

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION- 2025

Ph.D. (Mathematics)

COURSE CODE (CREDITS): 13P1WMA232 (3)

MAX. MARKS: 25

COURSE NAME: Mathematical Analysis

COURSE INSTRUCTOR: P K Pandey

MAX. TIME: 1 Hour 30 Min

Note: (a) All questions are compulsory.

(b) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

Q.No	Question	Marks
Q1	(a) Prove or disprove that $d(x, y) = \min\{x, y\}$ defines a metric on \mathbb{R} . (b) Compute the residue of $f(z) = \frac{e^z}{z^3}$ at the point $z = 0$.	3
Q2	State and prove the Mean Value Theorem (MVT).	3
Q3	Show that the function $f(x) = \begin{cases} x, & \text{if } x \text{ rational} \\ 0, & \text{if } x \text{ irrational} \end{cases}$ is continuous only at the point $x = 0$.	3
Q4	Find the Laurent series expansion of $f(z) = \frac{1}{(z-i)(z+2)}$ centered at $z_0 = 0$ valid in the annulus $A = \{z \in \mathbb{C} : 1 < z < 2\}$.	3
Q5	For $f(z) = \frac{z^3}{(z+3i)}$ compute the maximum value $ f(z) $ over the domain $ z \leq 2$.	3
Q6	Prove or disprove that $f(z) = e^z$ contradicts the Liouville theorem.	3
Q7	Evaluate $\oint_{\gamma} \frac{4}{(z^2+4)^3} dz$ where γ is a positively oriented contour given by $ z - i = 2$.	3.5
Q8	Evaluate $\oint_{\gamma} \frac{\cos z}{z(z^2+8)} dz$ where γ is a positively oriented (rectangular) contour whose sides are given by $x = \pm 2$ and $y = \pm 2$.	3.5
