Jaypee University of Information Technology, Waknaghat
Test-3 Examination, June 2022
B.Tech - II Semester (CSE/IT/ECE/ECM/CE)

Course Code/Credits: 18B11MA211/4
Course Title: Engineering Mathematics-II
Course Instructors: RAD, KAS, RKB, BKP

Max. Marks: 35

Max. Time: 2 Hrs.

Instructions: All questions are compulsory. Marks are indicated against each question.

1. Examine the convergence of the alternating series $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n^2+1}$.   (3 Marks) [CO-1]

2. Solve $y'' - 2y' = e^x \sin x$ using the method of variation of parameters.   (3 Marks) [CO-2]

3.   (4 Marks) [CO-3]

   (a) Express $\int x^{-3}\mathcal{J}_4(x)\, dx$ in terms of $\mathcal{J}_0(x)$ and $\mathcal{J}_1(x)$.

   (b) Evaluate the value of $\mathcal{P}_n(1)$ and write $x^3 - 2x^2 + 1$ using the Legendre's polynomials.

4. The vibrations of an elastic string is governed by the partial differential equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$. The length of the string is π and the ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0) = 2(\sin x + \sin 3x)$. Find the deflection $u(x,t)$ of the vibrating string for $t > 0$.   (4 Marks) [CO-4]

5. Consider the following function of complex variables: (3 Marks) [CO-5]

$$ f(z) = \begin{cases} \frac{x^3 y(y-ix)}{x^6+y^2} & , \ z \neq 0 \\ 0 & , \ z = 0 \end{cases} $$

   (a) Find $f'(0)$ by using limit definition along (i) $y = mx$ (ii) $y = x^3$.

   (b) Is $f(z)$ differentiable at $z = 0$? Give reasons.

6. Consider $v(x, y) = -2(e^y - e^{-y})\sin x$.   (5 Marks) [CO-5]

   (a) Show that $v(x, y)$ is harmonic.

   (b) Construct an analytic function $f(z) = u + iv$.

7. Evaluate the following integral by Cauchy integral formula: (6 Marks) [CO-6]

$$ \oint_c \frac{z-1}{(z+1)^2(z-2)}\, dz $$

   where $C : |z - i| = 2$.

8.   (7 Marks) [CO-7]

   (a) Expand $f(z) = \frac{z^2}{(z-3)(z-4)}$ in a Laurent series valid in (i) $|z| < 3$ (ii) $|z| > 4$.

   (b) Evaluate the real integral $\int_0^{2\pi} \frac{1}{2 + \sin\theta}\, d\theta$ using contour integration technique.