Jaypee University of Information Technology, Waknaghat

TEST-3 Examination - December 2024

Ph.D (Mathematics) - I Semester

Course Code/Credits: 18P1WGE101/3 — Max. Marks: 25
Course Title: Research Methodologies and Quantitative Methods and Computer Applications
Course Instructor: RAD — Max. Time: 2 Hours

Note: (a) ALL questions are compulsory.

(b) Scientific calculators are allowed.

(c) The candidate is allowed to make suitable numeric assumptions wherever required.

Q.No	Question	Marks
Q1	Answer the following questions: (a) Explain the difference between linear codes and block codes. Why are linear codes preferred for error correction in practical communication systems? (b) Consider a binary block code with generator matrix: $$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$ Encode the message vector $\mathbf{m} = [1\ 0\ 1]$.	5
Q2	(a) Define the generator matrix and the parity-check matrix of a linear code. How are these matrices related? (b) For a linear code with parity-check matrix: $$\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$ determine if the vector $\mathbf{v} = [1\ 0\ 1\ 0]$ is a valid codeword.	5
Q3	(a) What is a primitive polynomial, and why is it important in the construction of cyclic codes? (b) Verify whether the polynomial $p(x) = x^3 + x + 1$ over $\mathbb{F}_2$ is a primitive polynomial.	5

Q.No	Question	Marks
Q4	(a) Define the dual code of a linear code. What is the significance of the dual code in coding theory? (b) Given a linear code with generator matrix: $$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix},$$ find the parity-check matrix for the dual code.	5
Q5	(a) Explain the construction of Hamming codes and their error-detection and correction capabilities. (b) Construct a Hamming code for $n = 7$ and $k = 4$ using the parity-check matrix: $$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$ Encode the message $\mathbf{m} = [1\ 0\ 1\ 1]$.	5