

Jaypee University of Information Technology, Waknaghat

T-3 Examination, June 2022

Ph.D - II Semester (Mathematics)

Course Title: Mathematical Analysis  
Course Code (Credits): 13P1WMA232 (3)  
Course Instructor: RAD

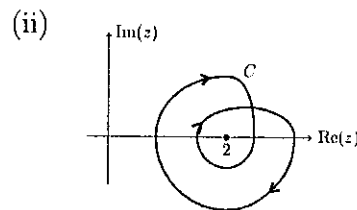
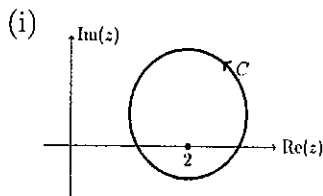
Program: Ph.D  
Marks: 35 Marks  
Time: 2 Hrs.

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**Instructions:** ALL questions are compulsory and carry equal marks.

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1. State and prove Bolzano-Weierstrass theorem.
2. (a) Show that  $f(z) = \sqrt{|xy|}$  is not analytic.  
(b) Show that  $f(z) = \log z$  is analytic everywhere except at origin and find its derivatives.
3. (a) State Cauchy's Integral formula.  
(b) Compute  $\int_C \frac{e^{z^2}}{z-2} dz$ , where  $C$  is as shown below:



4. State and prove Laurent's theorem.
5. Expand  $f(z) = \frac{1}{(z-2)(z-3)}$  in a Taylor series about  $z_0 = 1$ , valid through  $0 \leq |z-1| < 1$ .
6. Determine the Laurent series of  $f(z) = \frac{z^2-2z+3}{z-2}$  for  $|z-1| > 1$ .
7. State and prove Hahn-Banach theorem.

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