JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATION- 2025

MTech-I Semester (DS)

COURSE CODE (CREDITS): 22M11MA111 (3) MAX. MARKS: 35

COURSE NAME: Mathematical Foundations for Data Science

COURSE INSTRUCTORS: RVS MAX. TIME: 2 Hours

Note: *(a) All questions are compulsory.*

(b) A scientific calculator is allowed.

Q.No	Question	CO	Marks
Q1	(a) Explain with an example how closure under addition and closure under scalar multiplication ensure that a set qualifies as a vector space? (b) Check whether the following vectors in $\mathbb{R}^3$ are linearly independent: $u = (1, -2, 0), v = (3, -6, 1), w = (2, -4, -1)$. Show all steps clearly.	CO1	2+3
Q2	(a) What is the span of a set of vectors? How is span related to the concept of a subspace? Provide one example. (b) Let $S = \{(x, y, z) \in \mathbb{R}^3 \mid 4x - y + z = 0\}$. (i) Show that S is a subspace of $\mathbb{R}^3$. (ii) Find two linearly independent vectors that form a basis for S.	CO2	2+3
Q3	(a) Explain the geometric meaning of a basis. Why does choosing a different basis change the coordinate representation of vectors? (b) Let $T(x, y) = (2x - y, x+3y)$ (i) Find the matrix representation of T. (ii) Compute $T(3, -1)$.	CO2	2+3
Q4	(a) Define an invariant subspace for a linear operator. Give one example of an invariant subspace of a 2×2 matrix. (b) For $A = \begin{bmatrix} 7 & 2 \\ 0 & 3 \end{bmatrix}$, (i) Find the eigenvalues. (ii) Find one eigenvector for each eigenvalue. (iii) Decide whether A is diagonalizable.	CO3	2+3

Q5	(a) Explain supervised, unsupervised, and reinforcement learning. Give one real-world example for each. (b) Why is Machine Learning useful for solving complex real-life problems? Give two reasons.	CO3	3+2
Q6	(a) K-Means Numerical: Given points (1,1), (2,1), (4,3), (5,4)and initial centroids $C_1 = (1,1), C_2 = (5,4),$ perform one full iteration of K-Means (assignment + recompute centroids). (b) Explain the working intuition of Naive Bayes and Decision Tree classifiers.	CO4	3+2
Q7	a) A matrix with distinct eigenvalues is always diagonalizable. (T/F) b) K-Means clustering is an example of supervised learning. (T/F) c) Every subspace must contain the zero vector. (T/F) d) The zero vector cannot be part of a linearly independent set. (T/F) e) If $Av = \lambda v$, then v is an eigenvector of A. (T/F) f) The number of vectors in any basis is the ________ of the vector space. g) A set of vectors is linearly dependent if one vector can be written as a ________ of others. h) In Naive Bayes, features are assumed to be conditionally ________. i) The determinant of a diagonal matrix is the ________ of its diagonal entries. j) In a field, every non-zero element has a multiplicative ________.	CO1-4	5