

Course Code/Credits: 22B1WMA731/3

Max. Marks: 75

Course Title: Linear Algebra for Data Science & Machine Learning

Max. Time: 2 Hours

Course Instructor: RAD

Note: (a) ALL questions are compulsory.

(b) Scientific calculators are allowed.

(c) The candidate is allowed to make suitable numeric assumptions wherever required.

Q.No	Question	CO	Marks								
Q1	Determine whether the following sets are subspaces of $\mathbb{R}^3$ : (a) The set of all vectors $(x, y, z)$ satisfying $x^2 + y^2 + z^2 = 0$ . (b) The set of all vectors $(x, y, z)$ such that $x - 2y + z = 0$ .	CO-1	7								
Q2	Show that the polynomials $f(t) = 1$ , $g(t) = t - 1$ , and $h(t) = t^2$ form a basis for the vector space of all polynomials of degree at most 2. Hence, find the dimension of the space.	CO-1	8								
Q3	Let $W$ be the subspace of $\mathbb{R}^3$ spanned by $\mathbf{w}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{w}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ . Find a basis for $W^\perp$ and determine $\dim(W^\perp)$ .	CO-2	10								
Q4	Given the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \end{bmatrix}$ , find the null space of $A$ and determine its rank and nullity.	CO-2	10								
Q5	Let $\mathbf{u} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ , $\mathbf{v} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ , and $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ . Find the orthogonal projection of $\mathbf{w}$ onto the subspace spanned by $\mathbf{u}$ and $\mathbf{v}$ .	CO-2	10								
Q6	Find the least-squares solution of the system $Ax = b$ : $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$	CO-3	10								
Q7	Determine the singular value decomposition of $A = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$ .	CO-3	10								
Q8	Consider the following data representing the number of hours studied ( $x$ ) and the corresponding marks obtained ( $y$ ) by three students: <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <th>Hours Studied (<math>x</math>)</th> <th>Marks Obtained (<math>y</math>)</th> </tr> <tr> <td>1</td> <td>2</td> </tr> <tr> <td>2</td> <td>4</td> </tr> <tr> <td>3</td> <td>6</td> </tr> </table> (a) Compute the covariance matrix. (b) Identify the principal components of the data.	Hours Studied ( $x$ )	Marks Obtained ( $y$ )	1	2	2	4	3	6	CO-4	10
Hours Studied ( $x$ )	Marks Obtained ( $y$ )										
1	2										
2	4										
3	6										