Jaypee University of Information Technology, Waknaghat

TEST-2 Examination - October 2025

B.Tech - VII Semester (ALL)

Course Code/Credits: 22B1WMA731/3 — Max. Marks: 25
Course Title: Linear Algebra for Data Science & Machine Learning
Course Instructor: RAD — Max. Time: 90 mins

Note: (a) ALL questions are compulsory.

(b) The candidate is allowed to make suitable numeric assumptions wherever required.

Q.No	Question	CO	Marks
Q1	In the following problems, check to see if each set is a subspace of the corresponding vector space. Justify your answer. (a) $\mathbf{S}_1 = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 \ : \ x_1 \leq x_2 \right\} \subset \mathbb{R}^2$. (b) $\mathbf{S}_2 = \left\{ \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix} \ : \ a \in \mathbb{R} \right\} \subset \mathbb{R}^{2\times 2}$.	CO-1	4
Q2	Consider linear transformation $\mathbf{T} : \mathbb{R}^2 \to \mathbb{R}^3$ defined as $\mathbf{T}(x) = \mathbf{A}x$, where $\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{pmatrix}$. Find $x \in \mathbb{R}^2$ such that $\mathbf{T}(x) = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}$	CO-2	4
Q3	Let $\mathbf{W} = \text{span}\{\mathbf{w}_1, \mathbf{w}_2\}$ be a subspace of $\mathbb{R}^4$ generated by $\mathbf{w}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}, \mathbf{w}_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$. (a) Find a basis for $\mathbf{W}$. (b) Find a basis for the *orthogonal complement* of $\mathbf{W}$.	CO-2	4
Q4	In data science and engineering applications, one often needs to project a vector onto a subspace defined by linear constraints. Consider the vector $\mathbf{v} = (1, 1, 1)$ in $\mathbb{R}^3$. (a) Determine the subspace $\mathbf{W}$ defined by the equations: $x + y + z = 0$ $x - y - 2z = 0$ (b) Find *orthogonal projection* of $\mathbf{v}$ onto the subspace $\mathbf{W}$. This corresponds to finding the closest feasible vector to $\mathbf{v}$.	CO-2	4

Q.No	Question	CO	Marks
Q5	Consider the linear transformation $\mathbf{T} : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $\mathbf{T}(x_1, x_2, x_3) = (x_1 - x_2 + x_3,\ x_2 - x_3,\ x_1,\ 2x_1 - 5x_2 + 5x_3)$. (a) Determine the range space (image) of $\mathbf{T}$. (b) Compute the null space of $\mathbf{T}$. Verify Rank-Nullity Theorem.	CO-2	4
Q6	Consider the basis $\{u_1, u_2, u_3\}$ for a subspace $\mathbf{W}$ of $\mathbb{R}^4$: $u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad u_3 = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 1 \end{pmatrix}.$ (a) Use the Gram–Schmidt process to construct an *orthogonal basis* for the subspace $\mathbf{W}$. (b) Normalize the *orthogonal basis* to an *orthonormal basis*.	CO-3	5

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