JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -1 EXAMINATION- 2026

Ph.D.-II Semester (MATHEMATICS)

COURSE CODE (CREDITS): 13P1WMA232 (3) MAX. MARKS: 15

COURSE NAME: MATHEMATICAL ANALYSIS

COURSE INSTRUCTOR: PRADEEP KUMAR PANDEY MAX. TIME: 1 Hour

Note: *(a) All questions are compulsory.*

(b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.

(c) Use of calculator is not allowed.

Q.No.	Question	Marks
Q1	In a digital communication, binary messages of length 6 bits are transmitted. Consider $X = \{0,1\}^6$, and $\forall x, y \in X$, define $d(x,y) =$ the number of bit positions that differ in messages (strings) x and y. (i) Prove or disprove that (X, d) is a metric space. (ii) Verify triangle inequality for messages $x = 000000, y = 111000$, $z = 111111$.	3
Q2	Consider a metric space $(\mathbb{R}, d)$, where $d(x,y) = \frac{\|x-y\|}{1+\|x-y\|}$ for all $x, y \in \mathbb{R}$. Compute and explain the open ball and the closed ball with centre 0 and radius 0.9.	3
Q3	Define a complete metric space. Give one example and one non-example of a complete metric space.	3
Q4	State Bolzano Weierstrass theorem and explain the following: Given a sequence $(x_n)_{n=1}^{\infty}$ in $\mathbb{R}$, with $x_n = \begin{cases} 0, & \text{if } n \text{ is even} \\ n, & \text{if } n \text{ is odd} \end{cases}$ (i) Check whether sequence $(x_n)_{n=1}^{\infty}$ is bounded or not? (ii) The sequence $(x_n)_{n=1}^{\infty}$ has a convergent subsequence or not? (iii) Explain whether sequence $(x_n)_{n=1}^{\infty}$ violate the Bolzano Weierstrass theorem?	3
Q5	Compute the lim inf x_n and lim sup x_n for the sequence (x_n), where: $$x_n = (-1)^n + \sin\left(\frac{n\pi}{2}\right), n = 1,2,3,\cdots$$	3

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