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JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT
MID SEMESTER EXAMINATION-2015
B.Tech. II Semester (ECE/CSE/IT)

COURSE CODE: 10B11MA211
COURSE NAME: DISCRETE MATHEMATICS
COURSE CREDITS: 4

MAX. MARKS: 30
MAX. TIME: 2 HRS

***Note:** All questions are compulsory. Use of Calculator is not allowed.*

Section A **(1x6 = 6 Marks)**

1. If $I_n = \left(-1-\frac{1}{n}, 1+\frac{1}{n}\right)$, then find $\bigcup_{n=1}^{\infty} I_n$ and $\bigcap_{n=1}^{\infty} I_n$.
2. How many partitions are possible for the sets S_1 and S_2, where $|S_1 = 5|$ and $|S_2 = 8|$.
3. Find the dual of $A \subseteq (B \cup C)$.
4. Can a relation R on a set A be both symmetric and anti symmetric? Give an example to justify.
5. In view of a universal set, explain a set using its characteristic function.
6. Write the detailed form of contra positive of the following statement:
   $(\forall x \in A\ \exists y \in B\ \ p(x,y) \wedge q(x,y)) \Rightarrow (\exists x \in A\ \forall y \in B\ \ p(x,y) \Rightarrow q(x,y))$.

Section B **(9 Marks)**

7. Find the complexity of finding the sum of squares of n consecutive natural numbers in terms of Big O notation. (3 Marks)
8. Let **N** denote the set of all natural numbers and let R be a relation on **N X N** defined by
   $$(a,b)\ R\ (c,d)\ iff\ ad(b+c) = bc(a+d).$$ (3 Marks)
   Show that R is an equivalence relation and find equivalence class of (2, 3) and (3, 5).
9. Let $p(x): x^2 > x$ and $q(x): x^2 = x$, and the universe of discourse is set of integers. Determine the truth value of each quantified statement: (3 Marks)
   $(i)\ \exists x\ p(x) \wedge q(x)$ $(ii)\ \forall x\ p(x) \wedge q(x)$ $(iii)\ \exists x\ p(x) \vee q(x)$ $(iv)\ \forall x\ p(x) \vee q(x)$

Section C **(15 Marks)**

10. For a fixed positive integer n and let a,b,c,d are integers such that $a \equiv b (\bmod n)$ and $c \equiv d (\bmod n)$. Prove that $a + c \equiv b + d (\bmod n)$ and $ac \equiv bd (\bmod n)$. (3 Marks)
11. Using generating function, find a formula for numbers a_k where a_k satisfy the recurrence relation
    $$a_k - 7a_{k-1} + 10a_{k-2} = 0; \quad k \geq 2,\ a_0 = 1,\ a_1 = 8.$$ (3 Marks)
12. Using matrix multiplication, find the transitive closure of the relation R given by
    $$R = \{(1,2),(2,3),(3,4),(4,1)\} \text{ on } A = \{1, 2, 3, 4\}.$$ (3 Marks)
13. Use principle of mathematical induction to show that (3 Marks)
    $$1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} \leq 2 - \frac{1}{n}; \quad \forall n \in \text{ set of natural numbers.}$$
14. Using Truth table, check the validity of the following argument: (3 Marks)
    If the computer was down Saturday afternoon, then Mary went to a matinee. Either Mary went to a matinee or took a nap Saturday afternoon. Mary did not take a nap that afternoon. Therefore, the Computer was down Saturday afternoon.

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