

**JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT**

**TEST -2EXAMINATION- 2025**

**MTech-I Semester (DS)**

**COURSE CODE (CREDITS):22M11MA111 (3)**

**MAX. MARKS: 25**

**COURSE NAME: Mathematical Foundations for Data Science**

**COURSE INSTRUCTORS: RVS**

**MAX. TIME: 1 Hour 30 Min**

**Note:(a)All questions are compulsory.**

**(b) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems**

| Q.No | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | CO    | Marks |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|-------|
| Q1   | <p>(a) Define a vector and explain any three operations that can be performed on vectors. Give one real-world example of vector use in data science.</p> <p>(b) If</p> $u = (2, -1, 3) \text{ and } v = (1, 4, 2),$ <p>find:</p> <ol style="list-style-type: none"> <li><math>u + v</math></li> <li><math>2u - 3v</math></li> <li>The dot product <math>u \cdot v</math>.</li> </ol>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | CO2   | 2+3   |
| Q2   | <p>Define basis and dimension. Explain their importance in representing data in lower-dimensional spaces.</p> <p>(b) Find the basis and dimension of the subspace spanned by</p> $S = \{(1, 2, 3), (2, 4, 6), (1, 0, 1)\}.$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | CO2   | 2+3   |
| Q3   | <p>(a) Define linear transformation and explain how it can be represented by a matrix.</p> <p>(b) Let</p> $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \text{ be defined as } T(x, y) = (x + 2y, 3x + 4y).$ <p>Find the matrix representation of <math>T</math> and compute <math>T(1, 2)</math>.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | CO3   | 2+3   |
| Q4   | <p>Define eigenvalue and eigenvector. Explain their significance in data science or machine learning applications.</p> <p>(b) Find the eigenvalues and eigenvectors of the matrix</p> $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ <p>Also check whether <math>A</math> is diagonalizable.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | CO3   | 2+3   |
| Q5   | <ol style="list-style-type: none"> <li>The zero vector can be part of a basis. (True/False)</li> <li>A set of vectors is linearly independent if no vector is a linear combination of others. (True/False)</li> <li>The number of vectors in a basis is called the _____ of the vector space.</li> <li>Every subspace of a vector space must contain the _____ vector.</li> <li>The matrix of a linear transformation depends on the choice of _____.</li> <li>Eigenvectors corresponding to distinct eigenvalues are always _____.</li> <li>A matrix is diagonalizable if it has enough linearly _____ eigenvectors.</li> <li>The process of expressing a vector as a combination of basis vectors is called _____.</li> <li>A transformation <math>T</math> is linear if <math>T(au + bv) = aT(u) + bT(v)</math>. (True/False)</li> <li>The determinant of a diagonal matrix is the _____ of its diagonal elements.</li> </ol> | CO2-3 | 5     |