Jaypee University of Information Technology, Waknaghat

Comprehensive Examination - November 2025

Ph.D (MATHS)

Course Code/Credits: 17P1WMA131/3 | Max. Marks: 100
Course Title: Comprehensive Examination
Course Instructor: RAD, PKP, MDS | Max. Time: 3 hours

Note: (a) ALL questions are compulsory.
(b) The candidate is allowed to make suitable numeric assumptions wherever required.
(c) Scientific calculator is allowed.

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SECTIONA
(Research)

Q.N.	Question	Marks
Q1	Using the Newton–Raphson method, find the solution of the following system of non-linear equations up to two iterations $$x^3 + y^3 = 53, \quad 2y^3 + z^4 = 69, \quad 3x^5 + 10z^2 = 770$$ which is close to $x = 3, y = 3, z = 2$.”	9
Q2	Using Galerkin method, solve the following boundary value problem (BVP): $$\frac{d^2u}{dx^2} + u + x = 0, \quad 0 < x < 1,$$ subject to the boundary conditions: $$u(0) = 0, \quad \left.\frac{du}{dx}\right\vert_{x=1} = 0.$$	8
Q3	Use Crank-Nicolson method to find the numerical solution of the following parabolic partial differential equation after one-time step: $$T_t = T_{xx}, \quad 0 < x < 1$$ subject to initial condition $$T(x, 0) = 1, \quad 0 < x < 1$$ and the boundary conditions $$T(0, t) = T(1, t) = 0, \quad t0.$$ Compute the solution by taking $\Delta x = 1/4$ and $\Delta t = 1/32$.	9
Q4	Using Greens function, solve the boundary value problem $$\frac{d^2y}{dx^2} + y = x, \qquad y(0) = y(/2) = 0.$$	8

SECTIONB

(Mathematical Analysis)

Q.No	Question	Marks
Q1	Prove or disprove that $f(x) = \frac{1}{1+x^2}$ is uniformly continuous on $\mathbb{R}$.	9
Q2	Evaluate $\int_\gamma \frac{3z^3 + 4z^2 - 5z + 1}{(z^3 - z)(z - 2i)}\, dz$ where the contour $\gamma : \lvert z \rvert = 3$ is taken in the positive sense.	9
Q3	Define the inner product on $\mathbb{R}^2$, and prove or disprove that for $\mathbf{v}, \mathbf{w} \in \mathbb{R}^2$, the product given by $\langle \mathbf{v},\ \mathbf{w} \rangle = v_1 w_1 - v_1 w_2 - v_2 w_1 + 4 v_2 w_2$ is an inner product. Take $\mathbf{v} = (v_1, v_2)$, $\mathbf{w} = (w_1, w_2)$.	9
Q4	Consider a sequence (a_n), where $a_n = \frac{1}{2} + \frac{1}{2}\sin\left(\frac{n\pi}{3}\right)$. Compute $\liminf a_n$ and $\limsup a_n$.	6

SECTIONC

(Advanced Linear Algebra)

Q.No	Question	Marks
Q1	Consider the following system of equations: $$\begin{aligned} x + 2y + z &= 3 \\ 2x + 5y + 3z &= 8 \\ x + y + z &= 2 \end{aligned}$$ (a) Reduce the augmented matrix to *reduced row-echelon form.* (b) Determine whether the system is consistent.	6
Q2	Let $\mathbf{T} : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $$\mathbf{T}(x,\ y,\ z) = \begin{pmatrix} x + y \\ y + z \\ x + z \end{pmatrix}$$ (a) Find matrix representation of $\mathbf{T}$ with respect to the standard basis. (b) Determine the *rank* and *nullity* of $\mathbf{T}$. (c) Find a *basis* for the range and *kernel* of $\mathbf{T}$.	7

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Q3	Consider the following 3×3 diagonal matrix: $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$ (a) Find the *eigenvalues* and *eigenvectors* of the matrix A. (b) Determine whether A is *diagonalizable*.	7
Q4	Answer the following questions. (a) State and prove the *Cayley–Hamilton Theorem*. (b) Using it, find B^4 for $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$.	7
Q5	Consider the following 3×3 matrix: $C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$ (a) Find the *characteristic* and *minimal polynomials* of C. (b) Find the *Jordan canonical form* of C.	6

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