

**Note:** (a) All questions are compulsory.

(b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.

(c) Use of a scientific calculator is allowed.

Q. No	Question	CO	Marks
Q1	<p>(a) Classify the matrix <math>\begin{bmatrix} 3 &amp; 1 &amp; 2 \\ 1 &amp; -1 &amp; 3 \\ 2 &amp; 3 &amp; 2 \end{bmatrix}</math> as positive definite, negative definite, or indefinite matrix.</p> <p>(b) Let <math>M_{n \times n}</math> be the vector space of <math>n \times n</math> matrices. In each of the following parts determine whether the transformation is linear.</p> <p>(i) <math>T_1(A) = A^T</math></p> <p>(ii) <math>T_2(A) = \det(A)</math></p>	1	2+1.5+1.5
Q2	Using the method of Lagrange multipliers, find the greatest and smallest values that the function $xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$ .	3	5
Q3	<p>(a) Write Stochastic Gradient Descent algorithm.</p> <p>(b) Mention three differences between Gradient Descent and Stochastic Gradient Descent algorithms.</p> <p>(c) What happens if the learning rate is too high or too low?</p>	4	2+2+1
Q4	Using the Gradient Descent algorithm to minimize the loss function $f(x, y) = x^3 + 3y^2$ taking a learning rate of 0.2. Execute up to 2 iterations.	4	5

Q5	<p>(a) Describe soft margin support vector machine as an optimization problem.</p> <p>(b) Mention one major difference between hard and soft margin support vector machine classifiers.</p> <p>(c) Explain the significance of the regularization parameter.</p>	5	3+1+1															
Q6	<p>Compute the first principal components for the following data:</p> <table border="1" data-bbox="316 461 1152 577"> <thead> <tr> <th>Features</th> <th>Example 1</th> <th>Example 2</th> <th>Example 3</th> <th>Example 4</th> </tr> </thead> <tbody> <tr> <td><math>X_1</math></td> <td>4</td> <td>8</td> <td>13</td> <td>7</td> </tr> <tr> <td><math>X_2</math></td> <td>11</td> <td>4</td> <td>5</td> <td>14</td> </tr> </tbody> </table> <p>Given that eigenvalues are 6.61 and 30.38, and normalized eigenvectors are <math>\begin{bmatrix} 0.83 \\ 0.56 \end{bmatrix}</math> and <math>\begin{bmatrix} 0.56 \\ -0.83 \end{bmatrix}</math> respectively.</p>	Features	Example 1	Example 2	Example 3	Example 4	$X_1$	4	8	13	7	$X_2$	11	4	5	14	5	5
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$X_1$	4	8	13	7														
$X_2$	11	4	5	14														
Q7	<p>(a) What is a sigmoid function in logistic regression? Draw a diagram to illustrate.</p> <p>(b) Explain the meaning of <math>\log(\text{odds})</math> in logistic regression.</p> <p>(c) If a logistic regression model is given by <math>\log(\text{odds}) = -75.5 + 3 \times \text{input}</math>, then, find the input corresponding to a 95% chance for success.</p>	5	1+1+3															