Dr. Ravinder Bhatt

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATION (December 2018)

B.Tech (CSE&IT) VII Semester

COURSE CODE: 10B1WCI733 MAX. MARKS: 35

COURSE NAME: Graph Algorithms and Applications

COURSE CREDITS: 3 MAX. TIME: 2Hrs

Note: All questions are compulsory.

1. [3 + 3 Marks] [CO 1]

a. Draw all the three vertex tournaments whose vertices are u, v, x .

Prove or disprove: Let G and H be two tournaments on a vertex set V.

$d_G^+(v) = d_H^+(v)\ \forall v \in V$ if and only if G can be turned into H by a sequence of direction-reversals on cycles of length 3.

b. Show that the complete graph of four-vertices is self-dual.

Prove or disprove: A plane graph has a cut-vertex if and only if its dual has a cut-vertex.

2. [3 + 3 Marks] [CO 2]

a. Let G be a weighted connected graph with distinct edge weights. Prove that G has only one minimum-weight spanning tree.

b. Explain how to use Breadth-First Search to compute the girth of a graph (The girth of a graph with a cycle is the length of its shortest cycle. A graph with no cycle has infinite girth).

3. [3 + 3 Marks] [CO 3]

a. Write subroutine for converting adjacency matrix to incidence matrix in the graph.

b. Given a set of lines in the plane with no three meeting at a point, form a graph G whose vertices are the intersections of the lines, with two vertices adjacent if they appear consecutively on one of the lines. Prove that $\chi(G) \leq 3$.

4. [3 + 3 Marks] [CO 3]

a. State Kuratowski's theorem. Using Kuratowski's theorem, show that the graph shown in figure A (known as Petersen's graph) is nonplanar.

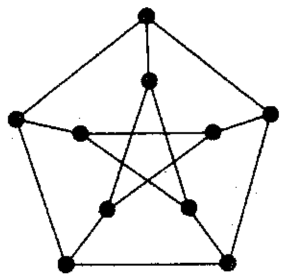

Figure A

b. Prove or disprove:

If x , y are vertices of a graph G and xy is not an edge in G, then the minimum size of an x, y cut equals the maximum number of pairwise internally disjoint x, y paths.

5. [3 + 3 Marks] [CO 4]

a. Two people play a game on a graph G, alternately choosing distinct vertices. Player 1 starts by choosing any vertex. Each subsequent choice must be adjacent to the preceding choice (of the other player). Thus together they follow a path. The last player able to move wins. Prove that the second player has a winning strategy if G has a perfect matching, and otherwise the first player has a winning strategy.

b. Suppose that you are required to make a class schedule in a university. There are a total of n courses to be taught in m available hours of the week. There are pairs of courses that cannot be taught at the same time because some students might like to take both. Explain how you will make the schedule. State the condition when it will be impossible to make the compatible schedule.

6. [2.5 + 2.5 Marks] [CO1]

Write short notes on

a. Graphs in Markov process

b. 5-color theorem.