JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATION- 2025

B. Tech.-III Semester (ECE/ECS/EEVLSI/MC)

COURSE CODE (CREDITS): 25B11MA311 (4) MAX. MARKS: 35

COURSE NAME: PROBABILITY AND RANDOM PROCESSES

COURSE INSTRUCTORS: SST MAX. TIME: 2 Hours

Note: (a) All questions are compulsory.

(b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.

(c) Use of a scientific calculator is allowed.

<table>
<tr><th>Q. No.</th><th>Question</th><th>CO</th><th>Marks</th></tr>
<tr><td>Q1</td><td>The joint probability density function for random variables X and Y is as follows:

$$f_{XY}(x,y) = \begin{cases} \frac{x(1+3y^2)}{4}, 0 < x < 2, 0 < y < 1 \\ 0, elsewhere \end{cases}$$

a) Find the marginal probability density function of X.
b) Find the conditional probability density $P_{Y|X}(y|x=1)$.</td><td>2</td><td>2+3</td></tr>
<tr><td>Q2</td><td>The life of a LED bulb follows exponential distribution with mean life of 500 hours. Let X denotes time-to-failure of this bulb.
a) Write the probability density function for X.
b) If the bulb has been used for 300 hours, what is the conditional probability that it will last for another 600 hours?</td><td>3</td><td>2+3</td></tr>
<tr><td>Q3</td><td>A study was done to study the effect of ambient temperature X on the electric power consumed by a chemical plant Y. Other factors were held constant, and the following data were collected from an experimental pilot plant:
<table>
<tr><th>Y (BTU)</th><th>X°(F)</th></tr>
<tr><td>250</td><td>27</td></tr>
<tr><td>285</td><td>45</td></tr>
<tr><td>320</td><td>72</td></tr>
<tr><td>295</td><td>58</td></tr>
<tr><td>265</td><td>31</td></tr>
<tr><td>298</td><td>60</td></tr>
<tr><td>267</td><td>34</td></tr>
<tr><td>321</td><td>74</td></tr>
</table></td><td>4</td><td>3+2</td></tr>
</table>

	a) Estimate the slope and intercept in a simple linear regression model. **b)** Predict power consumption for an ambient temperature of 65^oF.		
Q4	**a)** Write the logistic function and standard logistic function. **b)** What is the major difference between linear and logistic regression? **c)** Compute the predicted probability using the logistic regression model: $z = -2 + 0.5x_1 + x_2 - 0.5x_3$ for feature values $x_1 = 2, x_2 = 1, x_3 = 0$.	4	2+1+2
Q5	Mention one major difference between strict-sense stationary and wide-sense stationary random processes. Check whether $\{X(t)\}$, where $X(t) = Acos(\omega t)$, where, $t \geq 0$, ω is a constant, and A is uniformly distributed in $(0, \pi)$, is a wide-sense stationary random process or not.	5	1+4
Q6	A message transmission system is found to be Markovian with the transition probability of current message to next message as given by the matrix $\begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$. The initial probabilities are given by [0.4 0.3 0.3]. a) Draw the state transition diagram. b) Find the probability of the second message to be the next message. c) State the Chapman-Kolmogorov theorem and use it to find $\left[p_{ij}^{(3)}\right]$.	5	1+1+3
Q7	A wide-sense stationary Gaussian random process has an autocorrelation function $R_{XX}(\tau) = 16e^{-\lvert\tau\rvert/2}$. Determine the covariance matrix of the random variables $X(t), X(t+1), X(t+2)$, and $X(t+3)$. If $\mu_X(t) = 10$, evaluate, $P[X(10) \leq 12]$.	5	3+2

Areas under the Normal Curve:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879