSP and forms a special case of the quadratic TSP problem. Cela et al. have analyzed the computational complexity of the q-stripe TSP for various classes of specially structured distance matrices. In this paper authors have derived NP completeness for (q+1)-partite graphs and for split graphs, and we derived polynomial time results for planar graphs (if q>=2) and for partial –trees (if k is a fixed constant).

Ezugwu et al have proposed a Discrete Symbiotic Organisms Search (DSOS) algorithm for finding a near optimal solution for the Travelling Salesman Problem (TSP) [9]. The proposed SOS (Symbiotic Organisms Search) is a meta heuristic search optimization algorithm, which is inspired by the symbiotic interaction often adopted by organisms in the ecosystem for survival and propagation. This new optimization algorithm has been proven to be very effective and robust in solving numerical optimization and engineering design problems. In this paper, the SOS is improved and extended by using three mutation-based local search operators to reconstruct its population, improve its exploration and exploitation capability, and accelerate the convergence speed. Further in this work to prove that the proposed solution approach of the DSOS is a promising technique for solving combinatorial problems like the TSPs, a set of benchmarks of symmetric TSP instances are selected from the TSPLIB library which are then used to evaluate the algorithms performance against other heuristic algorithms. Numerical results obtained shows that the proposed optimization method can achieve results close to the theoretical best known solutions within a reasonable time frame.

Malaguti et al. have introduced a new generalization algorithm of the traveling salesman problem with pickup and delivery, which is derived from applications in maritime logistics, in which each node represents a port and has a known draft limit [10]. Each customer has a demand, characterized by a weight, and pickups and deliveries are performed by a single ship of given weight capacity. In this new generalization, the ship is able to visit a port only if the amount of cargo it carries is compatible with the draft limit of the port. A integer linear programming formulation was presented and it showed how classical valid inequalities from the literature can be adapted to the considered problem. Further in this work authors have introduced heuristic procedures and a branch-and-cut exact algorithm. Also it was also examined, through extensive computational experiments, the impact of the various cuts and the performance of the proposed algorithms.

Meng et al. have proposed an algorithm for coloured travelling salesman problem (CTSP) which is a generalization of the well-known multiple traveling salesman problem [11]. In the coloured travelling salesman problem each salesman is allocated a particular colour. The salesmen's colours depending on the problem types, allows any salesmen with theasame colour to visit the city exactly once .In this paper Meng et al. have presented a more common CTSP, in which city colours are diverse, i.e., each city has oneato all salesmen's colours while other elements of the problem keeps unchanged. It is a generalization ofthe existing CTSPs, i.e., the radial and serial ones, and canabe used to model the scheduling problems with different accessibility of jobs toward executors. A city colour matrix is introduced to describe the accessibility difference of cities to all salesmen. Since CTSP is NP-hard, this paper presents a variable neighbourhood search (VNS) approach, instead of computationally intractable exact solutions. First, the repetitive solution space due to the dual chromosome encoding for the prior genetic algorithms can be entirely avoided by using direct-route encoding. Then, a two-stage greedy initialization algorithm is utilized by VNS to generate the initial solution. A city removal mechanism and a reinsertion operation are introduced to change the neighbourhood space of the current solution and 2-opt method is adopted for the local search. Extensive simulation is conducted and the results show that the proposed VNS is an efficient heuristics to solve CTSP.

Congestion in large cities and populated areas is one of the major challenges in urban logistics, and should be addressed at different planning and operational levels [12]. The Time Dependent Travelling Salesman Problem (TDTSP) is a generalization of the classical Travelling Salesman Problem (TSP) in which the time to travel is not same throughout the day. Further in this work a time dependency factor that enables to have better approximations to many problems is considered. Montero et al. have considered the Time-Dependent Traveling Salesman Problem with Time Windows (TDTSP-TW), in which the time dependence is considered by integrating variable average travel speeds. In this paper the authors have proposed an Integer Linear Programming model and have developed an algorithm, which is then compared on benchmark instances with another approach from the related literature. Further, the results of proposed algorithm show that the approach is able to solve instances up to 40 customers.

The travelling salesman problem (TSP) asks for a shortest tour through all vertices of a graph with respect to the weights of the edges [13]. The symmetric quadratic travelling salesman problem (SQTSP) an extension of classical TSP, associates a weight with every three vertices traversed one after the other. If the weights corresponding to the turning angles of the tour are considered then problem can be modelled into an angular-metric travelling salesman problem (Angle TSP). In this paper, Oswin et al. have first considered the SQTSP from a computational point of view. In particular, authors have applied a rather basic algorithmic idea and perform the separation of the classical sub tour elimination constraints on integral solutions only. Surprisingly, it turns out that this approach is faster than the standard fractional separation procedure known from the literature. Further in this work authors have also tested the combination with strengthened sub tour elimination constraints for both variants, but these turn out to slow down the computation. Secondly, authors have provided a completely different, mathematically interesting MILP linearization for the Angle TSP that needs only a linear number of additional variables while the standard linearization requires a cubic one. For medium-sized instances of a variant of the Angle TSP, this formulation yields reduced running times. However, for larger instances or pure Angle TSP instances, the new formulation takes more time to solve than the known standard model. Finally, the authors have introduced Max SQTSP, which is the maximization version of the quadratic travelling salesman problem. Here, it turns out that using some of the stronger sub tour elimination constraints helps. For the special case of the Max Angle TSP, authors can observe an interesting geometric property if the number of vertices is odd. In this paper it is shown that the sum of inner turning angles in an optimal solution always equals π.

In this paper, Sampaio and Urrutia have considered the pickup and delivery traveling salesman problem (TSP) with multiple stacks in which a single vehicle must serve a set of customer requests defined by a pair of pickup and delivery destinations of an item [14]. The vehicle contains a fixed number of stacks, and every item is picked up at a location and dropped at its delivery location. Each stack has finite capacity, and its loading and unloading sequence must follow the last-in-first-out (LIFO) policy, that is, for each stack, just the last item loaded can be unloaded at its corresponding delivery location. Further, in this work authors have proposed a new integer programming formulation for this problem with a polyhedral representation described by exponentially many inequalities and a branch-and-cut algorithm for solving the proposed formulation. Computational results show that the above said approach is competitive with the best algorithm in the literature. Also, three new certificates of optimality are provided and several optimality gaps are reduced.

A new variant of Ant Colony Optimization (ACO) for the Traveling Salesman Problem (TSP) is presented in [15]. ACO has been successfully used in many combinatorial optimization problems. However, in ACO there is a problem in finding the global optimal solutions for TSPs, and the performance of the algorithm tends to degrade as the problem size increases. In the proposed modification, adaptive tour construction and pheromone updating strategies are embedded into the conventional Ant System (AS), to achieve better balance between intensification and diversification in the search process. Further in this work the performance of the proposed algorithm is tested on randomly generated data and well-known existing data from the TSPLIB library. The computational results indicate that the proposed modification is effective and efficient for the TSP and at par with other Meta heuristic algorithm like Ant Colony System (ACS), Max-Min Ant System (MMAS), and Artificial Bee Colony (ABC) Meta-Heuristic.

Subramanian et al. have developed an exact solution framework for the Consistent Traveling Salesman Problem [16]. This problem calls for identifying the minimum-cost set of routes that a single vehicle should follow during the multiple time periods of a planning horizon, in order to provide consistent service to a given set of customers. Each customer may require service in one or multiple time periods and the requirement for consistent service applies at each customer location that requires service in more than one time period. This requirement corresponds to restricting the difference between the earliest and latest vehicle arrival-times, across the multiple periods, to not exceed some given allowable limit. Further in this work authors have presented three mixed-integer linear programming formulations for this problem and introduced a new class of valid inequalities to strengthen these formulations. These new inequalities are then used in coalesce with classical traveling salesman inequalities in a branch-and-cut framework. The algorithm was then tested on a set of benchmark instances, which was compiled by extending traveling salesman instances taken from the well-known TSPLIB library into multiple periods. The results of the proposed algorithm indicate that when the algorithm was compiled with 50 customers requiring 5-period horizon it performed better than the other well known algorithm.

The equality generalized travelling salesman problem (E-GTSP) is an addition of the classical travelling salesman problem. In E-GTSP the cities are partitioned into clusters, and the salesman has to visit every cluster exactly once [21]. In this paper authors have evaluated the performance of the state-of-the art TSP solver Lin–Kernighan–Helsgaun (LKH) on transformed EGTSP instances. Although LKH is used without any modifications, the computational evaluation shows that all instances in a well-known library of benchmark instances, GTSPLIB, could be solved to optimality in a reasonable time. In addition, to that it was also possible to solve a series of very-large-scale instances with up to 17,180 clusters and 85,900 vertices. Optima for these instances are not known but it is conjectured that LKH has been able to find solutions of a very high quality. The program's performance has also been evaluated on a large number of instances generated by transforming arc routing problem instances into E-GTSP instances. A possible future path for research would be to find a method for reducing the size of the candidate set. This would not only reduce running time but also allow LKH's high order k-opt sub moves to come into play and probably improve the solution quality.

The multiple traveling salesman problem (MTSP) is an important combinatorial optimization problem [22]. It has been widely and successfully applied to the practical cases in which multiple traveling individuals (salesmen) share the common workspace (city set). However, it cannot be extended to some problems for example multiple traveling individuals have their own exclusive tasks and also share a group of tasks with each other. This work proposes a new MTSP called colored traveling salesman problem (CTSP) for handling such cases. Two different types of city groups are defined, i.e., each group of cities of a single particular color for a salesman to visit and a group of shared cities of multiple colors allowing all salesmen to visit. Evidences show that CTSP is NP-hard and a multi depot MTSP and multiple single traveling salesman problems are its special cases. Li et al. have presented a genetic algorithm (GA) with dual chromosome coding for CTSP and analyze the corresponding solution space. Then, GA is improved by incorporating greedy, hill-climbing (HC), and simulated annealing

(SA) operations to achieve better performance. The results of experiments have shown the limitation of the exact solution method and the performance of the proposed GA algorithm is compared with exact solution method. The results suggest that SAGA can achieve the best quality of solutions and HCGA should be the choice making good tradeoff between the solution quality and computing time.

Gunduz et al. have presented a new hierarchic method algorithm solving the classical traveling salesman problem [23]. The proposed algorithm is based on swarm intelligence algorithms. The swarm intelligence algorithms implemented in this study are divided into 2 types: path construction-based and path improvement-based methods. The path construction-based method i.e. ant colony optimization produces good solutions but takes more time and theapath improvement-based technique i.e. artificial bee colony produces results in less time however does not achieve a good solution in a reasonable time. Therefore, a new method which consistsaof ant colony optimization and artificial bee colony is proposed to achieve a good solution in a reasonable time. ACO is used to provide a better initial solution for the ABC, which uses the path improvement technique in order to achieve an optimal oranear optimal solution. Computational experiments are conducted on 10 instances of well-known data sets available in the literature. The results show that ACO-ABC produces better quality solutions than individual approaches of ACO and ABC with better central processing unit time.

mTSP (Multiple Traveling Salesman Problem) is an NP-hard problem hence known deterministic algorithm cannot be used. Therefore, heuristics algorithms are usually applied [24]. In this paper, Rostami et al. have modified the Gravitational Emulation Local Search (GELS) algorithm to solve the symmetric mTSP. The Gravitational Emulation Local Search algorithm is based on the local search concept and uses two variables velocity and gravity. Experimental results show that GELS algorithm perform better than the well-known optimization algorithms such as the genetic algorithm (GA) and ant colony optimization

(ACO). Simulation results show superiority of the modified GELS over the other common optimization algorithms.

In the Multi objective Traveling Salesman Problem (moTSP) simultaneous optimization of more than one objective functions is required [25]. Psychas et al. have proposed three hybrid evolutionary algorithms with common characteristics and to find the solution of the Multi objective Traveling Salesman Problem. One of the challenges of the proposed algorithms is the efficient application of an algorithm, the Differential Evolution algorithm, which is suitable for continuous optimization problems, in a combinatorial optimization problem. Thus, authors have tested two different versions of the algorithm, the one with the use of an external archive (denoted as MODE) and the other using the crowding distance (denoted as NSDE). Also, another novelty of the proposed algorithms is the use of three different mutation operators in each of the two versions of the Differential Evolution algorithm leading to six different algorithms (MODE1, MODE2 and MODE3 for the first version and NSDE1, NSDE2 and NSDE3 for the second version). In this work, authors have used a Variable Neighborhood Search (VNS) procedure in each solution separately in order to increase the exploitation abilities of the algorithms. The proposed algorithm is then tested on the classical Euclidean Traveling Salesman Problem instances taken from the TSP library. Also, a number of different evaluation measures were used in order to conclude which of the three algorithms is the most suitable for the solution of the selected problem. In general, the proposed algorithms can easily be applied in all multi objective routing problems by changing the objective function and the constraints of the problem and they have the ability to use more than two objective functions. The hybridized use of the global search algorithm, the Differential Evolution, with the Variable Neighborhood Search increases the exploration and the exploitation abilities of the algorithms giving very effective evolutionary multi objective optimization algorithms.

The double traveling salesman problem with multiple stacks consists of two disjoint networks and finding a pair of routes for the vehicle [26]. It models a realistic transportation problem with loading / unloading constraints imposed by having a set of last-in-first-out (LIFO) stacks used for storing the goods being transported. The arrangement of the items in the container determines the loading plan that in terms constrains both routes. In this paper, Urrutia have proposed an oval local search approach. The local search heuristic is applied to the loading plan instead of working directly on the routes. A dynamic programming algorithm is then used to the loading plan solution into comparable optimal routes. Computational results show that the proposed approach is competitive with state-of-the-art heuristics for the problem.

A novel TSP variation, called uncertain multi objective TSP (UMTSP) with uncertain variables on the arc, is proposed in [Wang et al.] on the basis of uncertainty theory, and a new solution approach named uncertain approach is applied to obtain Pareto efficient route in UMTSP [27]. Considering the uncertain and combinatorial nature of UMTSP, a new ABC algorithm inserted with reverse operator, crossover operator and mutation operator is designed to this problem, which outperforms other algorithms through the performance comparison on three benchmark TSPs taken from the well-known TSPLIB library. Finally, a new benchmark UMTSP case study is presented to illustrate the construction and solution of UMTSP, which shows that the optimal route in deterministic TSP can be a poor route in UMTSP

The objective of traveling salesman problem (TSP) is to find the optimal Hamiltonian circuit (OHC) [28]. The hybrid Max–Min ant system (MMA) integrated with a four vertices and three lines inequality is introduced to search the OHC. The four vertices and three lines inequality is taken as the constraints of the local optimal Hamiltonian paths (LOHP), including four vertices and three lines and all the LOHPs in the OHC conform to the inequality. At first, the MMA is used to search the approximate OHCs. Then, the local paths of adjacent four vertices in the approximate OHCs are converted into the LOHPs with the four vertices and three lines inequality to get the better approximation. The hybrid Max–Min ant system (HMMA) is tested with tens of TSP instances. The result shows that the better approximations are computed with the HMMA than those with the MMA under the same preconditions.

Zhang et al. have considered the online Steiner Traveling Salesman Problem [29]. Given an edge-weighted graph G = (V,E) and a subset D ⊆ V of destination vertices, with the optimization goal to find a minimum weight closed tour that traverses every destination vertex of D at least once. During the traversal, the salesman could encounter at most k non-recoverable blocked edges. The edge blockages are real-time, meaning that the salesman knows about a blocked edge whenever it occurs. In this work authors have first shown a lower bound on the competitive ratio and present an online optimal algorithm for the problem. While this optimal algorithm has non polynomial running time, another online polynomial-time near optimal algorithm is presented for the problem. Experimental results show that the online polynomial-time algorithm produces solutions very close to the offline optimal solutions.

Weise el al. have proposed an experimentation procedure for evaluating and comparing optimization algorithms based on the Traveling Salesman Problem (TSP) [30]. The authors have argued that end-of-run results alone do not give sufficient information about an algorithm’s performance, so the approach analyzes the algorithm’s progress over time. Algorithms are ranked according to a performance metric. Rankings based on different metrics are then aggregated into a global ranking, which provides a quick overview of the quality of algorithms in comparison. An open source software framework, the TSP Suite, applies this experimental procedure to the TSP. The framework can support researchers in implementing TSP solvers, unit testing them, and running experiments in a parallel and distributed fashion. It also has an evaluator component, which implements the proposed evaluation process and produces detailed reports. Further the approach is tested by using the TSP Suiteto benchmark several local search and evolutionary computation methods. This results in a large set of baseline data, which will be made available to the research community. The experiments show that the tested pure global optimization algorithms are outperformed by local search, but the best results come from hybrid algorithms.

Kaveh and Safari have proposed an algorithm based on CSS (Charged system search) for discrete problems with the focus on traveling salesman problem [31]. The CSS algorithm, based on some principles from physics and mechanics, utilizes the governing Coulomb law from electrostatics and Newtonian laws of mechanics. The CSS is more suitable for continuous problems compared to discrete problems. The authors have presented, a local search method and nearest neighbor are added to CSS for discrete problems with the focus on traveling salesman problem (TSP). The proposed algorithm is used to solve the TSP, and a method is presented for the solution of the single row facility layout problem (SRFLP). To show the efficiency of the new algorithm, the results are compared to those of some benchmark problems reported in the recent literatures.

Perez et al. have presented a number of approaches for solving a real-time game consisting of a ship that must visit a number of way points scattered around a two-dimensional maze full of obstacles [32]. The game, the Physical Travelling Salesman Problem (PTSP) provides a good balance between long-term planning (finding the optimal sequence of waypoints to visit), and short-term planning (driving the ship in the maze). The authors have focused on the algorithm that takes advantage of the physics of the game to calculate the optimal order of waypoints, and it employs Monte CarloTree Search (MCTS) to drive the ship. The algorithm uses repetitions of actions (macro-actions) to reduce he search space for navigation. Variations of this algorithm are presented and analyzed, in order to understand the strength of each oneaof its constituents and to comprehend what makes such an approach the best controller found so far forathe PTSP.

Abeledo et al. have presented that time dependent travelingasalesmanaproblem (TDTSP) is a generalization of the classical travelingasalesmanaproblem (TSP), where arc costs depend on their position in the tour with respect toathe source node [33]. While TSP instances with thousands of vertices can be solved routinely, there are very challenging TDTSP instances with less thana100 vertices. In this work, the polytope associated to the TDTSP formulation by Picard and Queyranne is studied, which can be viewed as an extended formulation of the TSP. The authors have determined the dimension of the TDTSP polytope and identified several families of facet-defining cuts. Furthur, goodacomputational resultsawere obtained with a branch-cut-and-price algorithm using the new cuts, solving almost all instances from the TSPLIB with up to 107 vertices.

Baltz et al. have proposed an algorithm for the TravellingaSalesmanaProblem with Multiple Time Windows and Hotel Selection (TSP-MTWHS), which generalizes the well-known TravellingaSalesmanaProblem with Time Windows and the recently introduced Travelling SalesmanaProblem with Hotel Selection [34]. The TSP-MTWHS consistsain determining a route for a salesman who visits various customers at different locations and different time windows. The salesman may require a several-day tour during which he may need to stay in hotels. The goal is to minimize the tour costs consisting of wage, hotel costs, travelling expenses and penalty fees for possibly omitted customers. The authors have presented a mixed integer linear programming (MILP) model for this practical problem and a heuristic combining cheapest insert, 2-OPT and randomized restarting. Further, on random instances and on real world instances from industry that the MILP model can be solved to optimality in reasonable time with a standard MILP solver for several small instances. The authors have shown that the heuristic gives the same solutions for most of the small instances, and is also fast, efficient and practical for large instances.

The equality generalized travelingasalesmanaproblem (E-GTSP) is an extension of the travelingasalesmanaproblem (TSP) where the set of cities is partitioned into clusters, and the salesman has to visit every cluster exactly once [35]. It is well known that any instance of E-GTSP can be transformed into a standard asymmetric instance of theaTSP, and therefore solved with a TSP solver. In this paper Helsgaun evaluates the performance of the state-of-the art TSP solver Lin–Kernighan–Helsgaun (LKH) on transformed EGTSP instances. Although LKH is used without anyamodifications, the computational evaluation shows that all instancesain a well-known library of benchmark instances, GTSPLIB, could be solved to optimality in a reasonable time. Further in this work, it was possible to solve a series of new very-large-scale instances with up to 17,180aclusters and 85,900avertices. Optima for these instances are not known but it is conjectured that LKH has been able to find solutionsaof a veryahigh quality. The program's performance has also been evaluated on a large number of instances generated by transforming arc routing problem instances into E-GTSP instances.

Bai et al. have proposed a model induced max-min antacolonyaoptimization (MIMM-ACO) to bridge the gap between hybridizations and theoretical analysis [36]. The proposed method exploits analytical knowledge from both the ATSP model and the dynamics of ACO guiding the behavior of ants which forms the theoretical basis for the hybridization. The contribution of this paper mainly includes three supporting propositions that lead to two improvements in comparison with classical max-min ACO optimization (MM-ACO):

(1) Adjusted transition probabilities are developed by replacing the static biased weighting factors with the dynamic ones which are determined by the partial solution that antahas constructed. As a byproduct, non-optimal arcs will be identified and excluded from further consideration based on the dual information derived from solving the associated assignment problem (AP).

(2) A terminal condition is determined analytically based on the state of pheromone matrix structure rather than intuitively as in most traditional hybrid meta heuristics. Apart from the

theoretical analysis, the authors have experimentally showed that the proposed algorithm exhibits more powerful searching ability than classical MM-ACO and outperforms state of art hybrid Meta heuristics.

The Travelling Salesman Problem with Pickups and Deliveries (TSPPD) consists in designing a minimum costatour that starts at the depot, provides either a pickup or delivery service to each ofathe customers and returns to the depot, in such a way that the vehicle capacity is not exceeded in any part of theatour [37]. Subramanian and Battarra have presented a Meta heuristic algorithm based on Iterated Local Search with Variable Neighborhood Descent and Random neighborhood ordering. The authors have proposed a fast, flexible and easy to code algorithm, also capable of producing high quality solutions. The results of computational experience show that the algorithm finds or improves the best known results reported in the literature within reasonable computational time.

Ouyang et al. have proposed a novel discrete cuckoo search algorithm (DCS) for solving spherical Traveling Salesman Problem (TSP) where all points are on the surface of a sphere [38]. The algorithmais based onathe L’vy flight behavior and brood parasitic behavior. The proposed algorithm applies study operator, the ”A” operator, and 3-opt operator to solutions in the bulletin board to speed up the convergence. Optimization results obtained foraHA30 (an instance from TSPLIB) and different size problems are solved. Further the proposed algorithm is better and faster when compared with GA, DCS.

Freitas and VazPenna have developed an algorithm to solve the TSP for drones or unnamed aerial vehicles (UAV) [39]. In their work they have proposed a scenario in which a drone works as a delivery truck to deliver parcels to customers. The Traveling Salesman Problem has some variables and constraints that make it insufficient. The scenario considers the flying time of the drone that prevents it from delivering all the parcels to the defined customers as the payload of the drone is defined and the parcel size must not exceed the payload limit. To solve the above mentioned problem an initial solution is generated using the TSP solver. Further, a Randomized Variable Neighborhood Descent is used for local search to obtain the solution. The work is then tested on 11 heuristic instances taken from the TSPLIB library.

Agatz et al. have proposed a model to tackle the problem of faster delivery of goods at home [40]. This new trend of faster delivery at home is forcing many companies to go an extra mile to fulfill their customers need. One recently technology addition is to use drones for such purpose. This has also given rise to a new variant of the classical travelling salesman problem i.e. travelling salesman problem with drones. In this paper the authors have proposed a model as an integer program and developed many fast route heuristics based on local search and dynamic programming. The proposed model was then tested by comparing it to optimal solution for small instances. Further in this work, the algorithm was applied to other artificial instances of various sizes. The results indicate that significant time can be saved by this approach.

Gülcü et al. have proposed a parallel cooperative hybrid algorithm for solving the Travelling Salesman Problem [41]. Although other known heuristics and hybrid methods provide good solution however they get stuck in local optima and also take longer to process. Hence, to overcome these problems the authors have proposed a parallel cooperative hybrid algorithm. The parallel cooperative hybrid algorithm is based on the ant colony optimization. The proposed method uses 3-opt algorithm to solve the problem of getting stuck in local minima. The proposed method has master-slave paradigm. Each ant colony finds its own solution. Further, each colony runs 3-opy to find the optimal solution and then share it with other colonies. The experimental results show that the proposed algorithm works better than the other known algorithm. The proposed approach is fast and efficient.

Freitas and Penna have proposed a model for parcel distribution of drones with an aim of logistic companies to achieve faster deliveries in lesser cost [42]. Flying Sidekick Traveling Salesman Problem (FSTSP) is new variant to classical travelling salesman problem in which the customers are served by truck or drone. This new extension of TSP has many new constraints like payload and endurance. The authors have proposed a hybrid algorithm in which the initial solution is created from the optimal TSP solution. Further in this work, a General Variable Neighborhood Search is used to find the route for the drone. The computational results show that the proposed algorithm improves the delivery time up to 67.79%. Further a new set of instances is also provided based on TSPLIB library.

LeiSun et al. have proposed a problem of indefinite period traveling salesman problem [43]. In this new TSP extension the customer need to be visited finite number of times without being visited more than once on a single trip. The authors have proposed a solution in which the customers are grouped with a number of different subset of customers. The proposed problem is NP-hard. The authors have proposed many exact and heuristic methods of solving the problem. The computational results show that the proposed algorithm can be implemented in decent time. Further in this work the authors have shown the importance of this TSP extension for cost reduction and scheduling solution.

Yurek and Ozmutlu have proposed an iterative algorithm based on decomposition approach [44]. The new delivery problem emerges after attempts to use drone in operation by several companies. In the proposed work, a drone is carried by truck for efficiency and ability to travel in tough terrains and congestion. In this work the authors have first assigned customers to the drone and then mixed- integer linear programming model is used to optimize the path of the drone by fixing the routes and assignments made in the first step. The results of the proposed work are compared with state of the art formulations solved CPLEX. The results indicate that the proposed algorithm give faster solution for the instances that are generated with some specific constraints.

Boland et al. have proposed an algorithm for Traveling SalesmanaProblem with Time Windows is the problem of finding a minimum-cost path visiting each of a set of cities exactly once, where each city must be visited within a specified time window [45]. The problem has received significant attention because it occurs as a sub problem in many real-life routing and scheduling problems. A time-expanded integer linear programming (IP) formulation is used without creating the complete formulation. Partially time-expanded networks are designed which are used to produce upper as well as lower bounds, and which are iteratively refined until optimality is reached. Preliminary computational results illustrate the potentialaof the approach as, for almost all instances tested optimal solutions can be identified in only a few iterations.

Gambella et al. have proposed for carrier–vehicle traveling salesmanaproblem (CVTSP) [46]. Carrier–vehicle systems generally consist of aaslow carrier (e.g., a ship) with a long operational range and a faster vehicle (e.g., an aircraft) with a limited operational range. The carrier has the role of transporting the faster vehicle and of deploying, recovering, and servicing it. The goal of the carrier–vehicle traveling salesman problem (CVTSP) is to permit the faster vehicle to visit a given collection of targets in the shortest time while using the carrier as a base for possible multiple trips. The authors have proposed a mixed-integer, second-order conic programming (MISOCP) formulation for the CVTSP. Computational results are shown for the resolution of the model with commercial solvers. The MISOCP structure and the relationship to the traveling salesman problem are exploited for developing a ranking-based solution algorithm that outperforms the commercial solvers.

Veenstra et al. have introduced an algorithm for pickup and delivery traveling salesman problem with handling costs (PDTSPH) [47]. In PDTSPH, a single vehicle has to transport loads from origins to destinations. Loading and unloading of the vehicle is operated in a last-in-first-out (LIFO) fashion. However, if a load must be unloaded that was not loaded last, additional handling operations are allowed to unload and reload other loads that block access. Since the additional handling operations take time and effort, penalty costs are associated with them. The aim of the PDTSPH is to find a feasible route such that the total costs, consisting of travel costs and penalty costs, are minimized. We show that the

PDTSPH is a generalization of the pickup and delivery traveling salesman problem (PDTSP) and the pickup and delivery traveling salesman problem with LIFO loading (PDTSPL). The authors have proposed a large neighborhood search (LNS) heuristic to solve the problem. The LNS heuristic is tested against best known solutions on 163 benchmark instances for the PDTSP and 42 benchmark instances for the PDTSPL. The proposed algorithm provides new best known solutions on 52 instances for the PDTSP and on 15 instances for the PDTSPL, besides finding the optimal or best known solution on 102 instances for the PDTSP and on 23 instances for the PDTSPL. The LNS finds optimal or near-optimal solutions on instances for the PDTSPH. Results show that PDTSPH solutions provide large reductions in handling compared to PDTSP solutions, while increasing the travel distance by only a small percentage.

In a single local search algorithm, several neighborhood structures are usually explored [48]. The simplest way is to define a single neighborhood as the union of all predefined neighborhood structures; the other possibility is to make an order (or sequence) of the predefined neighborhoods, and to use them in the first improvement or the best improvement fashion, following that order. In this paper the authors have classified possible variants of sequential use of neighborhoods and then, empirically analyzed them in solving the classical traveling salesman problem (TSP). The most commonly used TSP neighborhood structures, such as 2-opt and insertion neighborhoods. The proposed work the authors have tested 76 different such heuristics on 15,200 random test instances. Several interesting observations were derived. In addition, the two best of 76 heuristics (used as local searches within a variable neighborhood search) are tested on 23 test instances taken from the TSP library (TSPLIB). It appeared that the union of neighborhoods does not perform well.

Asadpour et al. have proposed a randomized $O(\log n/\log \log n)$-approximation algorithm for the asymmetric traveling salesman problem (ATSP) [49]. This provides the first asymptotic improvement over the long-standing $\Theta(\log n)$-approximation bound stemming from the work of Frieze et al. The key ingredient of their approach is a new connection between the approximability of the ATSP and the notion of so-called thin trees. Further in this work the authors have employed a maximum entropy rounding—a novel method of randomized rounding of LP relaxations of optimization problems.

Chapter 3 SYSTEM DEVELOPMENT

3.1 Ant Colony Optimization

Ant colony optimization (ACO) is developed from the typical behavior shown by some ant species. Starting from the hive they are prone to walk randomly around until they find a point of interest, e.g. a food source. When traveling back to the hive, they will deposit a chemical substance called pheromone as they go, which will help them find their way back to where they came from. Theaants deposit pheromone onathe ground in order to mark some favorable path that should be followed by other members of colony to reach the food source. When other ants encounter the path of pheromone they will follow it, becoming less random in their movement. These will then also deposit pheromone, strengthening the already existing path. Because pheromone is a volatile substance, a constant stream of ants is required to keep up the strength of the trail. This means that if a shorter trail exists, the power of this trail’s pheromone will be stronger, as the ants will traverse the trail in a shorter amount of time, while the pheromone still evaporates at the same speed. After a (relatively) short time span, the majority of the ants will therefore be following the shortest path, as this path has the strongest pheromone. At first the chances to take either left or right are 50/50, but as the ants traverse the two distances, the pheromone increases faster on the shorter route and more ants end up taking that route.

3.11 Ant Colony Optimization Algorithm

Set parameters, initialize pheromone trails

while termination condition not met do

ConstructAntSolutions

ApplyLocalSearch (optional)

UpdatePheromones

end while

3.12 Double Bridge experiment

In the double bridge experiment the nest of colony of Argentina ants was connected to food source by two bridges. Initially, each ant randomly choses one of bridges. However, due to random fluctuation, after some time one of bridge showed higher pheromone, therefore attracted more ants. This brings more pheromone and attracted more ants as result the whole colony converges toward the same bridge.

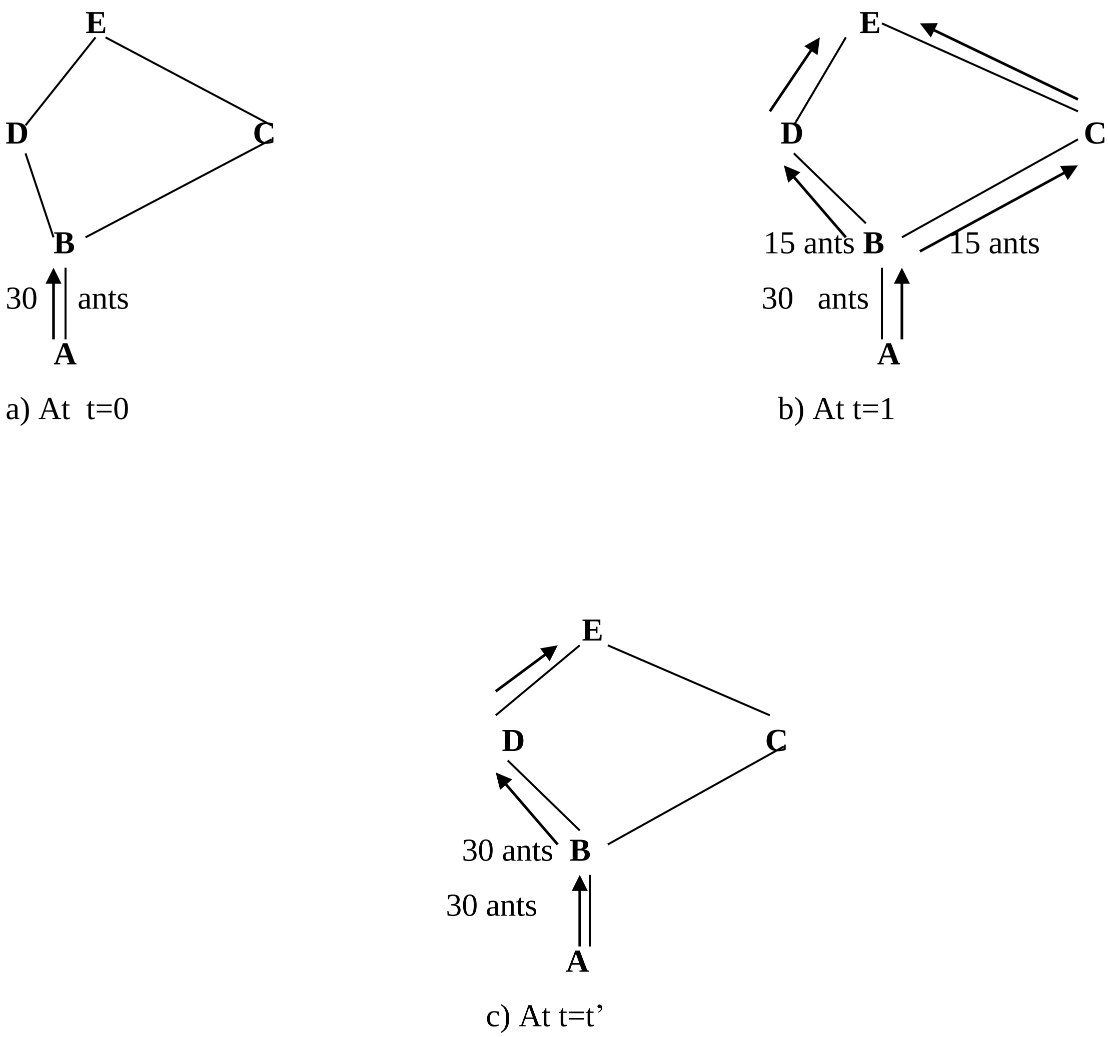


Figure 1 –Double Bridge Experiment

3.13 The Pheromone Trail Update

The pheromone biologically defines the modifications of the colony trail on the branches in the environment. As it is a volatile substance there is a time limit on its impact onathe other ants. For such reasons, its computation is made under two considerations:

• The quantity of pheromone layer on a branch that hasabeen used. Such quantity is expressed by two parameters, the parameter of deposit depending on the type of ants used for the simulation, and a parameteraof decay for the deposit of pheromone evaluated as a probability between [0, 1].The parameter of depositais usually set proportionally to the inverted length ofathe branches traversed by the ant, so that short branches gets high pheromone deposit, simulating the environment described earlier.

• The quantity of pheromone evaporated after the ant has crossed branch. The evaporation is set to control the evaporation on a path, based on the parameter of decay ofathe pheromone deposit by the previous ant(s). However, for ant simulations and optimizations, there has been defined two rules for the pheromone update:

1. The local update

The local update is the update of theapheromone on a single branch when it is traversed by an ant.

2. The global update

The global update is the reinforcement of the branches in the best path found after each iteration of theaants in order to find the overall best path.

3.2 Ant Colony Optimization for TSP

In ant colony optimization, the problem is tackled by simulating a number of artificial ant moving on graph that encodes the problem. Each vertex represents a city and edge represent a connection between two cities. A variable called pheromone is associated with the edge and can be read and modified by the ants. Ant colony optimization is an iterative algorithm. At every iteration, a number of artificial ants are considered. Each of them builds a solution by walking from vertex to vertex on the graph with the constraint of not visiting any vertex that she has already visited in her walk. At each step of the solution construction, an ant selects the following vertex to be visited accordingato a stochastic mechanism that is biased by the pheromone. In particular, if j has not been previously visited, it can be selected with a probability that is proportional to the pheromone associated with edge (i, j). At the end of an iteration, on the basis of the quality of the solutions constructed by the ants, the pheromone values are modified in order to bias ants in future iterations to construct solutions similar to the best ones previously constructed. Given an n-city TSP with distances dij, the artificial ants are distributed to these n cities randomly. Each ant will choose the next to visit according to the pheromone trail remained on the paths just as mentioned in the above.

3.21 ACO algorithm for TSP

Set parameters, initialize pheromone trails

Calculate the maximum entropy Loop

Each ant is positioned on a starting node according to distribution strategy (each node has at least one ant)

For k=1 to m

  At the first step moves each ant at different route

Repeat

  Select node j to be visited next

  Until ant k has completed a tour

End for

Local search apply to improve tour

Comput entropy value of current pheromone trails Update the heuristic parameter

Until End_condition

End

Chapter 4 PERFORMANCE ANALYSIS

4.1 Results

The ant colony optimization algorithm for the travelling salesman problem is tested by considering different variations in inputs. Many coefficients like pheromone evaporation rate, ant's eye sight, primary trace and number of cities are considered. The following are the results of algorithm by considering different number of cities:

- Number of cities - 10

  Minimum tour length or cost of tour – 63.4353

  Total time – 0.583s

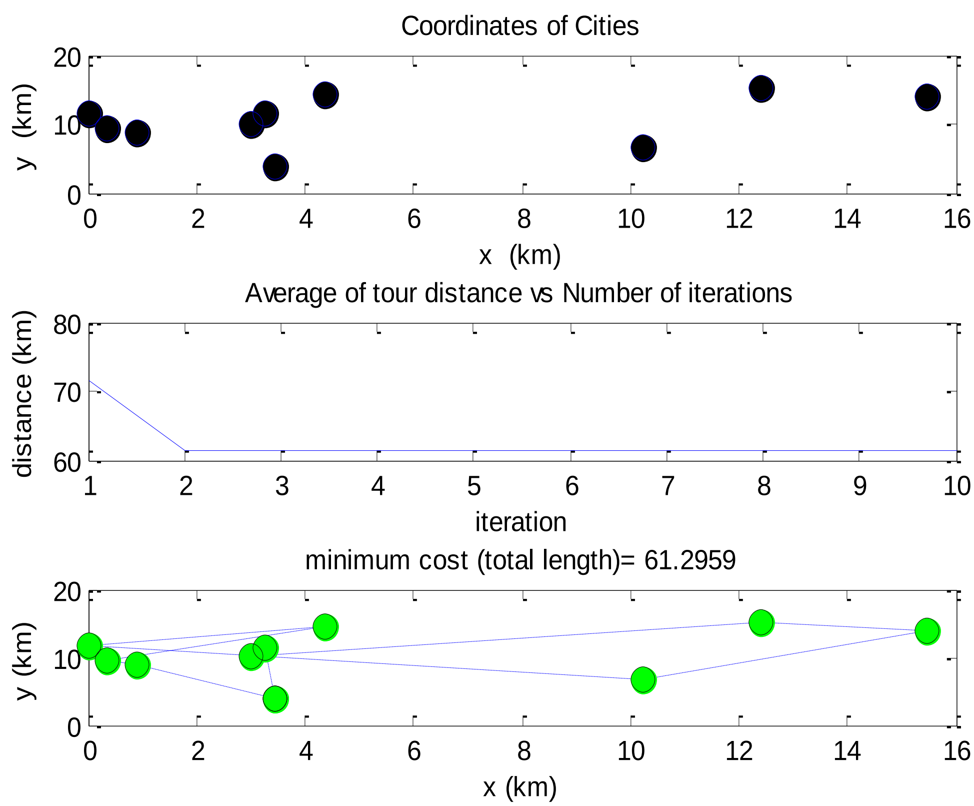


Figure 2

- Number of cities - 15

  Minimum tour length or cost of tour – 103.9245

  Total time – 0.445s

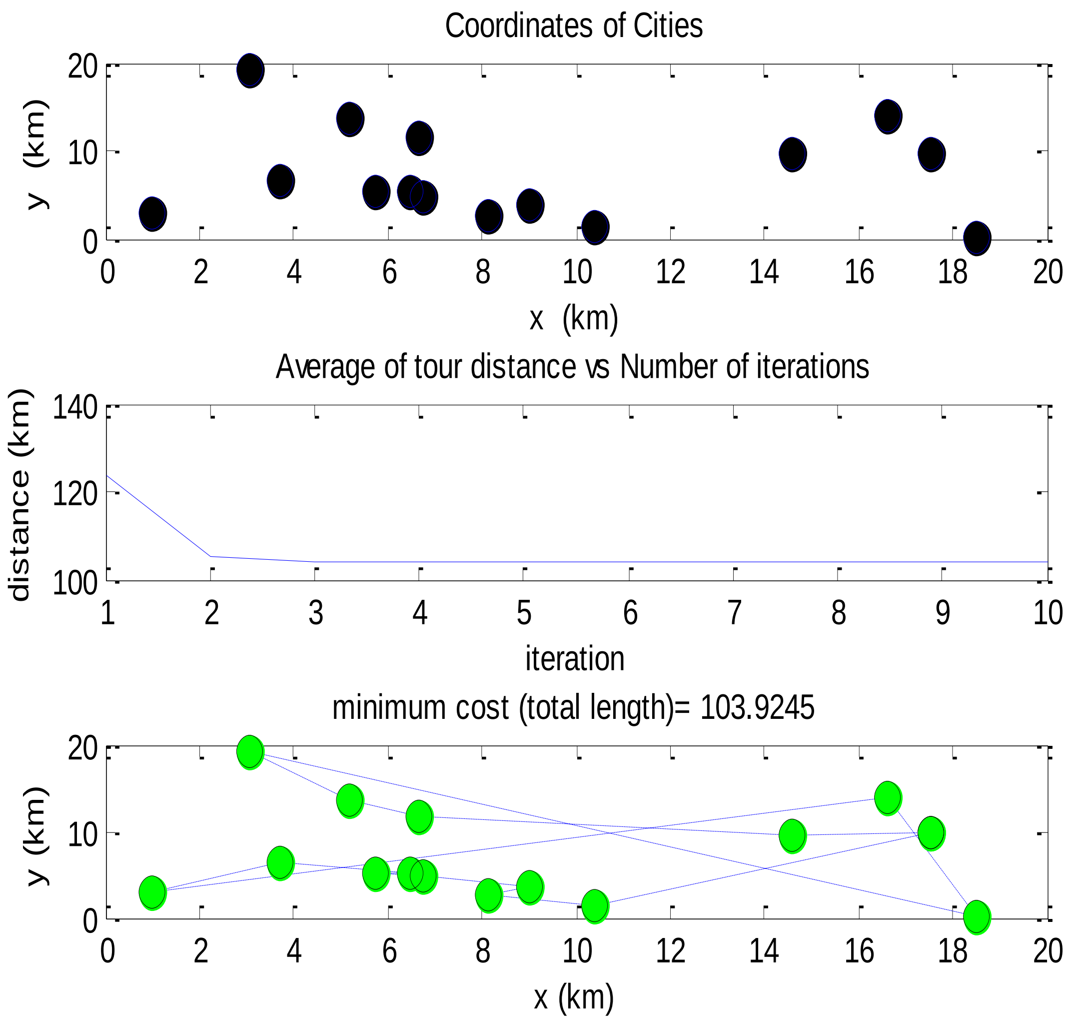


Figure 3

- Number of cities - 20

  Minimum tour length or cost of tour – 107.6694

  Total time – 0.472s

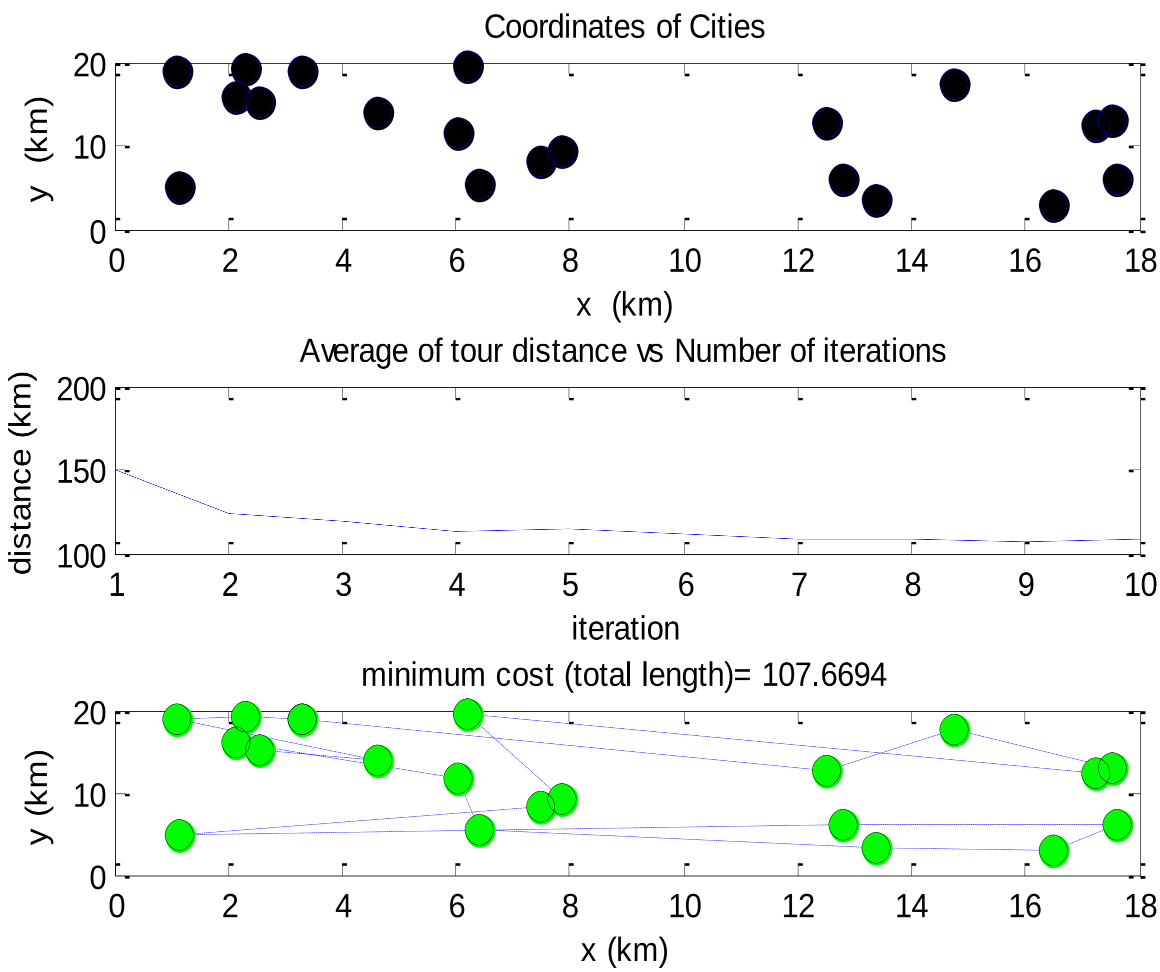


Figure 4

- Number of cities - 25

  Minimum tour length or cost of tour – 134.4631

  Total time – 0.449s

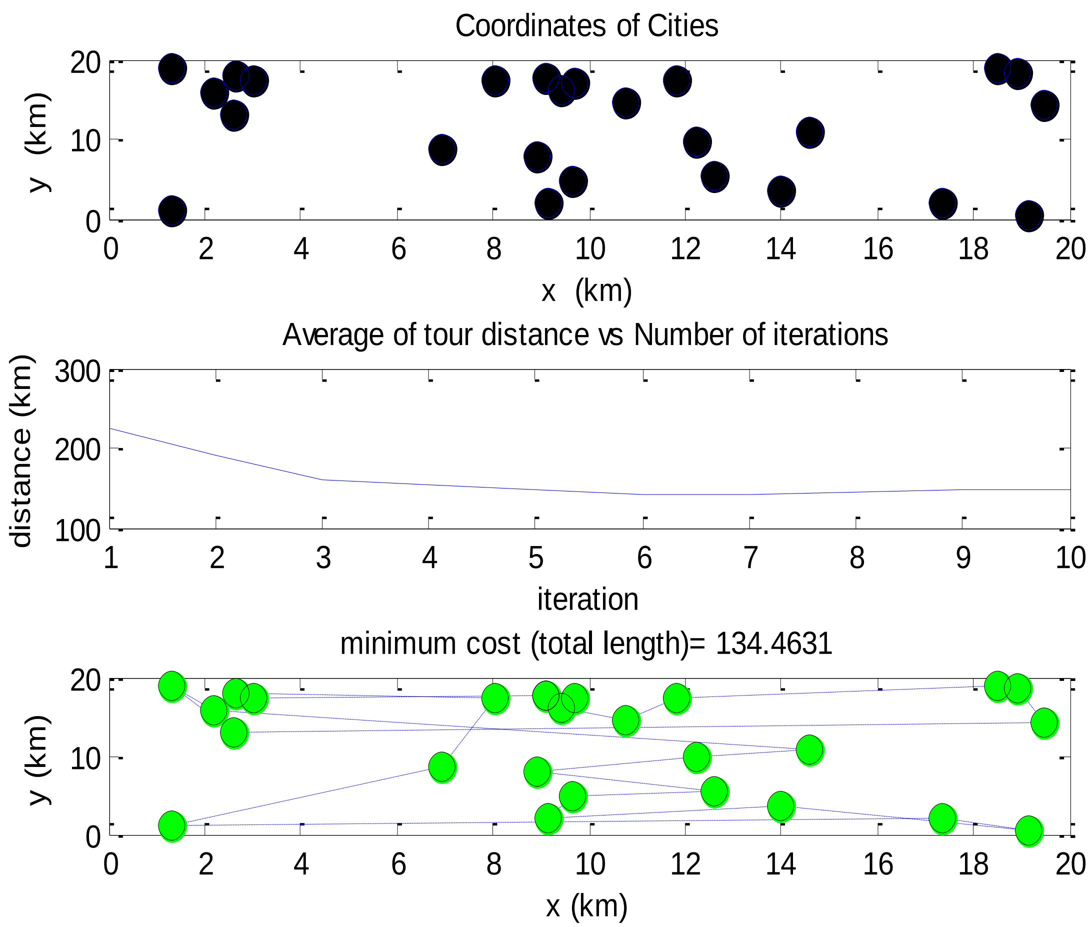


Figure 5

- Number of cities - 50

  Minimum tour length or cost of tour – 234.2743

  Total time – 0.942s

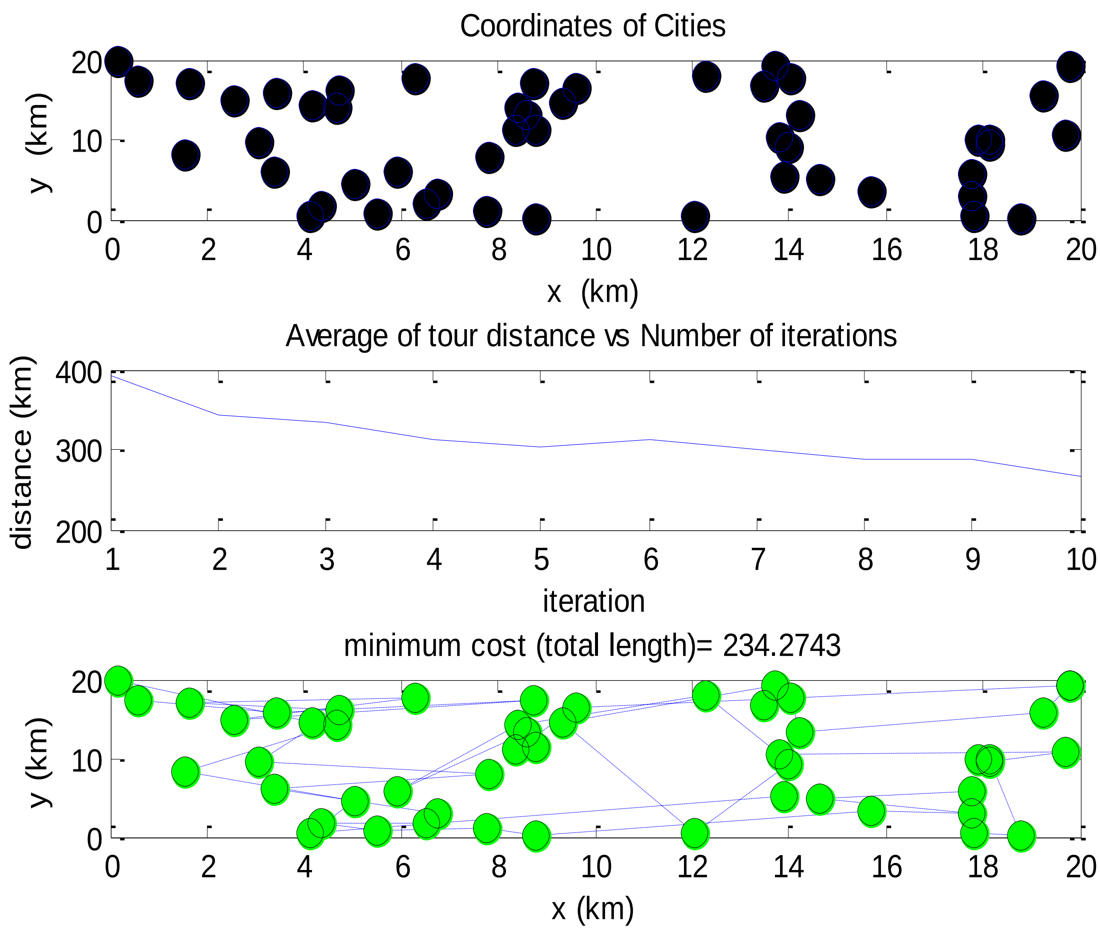


Figure 6

- Number of cities - 100

  Minimum tour length or cost of tour – 481.7417

  Total time – 2.189s

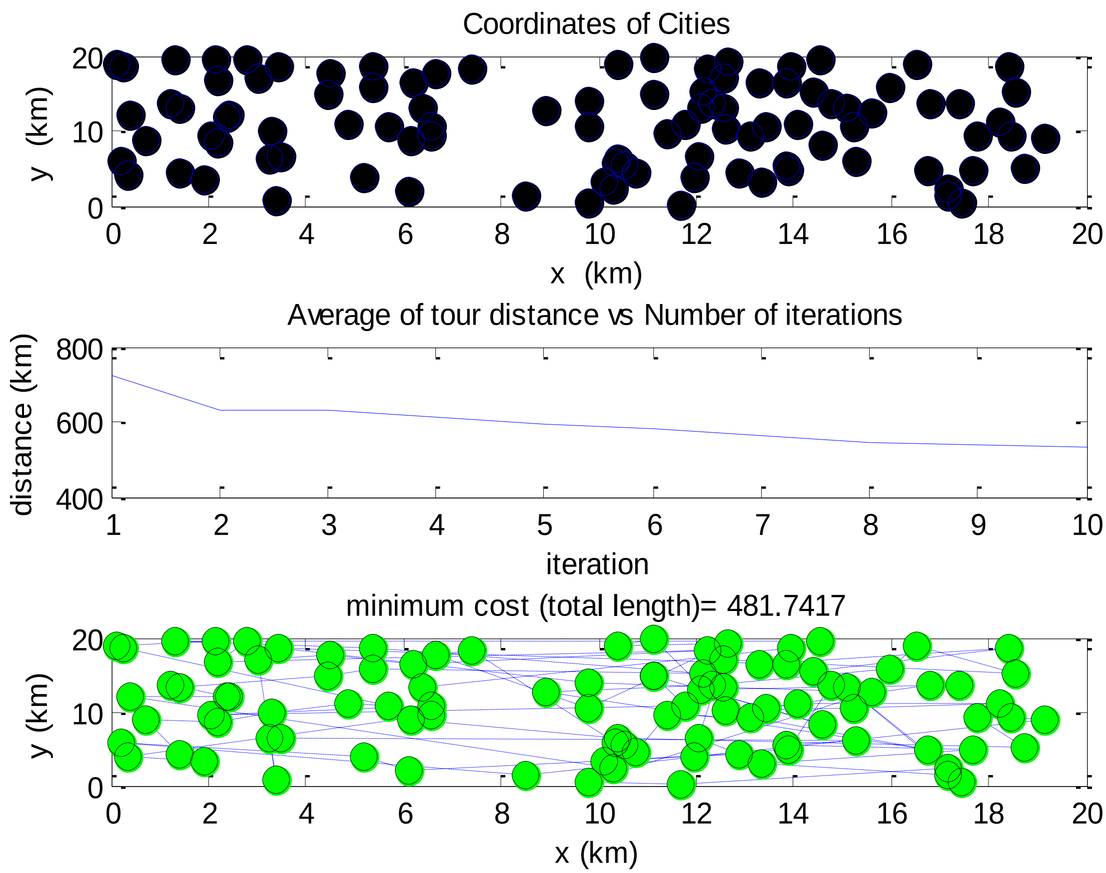


Figure 7

- Number of cities - 200

  Minimum tour length or cost of tour – 766.03

  Total time – 13.454s

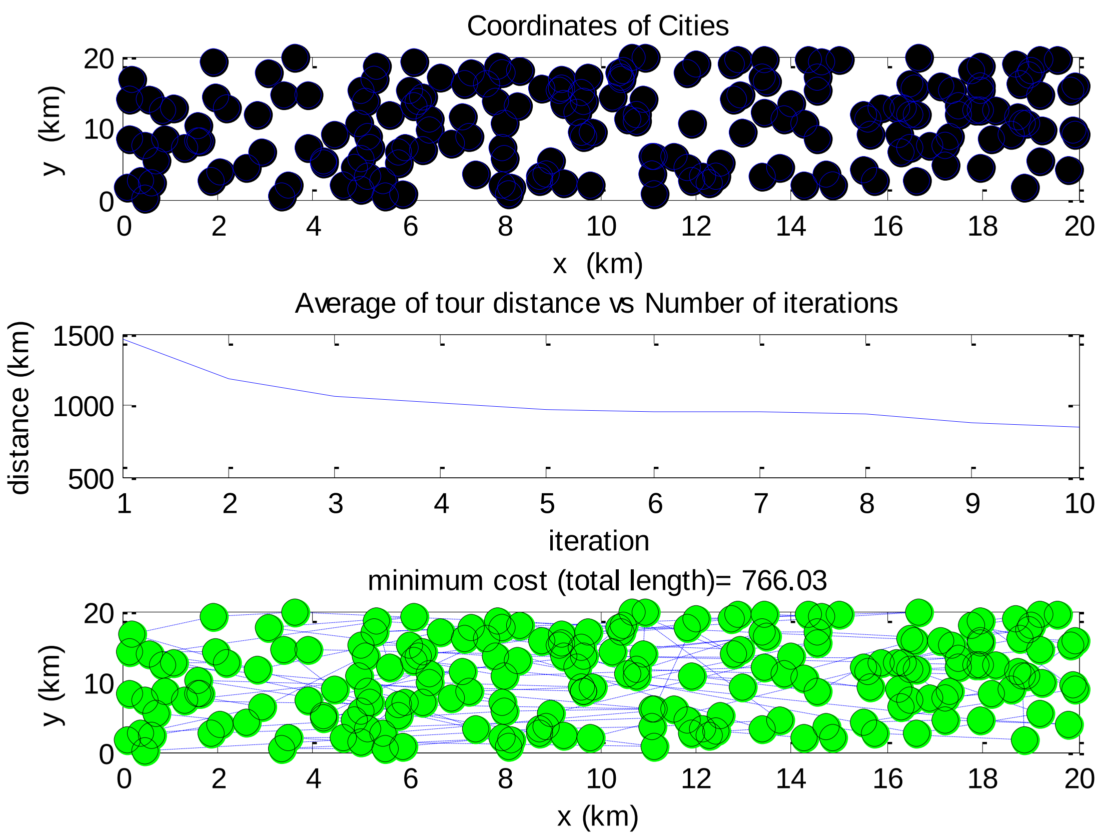


Figure 8

The advantage of ACO for TSP over the exact methods is that ant colony optimization provides good results with lesser number of iterations. Hence, ACO is better option to find a good solution in shorter time and is also useful and has its advantages for solving problems occurring in practical application.

4.2 Comparison with other methods

To compare the ACO with other known heuristics two sets of TSP problems are reviewed. The first set consists of five randomly generateda50-city problems and the second set consists of three geometric problems of betweena50 anda100 cities. The ant colony optimization algorithm is compared with other heuristics like simulated annealing (SA), elastic net (EN), and self-organizing map (SOM). ACO was run with considering ten ants and ACO almost always have the best average tour cost for every problem. ACO was compared with genetic algorithm (GA), evolutionary programming (EP), and simulated annealing (SA) on geometric instances. Again ACO gives almost the best result in nearly every case. Only for the Eil50aproblem ACO gives a slightly worse solution using real-valued distance as compared with EP, but the ACS only visits 1830 tours, while EP used 100 000 such evaluations.

Chapter 5 CONCLUSIONS

The Travelling Salesman Problem is a widely studied problem and has many applications in real world problems. Travelling Salesman Problem is a problem in which a salesman intends to visit a large number of cities exactly once and returning to starting point while minimizing the total distance travelled or the overall cost of the trip. TSP is also used as a benchmark for many optimization methods.The TSP has many applications, in the manufacture of microchips, planning and logistics The Ant Colony Optimization Algorithm is a Meta heuristic algorithm based on the behavior of some ant species. The ant colony optimization algorithm for the travelling salesman problem is tested by considering different variations in inputs. Many coefficients like pheromone evaporation rate, ant's eye sight, primary trace and number of cities are considered. The results for 10,15,20,25,50,100,200 cities are reported in the above work. For each of the input condition minimum length of tour and time to execute are also noted. The Ant Colony Optimization gave better results when compared to other heuristics.

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APPENDICES

Code of program

```
function myaco()
miter=10;
m=10;
n=10;
% parameters
e=.15;          % evaporation coefficient.
alpha=1;         % effect of ants' sight.
beta=4;          % trace's effect.
t=0.0001*ones(n); % primary tracing.
el=.97;          % common cost elimination.

for i=1:n
   x(i)=rand*20;
   y(i)=rand*20;
end
subplot(3,1,1);
plot(x,y,'o','MarkerFaceColor','k','MarkerEdgeColor','b','MarkerSize',10);
title('Coordinates of Cities');
xlabel('x  (km)');
ylabel('y  (km)');

for i=1:n
   for j=1:n
      d(i,j)=sqrt((x(i)-x(j))^2+(y(i)-y(j))^2);
```

```
    end
end
for i=1:n
    for j=1:n
        if d(i,j)==0
            h(i,j)=0;
        else
            h(i,j)=1/d(i,j);
        end
    end
end
for i=1:miter
for j=1:m
    start_places(j,1)=fix(1+rand*(n-1));
end

    [tour]=ant_tour(start_places,m,n,h,t,alpha,beta);
    tour=horzcat(tour,tour(:,1));
    [cost,f]=calculate_cost(m,n,d,tour,el);
    [t]=update_the_trace(m,n,t,tour,f,e);
    average_cost(i)=mean(cost);
    [min_cost(i),best_index]=min(cost);
    besttour(i,:)=tour(best_index,:);
    iteration(i)=i;
end

subplot(3,1,2);plot(iteration,average_cost);
title('Average of tour distance vs Number of iterations');
```

```
xlabel('iteration');
ylabel('distance (km)');
 [k,l]=min(min_cost);

for i=1:n+1
    X(i)=x(besttour(l,i));
    Y(i)=y(besttour(l,i));
End

subplot(3,1,3);plot(X,Y,'--o',...
                'MarkerEdgeColor','k',...
                'MarkerFaceColor','g',...
                'MarkerSize',10)
xlabel('x (km)');ylabel('y (km)');
title(['minimum cost (total length)= ',num2str(k)]);
end


function [t]=update_the_trace(m,n,t,tour,f,e);
for i=1:m
    for j=1:n
        dt=1/f(i);
        t(tour(i,j),tour(i,j+1))=(1-e)*t(tour(i,j),tour(i,j+1))+dt;
    end
end


function [new_places]=ant_tour(start_places,m,n,h,t,alpha,beta);
```

```
for i=1:m
    mh=h;
    for j=1:n-1
        c=start_places(i,j);
        mh(:,c)=0;
        temp=(t(c,:).^beta).*(mh(c,:).^alpha);
        s=(sum(temp));
        p=(1/s).*temp;
        r=rand;
        s=0;
        for k=1:n
            s=s+p(k);
            if r<=s
                start_places(i,j+1)=k;
                break
            end
        end
    end
end
new_places=start_places;


function [cost,f]=calculate_cost(m,n,d,at,el);
for i=1:m
    s=0;
    for j=1:n
        s=s+d(at(i,j),at(i,j+1));
    end
```

```
    f(i)=s;
end
cost=f;
f=f-el*min(f);
```