JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATIONS- 2026

Ph.D. II Semester (MATHEMATICS)

COURSE CODE (CREDITS): 13P1WMA232 (3) MAX MARKS: 35

COURSE NAME: MATHEMATICAL ANALYSIS

COURSE INSTRUCTOR: P K Pandey MAX. TIME: 2 Hours

Note: *(a) All questions are compulsory.*

(b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems (e.g. unless mentioned otherwise assume $\mathbb{F} = \mathbb{R}$).

(c) Use of calculator is not allowed.

Q.No.	Question	Marks
Q1	Explain Maximum modulus principle. Using the Maximum modulus principle find max $\lvert f(z)\rvert$, for $f(z) = e^z$ on the closed square $S = \{z = x + iy: 0 \leq x \leq 1, 0 \leq y \leq \pi\}$. Also specify points where max $\lvert f(z)\rvert$ is attained.	3.5
Q2	Evaluate $\oint_\Gamma \frac{1}{1+z^2} dz$ where $\Gamma: \left\lvert z - \frac{i}{2}\right\rvert = 1$ taken in positive sense.	3.5
Q3	Explain the difference between Cauchy's Integral Theorem & Cauchy's Integral Formula. Using Cauchy's Integral Formula evaluate: $\oint_\Gamma \frac{5\cos 2\pi z}{(2z-1)(z-3)} dz$ where $\Gamma: \lvert z\rvert = 1$ taken in positive sense.	3.5
Q4	State Liouville theorem and using it prove the Fundamental theorem of algebra.	3.5
Q5	State Schwarz lemma, and check whether Schwarz lemma applies to function $f(z) = \frac{z^2}{4} + \frac{z}{2}$. Also, estimate a bound for $\lvert f(0.6i)\rvert$.	3.5
Q6	Define a linear space. Consider the space of continuous functions $S = \{f \in C[0,1]: f(x) \geq 0, \forall x \in [0,1]\}$, prove or disprove that it is a linear space.	3.5
Q7	Define norm on a linear space. Consider $C^1[0,1]$ the space of continuously differentiable functions on $[0,1]$, define $\lVert f\rVert = \max_{x\in[0,1]} \lvert f(x)\rvert + \max_{x\in[0,1]} \lvert f'(x)\rvert$ prove or disprove that it is a norm.	3.5
Q8	Define Banach space. Consider linear space of rationals $\mathbb{Q}$ (over $\mathbb{Q}$) with $\lVert x\rVert = \lvert x\rvert$. Verify whether it is a Banach space or not?	3.5
Q9	Consider a discrete metric space (X, d). Say $X = \{a, b, c, d\}$ and $A = \{a, b, c\}$. Determine the set of all (i) Adherent points of A (ii) limit points of A, and (iii) isolated points of A.	3
Q10	What is a compact set? Check whether $K = \left\{\frac{1}{n} : n \in \mathbb{Z}^+\right\} \cup \{0\}$ is a compact set or not.	2
Q11	Write the statement of the following: (i) Open Mapping theorem (ii) Closed graph theorem.	2