(176)

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATIONS- 2026

PhD-II Semester (MATH)

COURSE CODE (CREDITS): 13P1WMA232 (3) MAX MARKS: 25

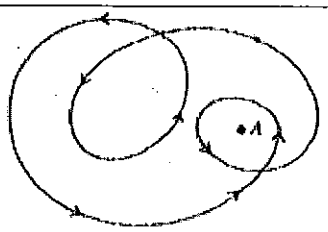
COURSE NAME: MATHEMATICAL ANALYSIS

COURSE INSTRUCTOR: P K Pandey MAX. TIME: 1 Hour 30 Min

Note: *(a) All questions are compulsory.*

(b) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems.

(c) Use of calculator is not allowed.

Q. No	Question	Marks
Q1	State mean value theorem and verify it for $f(x) = (x-1)(x-2)(x-3)$; $x \in [0,4]$.	4
Q2	Investigate the differentiability of following function at the point $x = 0$: $$f(x) = \begin{cases} x\sqrt{\lvert x\rvert}\sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$	4
Q3	Using Milne-Thomson method find a holomorphic function $f(z)$ whose imaginary part is $v(x,y) = \frac{y}{x^2+y^2}$; $(x,y) \neq (0,0)$.	4
Q4	State Weierstrass M-Test, and test the uniform convergence of $\sum_{n=1}^{\infty} \frac{cos\,(2^n x)}{3^n}$.	4
Q5	Using the generalised Cauchy–Goursat theorem evaluate the following: $$\oint_\gamma \frac{e^z}{z(z-2)}dz$$ Where γ: $\lvert z\rvert = 3$ taken in positive sense.	3
Q6	Use generalised Cauchy's Integral formula to evaluate: $$\oint_\gamma \frac{\cos z}{(z+2)^3}dz$$ where γ is the closed contour shown in the figure 1, and the marked point A represents $z = -2$.  Fig. 1	3
Q7	Find the Laurent series of $f(z) = \frac{1}{(1-z)(z-2)}$ about $z_0 = 0$.	3

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