COURSE CODE (CREDITS): 18B11MA413   MAX. MARKS: 35

COURSE NAME: DISCRETE MATHEMATICS

COURSE INSTRUCTORS: PKP   MAX. TIME: 2 Hours

Note: All questions are compulsory. Marks are indicated against each question in square brackets.

Q1. (a) Find the generating function for the sequence $1, 2, 2^2, 2^3, \cdots$ [CO-1][2M]

(b) Prove or disprove that $f(x) = 5x^3 - 2x^2 + 3x - 7$ is big-O of x^3 [2M]

(c) Write the negation of $\forall x \exists y \exists z (x^2 + y^2 > z^2)$.   [1M]

Q2. In the following Hasse diagram of a poset, write minimal, maximal, least and greatest elements? Give reasons in support of your answers (why they exist/ not exist).   [CO-2] [4M]
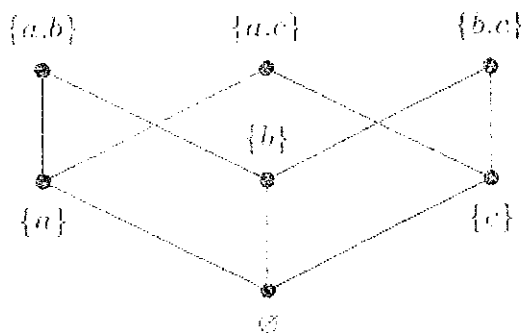
Q3. (a) State the Konigsberg bridge problem. Using a graph, model the Konigsberg bridge problem, and hence obtain its solution.   [CO-3] [3M]

(b) Draw K_6 graph, and using Kuratowski's theorem prove or disprove that it is a planar graph.   [CO-3] [3M]

Q4. Using Welsh Powell algorithm colour the following graph (vertices), and also write its chromatic number: [CO-3] [5M]
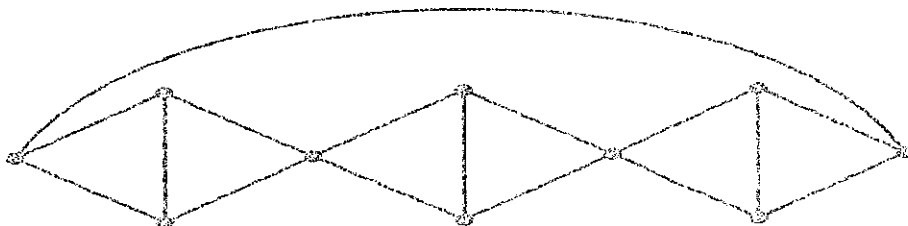
Q5. Give reasons to support/oppose the following: [CO-4] [5M]
    (a) Every graph has a Minimum Spanning Tree.
    (b) Cycle graph C_8 is both Hamiltonian, and Eulerian graph.
    (c) Graphs K_3 and C_4 are homeomorphic.
    (d) $(\mathbb{Z}^+, +)$ is a Monoid.
    (e) $(2\mathbb{Z}, +)$ is a Semigroup.

Q6. Prove or disprove that set of all real numbers with usual addition + is a group?
    Is it an Abelian group also?   [CO-4] [5M]

Q7.   [CO-5] [1+4M]
    (a) Give an example of a Language over the alphabet $\{\alpha, \beta, \gamma, \delta\}$, which begins with α.
    (b) Using Prim's algorithm find an MST for the following graph:

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