JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST - 3 EXAMINATIONS-2022

B.Tech - IV Semester (ECE)

COURSE CODE (CREDITS): 18B11EC412 (4) / 10B11EC301 (4) MAX. MARKS: 35

COURSE NAME: Fundamentals of Signals & Systems / Signals and Systems

COURSE INSTRUCTORS: Dr. Vikas Baghel MAX. TIME: 2 Hours

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*Note: All questions are compulsory. Marks are indicated against each question in square brackets.*

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**Q1.** **a)** What is the difference between continuous and discrete signal? **[1]** **[CO1]**

**b)** What are the conditions for the existence of Fourier transform? **[1]**

**c)** Explain Gibbs phenomenon with example. **[2]**

**d)** What are the limitations of Fourier transform and use of Laplace transform? **[1]**

**Q2.** **a)** Find $x(0^+)$ for given LT of $x(t)$ **[1]** **[CO4]**

$$X(s) = \frac{2s^3 + 5s^2 + 12s}{s^3 + 4s^2 + 14s + 20}$$

**b)** How many signals have a Laplace transform that may be expressed as **[4]**

$$X(s) = \frac{1}{(s+1)(s+2)}$$

in its region of convergence? Write all signals expression.

**Q3.** **a)** Suppose that we are given the following information about an LTI system: **[3]** **[CO4]**

i. The system is causal.

ii. The system function is rational and has only two poles, at $s = -2$ and $s = 4$.

iii. If $x(t) = 1$, then $y(t) = 0$

iv. The value of the impulse response at $t = 0^+$ is 4.

From this information determine the system function of the system.

**b)** Show that **[2]**

$$\frac{t^{n-1}}{(n-1)!} e^{-at} u(t) \quad \overset{LT}{\longleftrightarrow} \quad \frac{1}{(s+a)^n} \qquad \Re\{s\} > -a$$

**Q4.** **a)** Find inverse z-transform of following sequences. Sketch the pole-zero plot and indicate the region of convergence. **[3]** **[CO4]**

i. $x[n] = 7\left(\frac{1}{3}\right)^n u[n] - 6\left(\frac{1}{2}\right)^n u[n]$

ii. $x[n] = \left(\frac{1}{3}\right)^n \sin\left(\frac{\pi}{4}n\right) u[n]$

**b)** Find inverse z-transform of **[2]**

i. $X(z) = \log(1 + az^{-1}), \quad |z| > |a|$

ii. $X(z) = 4z^2 + 2 + 3z^{-1}, \quad 0 < |z| < \infty$

**Q5.** **a)** Determine whether following systems with given system functions are causal or not! **[2]** **[CO4]**

i. $H(z) = \frac{z^3 - 2z^2 + z}{z^2 + \frac{1}{4}z + \frac{1}{8}}$

ii. $H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{1}{1 - 2z^{-1}}, \quad |z| > 2$

**b)** Suppose that we are given the following information about an LTI system: **[3]**

i. If the input to the system is $x_1[n] = \left(\frac{1}{6}\right)^n u[n]$, then the output is

$$y_1[n] = \left[a\left(\frac{1}{2}\right)^n + 10\left(\frac{1}{3}\right)^n\right] u[n]$$

where $a$ is a real number.

ii. If the input to the system is $x_2[n] = (-1)^n$, then the output is

$$y_2[n] = \frac{7}{4}(-1)^n$$

Determine the system function using given information.

**Q6.** **a)** Show that **[2]** **[CO4]**

$$na^n u[n] \xleftrightarrow{ZT} \frac{az^{-1}}{(1 - az^{-1})^2} \qquad |z| > |a|$$

**b)** Consider a causal and stable LTI system for which the input $x[n]$ and output $y[n]$ satisfy the linear constant-coefficient difference equation **[3]**

$$y[n] - \frac{1}{2}y[n-1] = x[n] + \frac{1}{3}x[n-1]$$

Determine impulse response of the system.

**Q7.** **a)** Let $x(t)$ be a signal with Nyquist rate $\omega_0$. Determine the Nyquist rate for each of the following signals: **[3]** **[CO5]**

i. $\frac{dx(t)}{dt}$

ii. $x(t) + x(t-1)$

iii. $x(t)\cos(\omega_0)$

**b)** Impulse-train sampling of $x[n]$ is used to obtain **[2]**

$$g[n] = \sum_{k=-\infty}^{\infty} x[n]\delta[n - kN]$$

If $X(e^{jw}) = 0$ for $\frac{3\pi}{7} \leq |\omega| \leq \pi$, determine the largest value for the sampling interval which ensures that no aliasing takes place while sampling $x[n]$.