Jaypee University of Information Technology, Waknaghat

Comprehensive Examination - November 2025

Ph.D (CSE/ECE/CE/BT/BI/PMS/MATHS/HSS)

Course Code/Credits: 17P1WMA131/3
Course Title: Comprehensive Examination
Course Instructor: RAD

Max. Marks: 100
Max. Time: 3 hours

Note: (a) ALL questions are compulsory.

(b) The candidate is allowed to make suitable numeric assumptions wherever required.

SECTION–A

(Number Theory/Weyl Algebra)

Q.No	Question	Marks
Q1	Let a and b be algebraic numbers with $a \neq 0, 1$ and b irrational. (a) State and prove the Gelfond–Schneider Theorem. (b) Deduce that the numbers $2^{\sqrt{2}}$ and e^{π} are transcendental.	7
Q2	Consider the multiplicative group $(\mathbb{Z}/p\mathbb{Z})^{\times}$. Prove that $(\mathbb{Z}/p\mathbb{Z})^{\times}$ is cyclic.	7
Q3	Answer the following questions. 1. Define the Euler's Totient Function $\varphi(n)$. 2. prove that for any integer a coprime to n, $a^{\varphi(n)} \equiv 1 \pmod{n}$. 3. Use this to explain the mathematical principle behind the RSA.	7
Q4	Let $A_1(\mathbb{F}) = \mathbb{F}\langle x,\ y \mid [y,\ x] = 1\rangle$ be the first Weyl algebra over a field $\mathbb{F}$ of characteristic 0. P.T $A_1(\mathbb{F})$ is non-commutative and has no zero divisors.	7
Q5	Consider the automorphism ϕ of $A_1(\mathbb{F})$ defined by $\phi(x) = x$, $\phi(y) = y + p(x)$, where $p(x) \in \mathbb{F}[x]$. S.T ϕ preserves the commutation relation $[y,\ x] = 1$.	6

SECTION–B

(Mathematical Analysis)

Q.No	Question	Marks
Q6	Prove or disprove that $f(x) = \frac{1}{1+x^2}$ is uniformly continuous on $\mathbb{R}$.	9
Q7	Evaluate $\displaystyle\int_{\gamma} \frac{3z^3 + 4z^2 - 5z + 1}{(z^3 - z)(z - 2i)}\, dz$ where the contour $\gamma : \lvert z \rvert = 3$ is taken in the positive sense.	9
Q8	Define the inner product on $\mathbb{R}^2$, and prove or disprove that for $\mathbf{v}, \mathbf{w} \in \mathbb{R}^2$, the product given by $\langle \mathbf{v},\ \mathbf{w} \rangle = v_1w_1 - v_1w_2 - v_2w_1 + 4v_2w_2$ is an inner product. Take $\mathbf{v} = (v_1, v_2)$, $\mathbf{w} = (w_1, w_2)$.	9
Q9	Consider a sequence (a_n), where $a_n = \frac{1}{2} + \frac{1}{2}\sin\left(\frac{n\pi}{3}\right)$. Compute $\liminf a_n$ and $\limsup a_n$.	6

SECTION–C

(Advanced Linear Algebra)

Q.No	Question	Marks
Q10	Consider the following system of equations: $$\begin{aligned} x+2y+z &= 3 \\ 2x+5y+3z &= 8 \\ x+y+z &= 2 \end{aligned}$$ (a) Reduce the augmented matrix to *reduced row-echelon form.* (b) Determine whether the system is consistent.	6
Q11	Let $\mathbf{T} : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $$\mathbf{T}(x,\ y,\ z) = \begin{pmatrix} x+y \\ y+z \\ x+z \end{pmatrix}$$ (a) Find matrix representation of $\mathbf{T}$ with respect to the standard basis. (b) Determine the *rank* and *nullity* of $\mathbf{T}$. (c) Find a *basis* for the range and *kernel* of $\mathbf{T}$.	7
Q12	Consider the following 3×3 diagonal matrix: $$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$ (a) Find the *eigenvalues* and *eigenvectors* of the matrix A. (b) Determine whether A is *diagonalizable.*	7
Q13	Answer the following questions. (a) State and prove the *Cayley–Hamilton Theorem.* (b) Using it, find B^4 for $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$.	7
Q14	Consider the following 3×3 matrix: $$C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$ (a) Find the *characteristic* and *minimal polynomials* of C. (b) Find the *Jordan canonical form* of C.	6

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