JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

Comprehensive Examination - 2025

Ph. D. (MATHS)

COURSE CODE (CREDITS): MATHS MAX. MARKS: 100

COURSE NAME: Comprehensive Examination

COURSE INSTRUCTORS: RAD, PKP, SST MAX. TIME: 3 Hours

Note: *(a) All questions are compulsory.*

(b) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

Sec A (MOO) [34 Marks]

Q. No.	Question	Marks
Q1	Answer the following: **a)** What is the weighted sum method to solve multi-objective optimization problem? Is this method applicable to both unconstrained and constrained problems? What are major drawbacks of this method? **b)** Consider the following problem: $Min\ (f_1, f_2, f_3)$ s.t. $(x,y) \in [-2,2] \times [-2,2]$, $f_1(x,y) = x^2 + (y-1)^2$, $f_2(x,y) = (x-1)^2 + y^2 + 2$, $f_3(x,y) = x^2 + (y+1)^2 + 1$. Convert this problem into a single objective optimization problem using weights.	5+5
Q2	Answer the following: **a)** What are the main differences between the multi-objective and many-objective optimization problems? **b)** Name two algorithms each for both types of problems with their pros and cons. **c)** Explain the terms, objective reduction and dimensionality handling in many-objective optimization problems.	4+4+4
Q3	Answer the following: **a)** How are indicators useful in multi-objective optimization? **b)** Explain the following indicators mentioning measure formula, utility in terms of convergence, diversity, etc. i. Hypervolume (HV) ii. R2 **c)** Consider the true Pareto front (reference point) as: $P = \{(2,6),(3,4),(4,3)\}$, and algorithm output is $S = \{(2,7),(3,5),(5,3)\}$ for a bi-objective optimization problem. Compute the following using Euclidean distance: GD, and IGD.	4+4+4

Sec B (ANALYSIS) [33 Marks]

Q. No.	Question	Marks
Q1	Prove or disprove that $f(x) = \frac{1}{1+x^2}$ is uniformly continuous on $\mathbb{R}$.	9
Q2	Evaluate $\int_\gamma \frac{3z^3+4z^2-5z+1}{(z^3-z)(z-2i)}dz$ where contour $\gamma: \lvert z\rvert = 3$ is taken in positive sense.	9
Q3	Define the inner product on $\mathbb{R}^2$, and prove or disprove that for $v, w \in \mathbb{R}^2$ the product given by: $\langle v,w\rangle = v_1w_1 - v_1w_2 - v_2w_1 + 4v_2w_2$ is an inner product. Take $v = (v_1, v_2), w = (w_1, w_2)$.	9
Q4	Consider a sequence (a_n), where $a_n = \frac{1}{2} + \frac{1}{2}\sin\left(\frac{n\pi}{3}\right)$. Compute, lim inf a_n, and lim sup a_n.	6

Sec C (ALA) [33 Marks]

Q. No.	Question	Marks
Q1	Consider the following system of equations: $x + 2y + z = 3$ $2x + 5y + 3z = 8$ $x + y + z = 2$ (a) Reduce the augmented matrix to *reduced row-echelon form* using elementary row operations. (b) Determine whether the system is consistent.	6
Q2	Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x, y, z) = \begin{pmatrix} x+y \\ y+z \\ x+z \end{pmatrix}$. (a) Find the matrix representation of T with respect to the standard basis. (b) Determine the *rank* and *nullity* of T. (c) Find a *basis* for the range and *kernel* of T.	7

Q3	Consider the following 3×3 diagonal matrix: $$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$ (a) Find the *eigenvalues* and *eigenvectors* of the matrix A. (b) Determine whether A is *diagonalizable.*	7
Q4	Answer the following question. (a) State and prove the *Cayley–Hamilton Theorem.* (b) Using it, find B^4 for $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$.	7
Q5	Consider the following 3×3 matrix: $$C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$ (a) Find the *characteristic* and *minimal polynomials* of C. (b) Find the *Jordan canonical form* of C.	6