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JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT
END SEMESTER EXAMINATION – MAY 2015
B.Tech. II Semester (ECE/CSE/IT)

COURSE CODE: 10B11MA211 MAX. MARKS: 45
COURSE NAME: DISCRETE MATHEMATICS
COURSE CREDITS: 4 MAX. TIME: 3 HRS

Note: *All questions are compulsory. Carrying of mobile phone during examinations will be treated as case of unfair means. Use of Calculator is not allowed.*

Section A **(12 Marks)**

1. Let $P(S)$ denotes the power set of S. Write the cardinality of $\{\phi\} \times P(\phi)$.
2. Suppose R is a relation on the set of integers such that $(a,b) \in R$ iff $a^2 = b^2$. Give reasons in support/oppose of the statement "R is an equivalence relation."
3. Consider the following recursive function:

$$M(n) = \begin{cases} n-10 & , \text{ if } n > 100 \\ M(M(n+11)) & , \text{ if } n \le 100 \end{cases}, \quad \forall n \in Z^+.$$

Find $M(100)$.

4. Let $p(n): 5n+3$ is even; $q(n): 4n+3$ is even. Give an example of an integer such that $p(n) \Rightarrow q(n)$ is true.
5. How many Hamiltonian circuits are there in a complete graph with 6 vertices? Justify your answer.
6. Draw the dual graph of Q_3 i.e. 3-cube.
7. Suppose that a connected planar graph has eight vertices, each of degree three. Into how many regions is the plane divided by a planar representation of this graph.
8. Find the chromatic number of a wheel graph in general.
9. Consider the binary operation $*$ defined on the set of positive integers Z^+ as $a*b = a+b+2; \forall a,b \in Z^+$. Determine whether $*$ is associative or not.
10. Give an example of a non-abelian group.
11. Show that D_{105} is a Boolean Algebra.
12. Give an example of Type 2 grammar.

Section B **(18 Marks)**

13. Let $A = \{0,1,2,3,4\}$ and R defines an equivalence relation on A given by

R = {(0, 0), (0,4), (1, 1), (1, 3), (2, 2), (3, 1), (3, 3), (4, 0), (4, 4)}. Find the equivalence class of every element of A and show that distinct equivalence classes partition the set A.

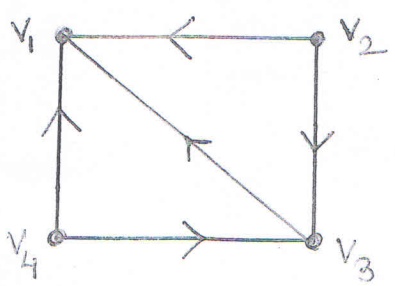
14. (a) Define a bijective function from the set of integers to the set of natural numbers. Also justify.

(b) Explain the Big O notation and give an illustrative example.

15. (a) Using truth table, verify that $[p \Rightarrow (q \Rightarrow r)] \Rightarrow [(p \Rightarrow q) \Rightarrow (p \Rightarrow r)]$ is a tautology.

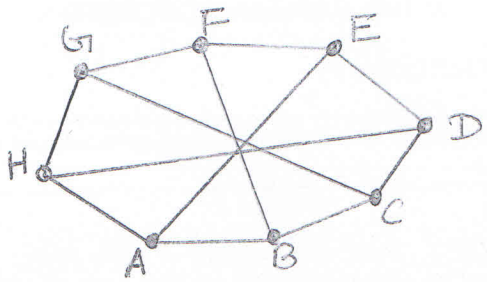
(b) Write the condition for the validity of an argument.

16. Define a r-regular graph with an example. Prove that if a graph of order (number of vertices) $3n$, $(n \ge 1)$ has equal number of vertices of degrees $n-1$, n and $n+1$, then n is even.
17. Using adjacency matrix, find the path matrix of the following directed graph:



[PTO]

18. State Kuratowski's theorem. What are Kuratowski's graphs? Using theorem, investigate the planarity of the following graph:



Section C **(15 Marks)**

19. Distinguish between Prim's and Kruskal's algorithm with the help of an example of a weighted graph. A certain tree T with 21 vertices has only vertices of degree 1, 3, 5 and 6. If T has exactly 15 end vertices and 1 vertex of degree 6, how many vertices of T have degree 5?

20. Give examples of following types of algebraic structures:
    (a) Finite cyclic group with 7 elements.
    (b) Finite Integral domain
    (c) Non-commutative ring
    (d) A ring with zero divisors
    (e) A ring without unity
    (f) A ring with zero divisors

21. Consider the lattice L given below:
    (a) Which non zero elements are join irreducible?
    (b) Which elements are atoms?
    (c) Is L distributive?
    (d) Is L a complemented lattice?
    (e) Find the least and greatest element of the lattice.
    (f) Find the minimal and maximal elements of the lattice. Privileged auspicious wish a hearty co

22. What is the Kleene closure of a language? Find the language of the following grammar:
    $G = (V, T, S, P)$, where $V = \{S, W, a, b, c\}, T = \{a, b, c\}, S = \{S\}, P = \{S \rightarrow aW, W \rightarrow bbW, W \rightarrow c\}$.

23. Let M be a finite state machine whose state table is as follows:
    (a) Find the input set I, set of states S, and output set O and draw the state diagram.
    (b) Suppose $w = aababaabbab$ is an input string. Find the corresponding output string.

	a	b	a	b
s_0	s_1, s_2		x, y	
s_1	s_3, s_1		y, z	
s_2	s_1, s_0		z, x	
s_3	s_0, s_2		z, x	

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