

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION- 2025

B.Tech. -III Semester (CSE/IT/CSECS/FSSD/AIDS/AIML/UXUI)

COURSE CODE (CREDITS): 25B11MA314 (4)

MAX. MARKS: 25

COURSE NAME: Mathematical Foundations for Artificial Intelligence and Data Science

COURSE INSTRUCTORS: RAD, BKP, SST

MAX. TIME: 1 Hour 30 Min

Note: (a) All questions are compulsory.

(b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems

(c) Use of a scientific calculator is allowed.

Q. No.	Question	CO	Marks
Q1	Consider the subspace $W = \text{Span} \{w_1, w_2, w_3\}$ of $\mathbb{R}^4$. $w_1 = \begin{pmatrix} 3 \\ 1 \\ -1 \\ 3 \end{pmatrix}, \quad w_2 = \begin{pmatrix} -5 \\ 1 \\ 5 \\ -7 \end{pmatrix}, \quad w_3 = \begin{pmatrix} 1 \\ 1 \\ -2 \\ 8 \end{pmatrix}$ Using the Gram-Schmidt process, construct an orthogonal basis for W .	1	5
Q2	Consider the following 3×2 rectangular matrix: $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ a) Determine the singular values of A . b) Compute singular value decomposition (SVD) of A .	1	4
Q3	Successful implementation of a new system is based on three independent modules. Module-1 works properly with probability 0.96. For module-2 and module-3, these probabilities are 0.95 and 0.90 respectively. Compute the probability that at least one of these three modules fails to work properly.	2	4
Q4	The Reviews editor for a certain scientific journal decides whether the review for any particular book should be short (1–2 pages), medium (3–4 pages), or long (5–6 pages). Data on recent reviews indicates that 60 % of them are short, 30 % are medium, and the other 10 % are long. Reviews are submitted in either Word or LaTeX. For short reviews, 80 % are in Word, whereas 50 % of medium reviews are in Word and 30 % of long reviews are in Word. Suppose a recent review is randomly selected. a) What is the probability that the selected review was submitted in Word format? b) If the selected review was submitted in Word format, what is the probability of it being long?	2	4

Q5	A web server receives requests from users that can be of three types: static page request ($X = 1$) with probability 0.5; database query ($X = 2$) with probability 0.3, file upload ($X = 3$) with probability 0.2. The random variable X denotes the type of request received in a time unit. Evaluate the following: a) Expectation of X. b) Variance of X.	2	4
Q6	A web application records the response time T (in seconds) of a server for processing a request. The response time is modeled as a continuous random variable with the following probability density function: $f(t) = \begin{cases} 2t, & 0 < t < 1 \\ 0, & \text{elsewhere} \end{cases}$ a) Verify that $f(t)$ is a valid probability density function. b) Find the cumulative distribution function of the response time.	2	4