DESIGN OF UNITY GAIN BIQUAD FILTERS

Project Report submitted in partial fulfillment of the requirement for the degree of

Bachelor of Technology

In

Electronics and Communication Engineering

By

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Certificate

This is to certify that the project report entitled "Design of Unity Gain Biquad Filters", submitted by Anurag Sharma, Ashish Kumar and Ashish Thakur in partial fulfillment for the award of degree of Bachelor of Technology in Electronics and Communication Engineering to Jaypee University of Information Technology, Waknaghat, Solan has been carried out under my supervision.

Signature

Date: 18th May, 2010

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Certified that this work has not been submitted partially or fully to any other University or Institute for the award of this or any other degree or diploma

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Abstract

Filters have numerous applications in communication be it transferring data on a communication channel or storing and retrieving information. Signal processing techniques involves methods to extract information from various types of signal sources but also methods to protect, store, and retrieve the information at a later date. In, for example, a radio system, we need to generate different types of signals and modify the signals so that the information can be transmitted over a radio channel, e.g., by frequency modulation of a high frequency carrier. Analog filters are key components in these applications. In a telecommunication system we are interested in transmitting information from one place to another, whereas in other applications, e.g., MP3 players etc., we are interested in efficient storing and retrieving of the information. In Mp3 players it is the analog filters that are used to remove non audible signals. A biquad filter is a filter of second order which has two poles. It is very useful block to realize high-order filters. So, in this project we have designed three basic types of biquad filters namely the high pass filter, the low pass and the band pass filter. Using these biquad filters, filters of higher order can be designed.

CHAPTER 1

INTRODUCTION

Signal processing techniques involves methods to extract information from various types of signal sources but also methods to protect, store, and retrieve the information at a later date. In, for example, a telecommunication system we are interested in transmitting information from one place to another, whereas in other applications, e.g., MP3 players, we are interested in efficient storing and retrieving of information... Note that storing information for later retrieval can be viewed as transmitting the information over a transmission channel with an arbitrary long time delay. In many cases, for example in MP3 format, signal processing techniques have been used to remove non audible (redundant) information in order to reduce the amount of information that needs to be stored.

In, for example, a radio system, we need to generate different types of signals and modify the signals so that the information can be transmitted over a radio channel, e.g., by frequency modulation of a high frequency carrier. Analog filters are key components in these applications.

In circuit theory, a filter is an electrical network that alters the amplitude and/or phase characteristics of a signal with respect to frequency. Ideally, a filter will not add new frequencies to the input signal, nor will it change the component frequencies of that signal, but it will change the relative amplitudes of the various frequency components and/or their phase relationships. Filters are often used in electronic systems to emphasize signals in certain frequency ranges and reject signals in other frequency ranges. Such a filter has a gain which is dependent on signal frequency. As an example, consider a situation where a useful signal at frequency f1 has been contaminated with an unwanted signal at f2. If the contaminated signal is passed through a circuit (Figure 1) that has very low gain at f2 compared to f1, the undesired signal can be removed, and the useful signal will remain. Note that in the case of this simple example, we are not concerned with the gain of the filter at any frequency other than f1 and f2. As long as f2 is sufficiently attenuated relative to f1, the performance of this filter will be satisfactory. In general, however, a filter's gain may be specified at several different frequencies, or over a band of frequencies. Since filters are defined by their frequency-domain effects on signals, it makes sense that the most useful analytical and graphical descriptions of filters also fall into the frequency domain. Thus, curves of gain vs. frequency and phase vs. frequency are commonly used to illustrate filter characteristics, and the most widely-used mathematical tools are based in the frequency domain.

1.1 The Transfer Function

The frequency-domain behavior of a filter is described mathematically in terms of its transfer function or network function. This is the ratio of the Laplace transforms of its output and input signals. The voltage transfer function H(s) of a filter can therefore be written as:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} \tag{1.1}$$

Where $V_{in}(s)$ and $V_{out}(s)$ are the input and output signal voltages and s is the complex frequency variable.

The transfer function defines the filter's response to any arbitrary input signal, but we are most often concerned with its effect on continuous sine waves. Especially important is the magnitude of the transfer function as a function of frequency, which indicates the effect of the filter on the amplitudes of sinusoidal signals at various frequencies. Knowing the transfer function magnitude (or gain) at each frequency allows us to determine how well the filter can distinguish between signals at different frequencies. The transfer function magnitude versus frequency is called the amplitude response or sometimes, especially in audio applications, the frequency response. Similarly, the phase response of the filter gives the amount of phase shift introduced in sinusoidal signals as a function of frequency. Since a change in phase of a signal also represents a change in time, the phase characteristics of a filter become especially important when dealing with complex signals where the time relationships between signal components at different frequencies are critical.

By replacing the variable s in (1.1) with jω, where jω is equal to b1, and ω is the radian frequency (2qf), we can find the filter's effect on the magnitude and phase of the input signal. The magnitude is found by taking the absolute value of (1.1):

$$|H(j\omega)| = \frac{|V_{out}(j\omega)|}{|V_{in}(j\omega)|} \tag{1.2}$$

and the phase is:

$$\arg H(j\omega) = \arg \frac{V_{out}(j\omega)}{V_{in}(j\omega)} \tag{1.3}$$

1.2 The Second Order Filter

As an example, the network of Figure 1.1 has the transfer function:

$$H(s) = \frac{s}{s^2+s+1}$$

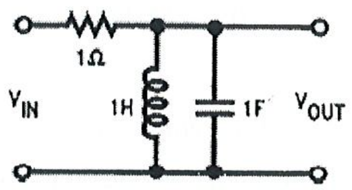


Figure 1.1

This is a 2nd order system. The order of a filter is the highest power of the variable s in its transfer function. The order of a filter is usually equal to the total number of capacitors and inductors in the circuit. (A capacitor built by combining two or more individual capacitors is still one capacitor.) Higher-order filters will obviously be more expensive to build, since they use more components, and they will also be more complicated to design.

However, higher-order filters can more effectively discriminate between signals at different frequencies.

Before actually calculating the amplitude response of the network, we can see that at very low frequencies (small values of s), the numerator becomes very small, as do the first two terms of the denominator. Thus, as s approaches zero, the numerator approaches zero, the denominator approaches one, and H(s) approaches zero. Similarly, as the input frequency approaches infinity, H(s) also becomes progressively smaller, because the denominator increases with the square of frequency while the numerator increases linearly with frequency. Therefore, H(s) will have its maximum value at some frequency between zero and infinity, and will decrease at frequencies above and below the peak.

To find the magnitude of the transfer function, replace s with jω to yield:

$$\mathbf{A(\omega) = |H(s)| = \left|\frac{j\omega}{-\omega^2+j\omega+1}\right|} \qquad (1.4)$$

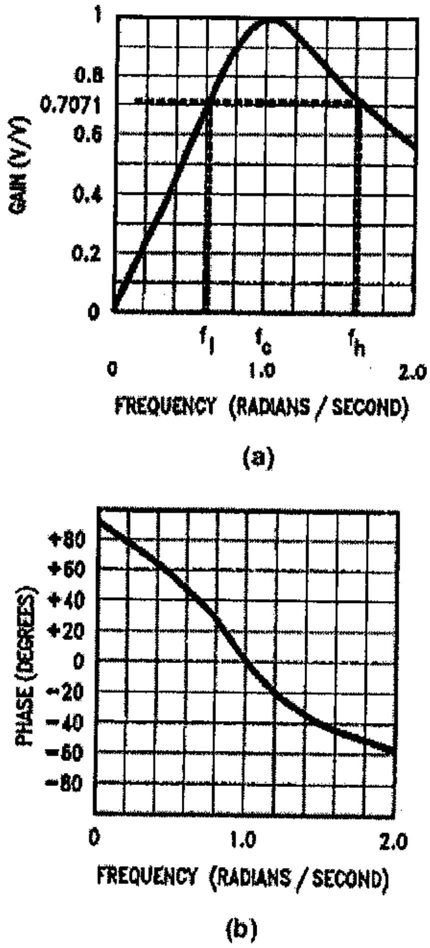
$$\mathbf{A(\omega) = |H(s)| = \frac{\omega}{\sqrt[2]{\omega^2+(1-\omega^2)^2}}} \qquad (1.5)$$

The phase is:

$$\mathbf{\Theta(\omega) = \arg H(s) = 90^o - \tan^{-1}\frac{\omega^2}{1-\omega^2}} \qquad (1.6)$$

The above relations are expressed in terms of the radian frequency ω, in units of radians/second. A sinusoid will complete one full cycle in 2q radians. Plots of magnitude and phase versus radian frequency are shown in Figure 1.2. When we are more interested in knowing the amplitude and phase response of a filter in units of Hz (cycles per second), we convert from radian frequency using ω e 2qf, where f is the frequency in Hz. The variables f and ω are used more or less interchangeably, depending upon which is more appropriate or convenient for a given situation.

Figure 1.2(a) shows that, as we predicted, the magnitude of the transfer function has a maximum value at a specific frequency (ω_0) between 0 and infinity, and falls off on either side of that frequency. A filter with this general shape is known as a band-pass filter because it passes signals falling within a relatively narrow band of frequencies and attenuates signals outside of that band. The range of frequencies passed by a filter is known as the filter's passband. Since the amplitude response curve of this filter is fairly smooth, there are no obvious boundaries for the passband. Often, the passband limits will be defined by system requirements. A system may require, for example, that the gain variation between 400 Hz and 1.5 kHz be less than 1 dB. This specification would effectively define the passband as 400 Hz to 1.5 kHz. In other cases though, we may be presented with a transfer function with no passband limits specified. In this case, and in any other case with no explicit passband limits, the passband limits are usually assumed to be the frequencies where the gain has dropped by 3 decibels (to 02/2 or 0.707 of its maximum voltage gain). These frequencies are therefore called the -3dB frequencies or the cutoff frequencies. However, if a passband gain variation (i.e., 1 dB) is specified, the cutoff frequencies will be the frequencies at which the maximum gain variation Specification is exceeded.



Amplitude (a) and phase (b) response curves for example filter. Linear frequency and gain scales.

Figure 1.2

The precise shape of a band-pass filter's amplitude response curve will depend on the particular network, but any 2nd order band-pass response will have a peak value at the filter's center frequency. The center frequency is equal to the geometric mean of the -3dB frequencies:

$$f_c = \sqrt[2]{f_1 f_h} \qquad (1.7)$$

where f_c is the center frequency

f_1 is the lower b3 dB frequency

f_h is the higher b3 dB frequency

1.3 The Quality Factor Q

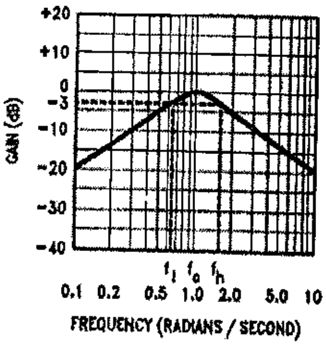
Another quantity used to describe the performance of a filter is the filter's ``Q". This is a measure of the ``sharpness" of the amplitude response. The Q of a band-pass filter is the ratio of the center frequency to the difference between the -3 dB frequencies (also known as the -3 dB bandwidth).

Therefore:

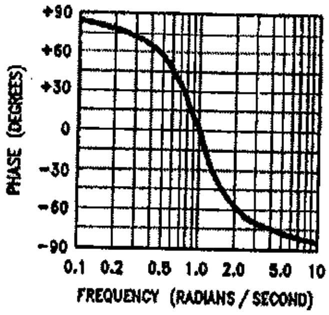
$$Q = \frac{f_c}{f_h - f_1} \tag{1.8}$$

When evaluating the performance of a filter, we are usually interested in its performance over ratios of frequencies. Thus we might want to know how much attenuation occurs at twice the center frequency and at half the center frequency. (In the case of the 2nd-order band pass above, the attenuation would be the same at both points). It is also usually desirable to have amplitude and phase response curves that cover a wide range of frequencies. It is difficult to obtain a useful response curve with a linear frequency scale if the desire is to observe gain and phase over wide frequency ratios. For example, if f0 e 1 kHz and we wish to look at response to 10 kHz, the amplitude response peak will be close to the left-hand side of the frequency scale. Thus, it would be very difficult to observe the gain at 100 Hz, since this would represent only 1% of the frequency axis. A logarithmic frequency scale is very useful in such cases, as it gives equal weight to equal ratios of frequencies.

Since the range of amplitudes may also be large, the amplitude scale is usually expressed in decibels (20loglH (jω) l). Figure 4 shows the curves of Figure 3 with logarithmic frequency scales and a decibel amplitude scale. Note the improved symmetry in the curves of Figure 4 relative to those of Figure 3.



(a)



(b)

Amplitude (a) and phase (b) response curves for example bandpass filter. Note symmetry of curves with log frequency and gain scales.

CHAPTER 2

PASSIVE FILTERS

2.1 Approach to implement Passive filters

Passive implementations of linear filters are based on combinations of resistors (R), inductors (L) and capacitors (C). These types are collectively known as *passive filters*, because they do not depend upon an external power supply and/or they do not contain active components such as transistors.

Inductors block high-frequency signals and conduct low-frequency signals, while capacitors do the reverse. A filter in which the signal passes through an inductor, or in which a capacitor provides a path to ground, presents less attenuation to low-frequency signals than high-frequency signals and is a *low-pass filter*. If the signal passes through a capacitor, or has a path to ground through an inductor, then the filter presents less attenuation to high-frequency signals than low-frequency signals and is a *high-pass filter*. Resistors on their own have no frequency-selective properties, but are added to inductors and capacitors to determine the *time-constants* of the circuit, and therefore the frequencies to which it responds.

The inductors and capacitors are the reactive elements of the filter. The number of elements determines the order of the filter. In this context, an LC tuned circuit being used in a band-pass or band-stop filter is considered a single element even though it consists of two components.

At high frequencies (above about 100 megahertz), sometimes the inductors consist of single loops or strips of sheet metal, and the capacitors consist of adjacent strips of metal. These inductive or capacitive pieces of metal are called stubs.

2.2 The Basic Filter Types

2.2.1Bandpass

A **band-pass filter** is a device that passes frequencies within a certain range and rejects (attenuates) frequencies outside that range. An example of an analogue electronic band-pass filter is an RLC circuit (a resistor–inductor–capacitor circuit). These filters can also be created by combining a low-pass filter with a high-pass filter.

Band pass is an adjective that describes a type of filter or filtering process; it is frequently confused with passband, which refers to the actual portion of affected spectrum. The two words are both compound words that follow the English rules of formation: the primary meaning is the latter part of the compound, while the modifier is the first part. Hence, one may correctly say 'A dual band pass filter has two passbands.

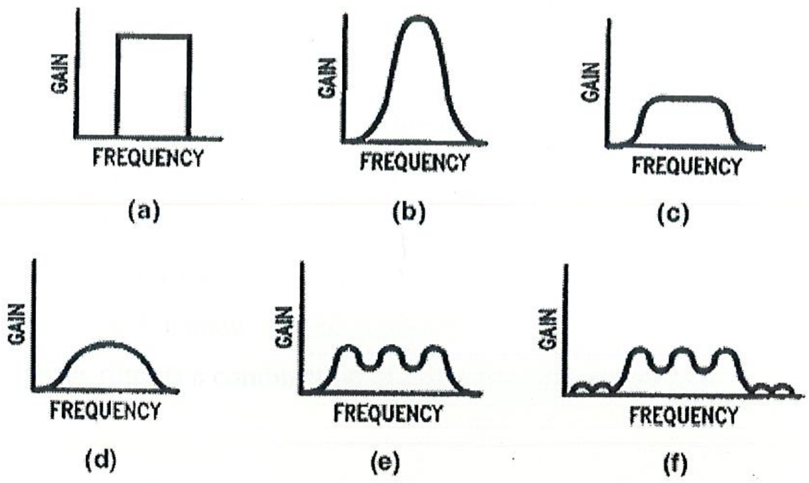
An ideal band pass filter would have a completely flat passband (e.g. with no gain/attenuation throughout) and would completely attenuate all frequencies outside the passband. Additionally, the transition out of the passband would be instantaneous in frequency. In practice, no band pass filter is ideal. The filter does not attenuate all frequencies outside the desired frequency range completely; in particular, there is a region just outside the intended passband where frequencies are attenuated, but not rejected. This is known as the filter roll-off, and it is usually expressed in dB of attenuation per octave or decade of frequency. Generally, the design of a filter seeks to make the roll-off as narrow as possible, thus allowing the filter to perform as close as possible to its intended design. Often, this is achieved at the expense of pass-band or stop-band ripple.

The bandwidth of the filter is simply the difference between the upper and lower cutoff frequencies. The shape factor is the ratio of bandwidths measured using two different attenuation values to determine the cutoff frequency, e.g., a shape factor of 2:1 at 30/3 dB means the bandwidth measured between frequencies at 30 dB attenuation is twice that measured between frequencies at 3 dB attenuation.

Outside of electronics and signal processing, one example of the use of band-pass filters is in the atmospheric sciences. It is common to band-pass filter recent meteorological data with a period range of, for example, 3 to 10 days, so that only cyclones remain as fluctuations in the data fields.

In neuroscience, visual cortical simple cells were first shown by David Hubel and Torsten Wiesel to have response properties that resemble Gabor filters, which are band-pass.

The number of possible band pass response characteristics is infinite, but they all share the same basic form. Several examples of band pass amplitude response curves are shown in Figure 2.1. The curve in 2.1(a) is what might be called an ideal band pass response, with absolutely constant gain within the passband, zero gain outside the passband, and an abrupt boundary between the two. This response characteristic is impossible to realize in practice, but it can be approximated to varying degrees of accuracy by real filters. Curves 2.1(b) through 2.1(f) are examples of a few band pass amplitude response curves that approximate the ideal curves with varying degrees of accuracy. Note that while some band pass responses are very smooth, other have ripple (gain variations in their passbands. Other have ripple in their stopbands as well. The stop band is the range of frequencies over which unwanted signals are attenuated. Band pass filters have two stopbands, one above and one below the passband.



Examples of Bandpass Filter Amplitude Response

FIGURE 2.1

Just as it is difficult to determine by observation exactly where the passband ends, the boundary of the stop band is also seldom obvious. Consequently, the frequency at which a stop band begins is usually defined by the requirements of a given system for example, a system specification might require that the signal must be attenuated at least 35 dB at 1.5 kHz. This would define the beginning of a stop band at 1.5 kHz.

The rate of change of attenuation between the passband and the stop band also differs from one filter to the next. The slope of the curve in this region depends strongly on the order of the filter, with higher-order filters having steeper cutoff slopes. The attenuation slope is usually expressed in dB/octave (an octave is a factor of 2 in frequency) or dB/decade (a decade is a factor of 10 in frequency). Band pass filters are used in electronic systems to separate a signal at one frequency or within a band of frequencies from signals at other frequencies. In 1.1 an example was given of a filter whose purpose was to pass a desired signal at frequency f1, while attenuating as much as possible an unwanted signal at frequency f2. This function could be performed by an appropriate band pass filter with center frequency f1. Such a filter could also reject unwanted signals at other frequencies outside of the passband, so it could be useful in situations where the signal of interest has been contaminated by signals at a number of different frequencies.

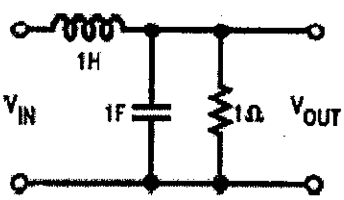
2.2.2 Low-Pass

A third filter type is the low-pass. A **low-pass filter** is a filter that passes low-frequency signals but attenuates (reduces the amplitude of) signals with frequencies higher than the cutoff frequency. The actual amount of attenuation for each frequency varies from filter to filter. It is sometimes called a **high-cut filter**, or **treble cut filter** when used in audio applications. A low-pass filter is the opposite of high, and a band-pass filter is a combination of a low-pass and a high-pass.

The concept of a low-pass filter exists in many different forms, including electronic circuits (like a hiss filter used in audio), digital algorithms for smoothing sets of data, acoustic barriers, blurring of images, and so on. Low-pass filters play the same role in signal processing that moving averages do in some other fields, such as finance; both tools provide a smoother form of a signal which removes the short-term oscillations, leaving only the long-term trend.

If the components of our example circuit are rearranged as in Figure 9, the resultant transfer function is:

$$\mathbf{H_{LP}(s)} = \frac{1}{s^2+s+1} \tag{2.1}$$



Example of a Simple Low-Pass Filter

FIGURE 2.2

It is easy to see by inspection that this transfer function has more gain at low frequencies than at high frequencies. As ω approaches 0, HLP approaches 1; as ω approaches infinity, HLP approaches 0.

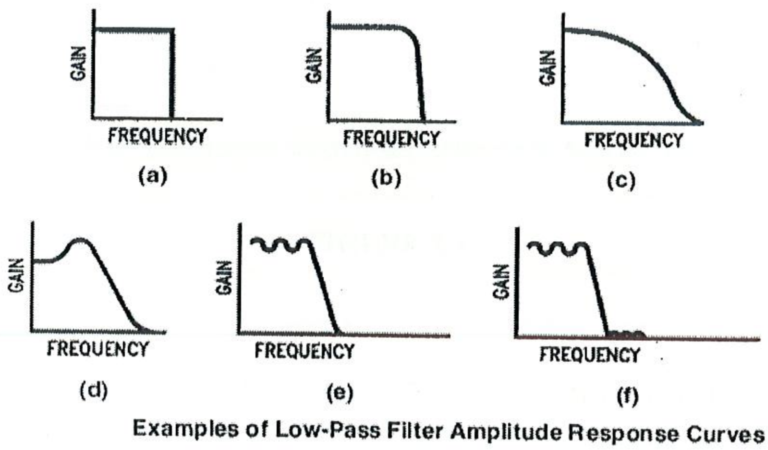
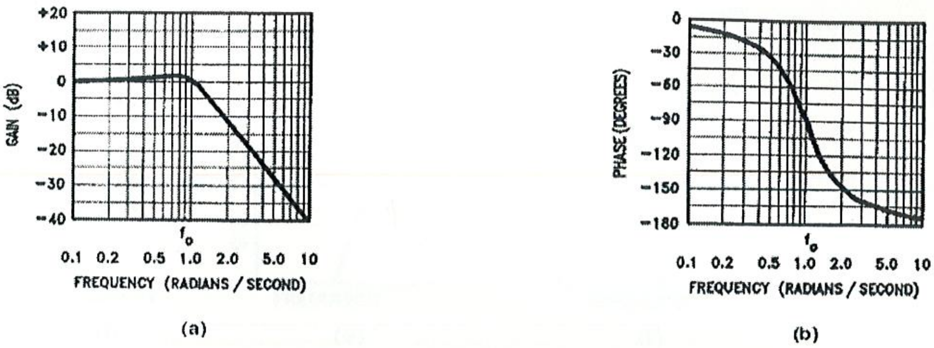


FIGURE 2.3

Amplitude and phase response curves are shown in Figure 2.4, with an assortment of possible amplitude response curves in Figure 2.4. Note that the various approximations to the unrealizable ideal low-pass amplitude characteristics take different forms, some being monotonic (always having a negative slope), and others having ripple in the passband and/or stop band.

Low-pass filters are used whenever high frequency components must be removed from a signal. An example might be in a light-sensing instrument using a photodiode. If light levels are low, the output of the photodiode could be very small, allowing it to be partially obscured by the noise of the sensor and its amplifier, whose spectrum can extend to very high frequencies. If a low-pass filter is placed at the output of the amplifier, and if its cutoff frequency is high enough to allow the desired signal frequencies to pass, the overall noise level can be reduced.



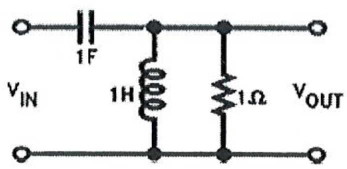
Amplitude (a) and Phase (b) Response Curves for Example Low-Pass Filter

FIGURE 2.4

2.2.3 High-Pass

The opposite of the low-pass is the high-pass filter, which rejects signals below its cutoff frequency. A high-pass filter can be made by rearranging the components of our example network as in Figure 2.5. The transfer function for this filter is:

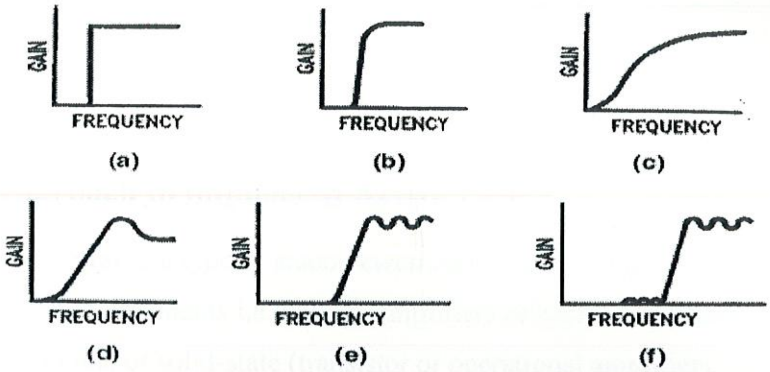
$$H_{HP}(s) = \frac{s^2}{s^2+s+1} \tag{2.2}$$



Example of Simple High-Pass Filter

FIGURE 2.5

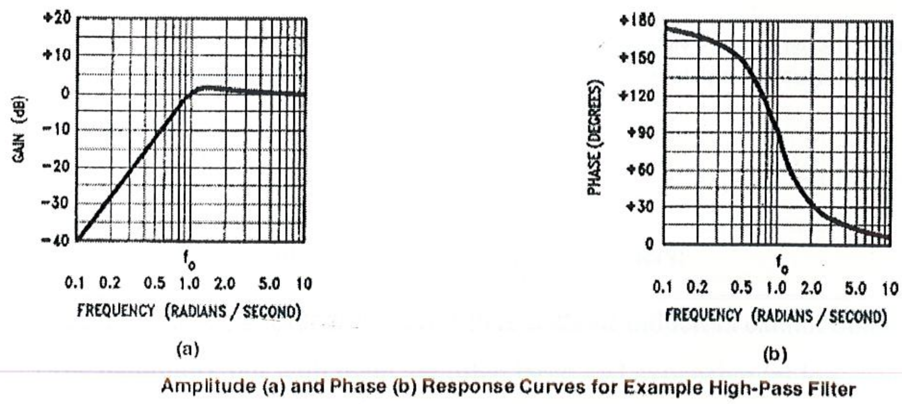
and the amplitude and phase curves are found in Figure 2.6. Note that the amplitude response of the high-pass is a mirror image of the low-pass response. Further examples of high-pass filter responses are shown in Figure 2.6, with the ``ideal" response in 2.6(a) and various approximations to the ideal shown in 2.6(b) through 2.6(f).



Examples of High-Pass Filter Amplitude Response Curves

FIGURE 2.6

High-pass filters are used in applications requiring the rejection of low-frequency signals. One such application is in high-fidelity loudspeaker systems. Music contains significant energy in the frequency range from around 100 Hz to 2 kHz, but high-frequency drivers (tweeters) can be damaged if low-frequency audio signals of sufficient energy appear at their input terminals. A high-pass filter between the broadband audio signal and the tweeter input terminals will prevent low-frequency program material from reaching the tweeter. In conjunction with a low-pass filter for the low-frequency driver (and possibly other filters for other drivers), the high-pass filter is part of what is known as a crossover network.



Amplitude (a) and Phase (b) Response Curves for Example High-Pass Filter

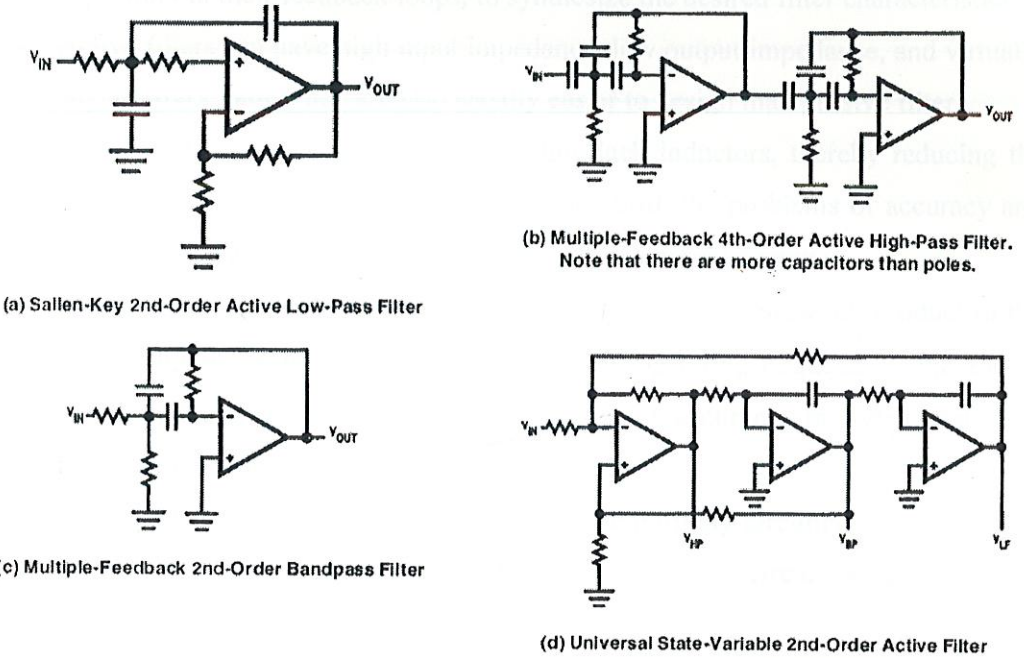
FIGURE 2.7

CHAPTER 3

ACTIVE FILTERS

3.1 Approach to implement Active Filters

An **active filter** is a type of analog electronic filter, distinguished by the use of one or more active components i.e. voltage amplifiers or buffer amplifiers. Typically this will be a vacuum tube, or solid-state (transistor or operational amplifier).



(a) Sallen-Key 2nd-Order Active Low-Pass Filter

(b) Multiple-Feedback 4th-Order Active High-Pass Filter. Note that there are more capacitors than poles.

(c) Multiple-Feedback 2nd-Order Bandpass Filter

(d) Universal State-Variable 2nd-Order Active Filter

Examples of Active Filter Circuits Based on Op Amps, Resistors, and Capacitors

FIGURE 3.1

Active filters have three main advantages over passive filters:

- Inductors can be avoided. Passive filters without inductors cannot obtain a high Q (low damping), but with them are often large and expensive (at low frequencies), may have significant internal resistance, and may pick up surrounding electromagnetic signals.

- The shape of the response, the Q (Quality factor), and the tuned frequency can often be set easily by varying resistors, in some filters one parameter can be adjusted without affecting the others. Variable inductances for low frequency filters are not practical.
- The amplifier powering the filter can be used to buffer the filter from the electronic components it drives or is fed from, variations in which could otherwise significantly affect the shape of the frequency response.

Other characteristics of active filters are:

- Active filters use amplifying elements, especially op amps, with resistors and capacitors in their feedback loops, to synthesize the desired filter characteristics.
- Active filters can have high input impedance, low output impedance, and virtually any arbitrary gain. They are also usually easier to design than passive filters.
- Their most important attribute is that they lack inductors, thereby reducing the problems associated with those components. Still, the problems of accuracy and value spacing also affect capacitors, although to a lesser degree.
- Performance at high frequencies is limited by the gain-bandwidth product of the amplifying elements, but within the amplifier's operating frequency range, the op amp-based active filter can achieve very good accuracy, provided that low-tolerance resistors and capacitors are used.
- Active filters will generate noise due to the amplifying circuitry, but this can be minimized by the use of low-noise amplifiers and careful circuit design.

The figure above shows a few common active filter configurations (There are several other useful designs; these are intended to serve as examples).

- The second-order Sallen-Key low pass filter in (a) can be used as a building block for higher order filters. By cascading two or more of these circuits, filters with orders of four or greater can be built.
- The two resistors and two capacitors connected to the op amp's non-inverting input and to VIN determine the filter's cutoff frequency and affect the Q; the two resistors connected to the inverting input determine the gain of the filter and also

affect the Q. Since the components that determine gain and cutoff frequency also affect Q, the gain and cutoff frequency can't be independently changed.

- Figures 3.1(b) and 3.1(c) are multiple-feedback filters using one op amp for each second-order transfer function. Note that each high-pass filter stage in Figure 3.1(b) requires three capacitors to achieve a second-order response. As with the Sallen-Key filter, each component value affects more than one filter characteristic, so filter parameters can't be independently adjusted.
- The second-order state-variable filter circuit in Figure 3.1(d) requires more op amps, but provides high-pass, low-pass, and band pass outputs from a single circuit. By combining the signals from the three outputs, any second-order transfer function can be realized. When the center frequency is very low compared to the op amp's gain- bandwidth product, the characteristics of active RC filters are primarily dependent on external component tolerances and temperature drifts. Predictable results in critical filter circuits, external components with very good absolute accuracy and very low sensitivity to temperature variations must be used, and these can be expensive.
- When the center frequency multiplied by the filter's Q is more than a small fraction of the op amp's gain-bandwidth product, the filter's response will deviate from the ideal transfer function. The degree of deviation depends on the filter topology; some topologies are designed to minimize the effects of limited op amp bandwidth.

3.2 ACTIVE REALIZATIONS

3.2.1Low pass

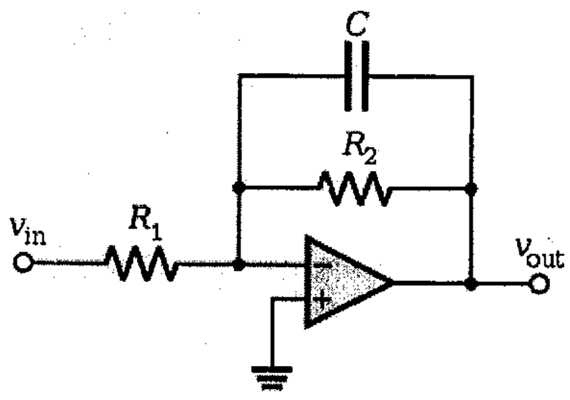


FIGURE 3.2

An active low-pass filter

Another type of electrical circuit is an *active* low-pass filter.

In the operational amplifier circuit shown in the figure, the cutoff frequency (in hertz) is defined as:

$$f_c = \frac{1}{2\pi R_2 C} \tag{3.1}$$

or equivalently (in radians per second):

$$\omega_c = \frac{1}{R_2 C} \tag{3.2}$$

The gain in the passband is $\frac{-R_2}{R_1}$, and the stop band drops off at −6 dB per octave as it is a first-order filter.

Sometimes, a simple gain amplifier (as opposed to the very-high-gain operation amplifier) is turned into a low-pass filter by simply adding a feedback capacitor C. This feedback decreases the frequency response at high frequencies via the Miller Effect, and helps to avoid oscillation in the amplifier. For example, an audio amplifier can be made into a low-pass filter with cutoff frequency 100 kHz to reduce gain at frequencies which would otherwise oscillate. Since the audio band (what we can hear) only goes up to 20 kHz or so, the frequencies of interest fall entirely in the passband, and the amplifier behaves the same way as far as audio is concerned.

3.2.2 HIGH PASS

The simple first-order electronic high-pass filter shown in Figure 1 is implemented by placing an input voltage across the series combination of a capacitor and a resistor and using the voltage across the resistor as an output. The product of the resistance and capacitance ($R \times C$) is the time constant (τ); it is inversely proportional to the cutoff frequency f_c, at which the output power is half the input power. That is,

$$f_c = \frac{1}{2\pi\tau} = \frac{1}{2\pi RC}, \quad (3.3)$$

where f_c is in hertz, τ is in seconds, R is in ohms, and C is in farads.

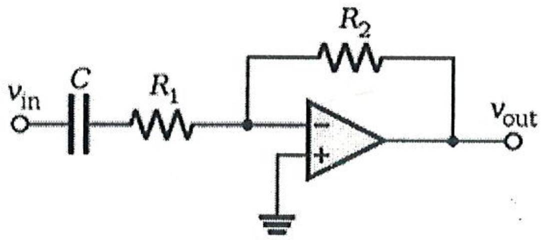


Figure 3.3 An active high-pass filter

$$\omega_c = \frac{1}{R_2 C} \quad (3.4)$$

The gain in the passband is $\frac{-R_2}{R_1}$, and the stop band drops off at −6 dB per octave as it is a first-order filter.

Figure 2 shows an active electronic implementation of a first-order high-pass filter using an operational amplifier. In this case, the filter has a passband gain of $-R_2/R_1$ and has a corner frequency of

$$f_c = \frac{1}{2\pi\tau} = \frac{1}{2\pi R_1 C} \quad (3.5)$$

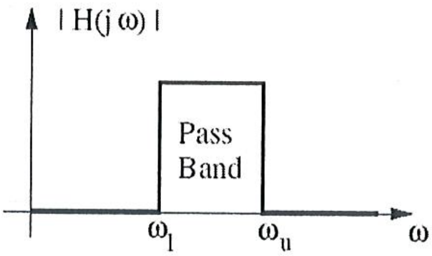
Because this filter is active, it may have non-unity passband gain. That is, high-frequency signals are inverted and amplified byR_2/R_1.

3.2.3 BAND PASS

A band pass Filter allows signals with a range of frequencies (pass band) to pass through and

Attenuates signals with frequencies outside this range.

ω_l :	Lower cut-off frequency;
ω_u :	Upper cut-off frequency;
$\omega_0 \equiv \sqrt{\omega_l \omega_u}$:	Center frequency;
$B \equiv \omega_u - \omega_l$:	Band width;
$Q \equiv \frac{\omega_0}{B}$:	Quality factor.



With practical low- and high-pass filters, upper and lower cut-off frequencies of practical Band pass Filter are defined as the frequencies at which the magnitude of the voltage transfer function is reduced by 1=p2 (or -3 dB) from its maximum value.

In a band pass filters we have high pass filter and a low pass filter cascaded to produce the resulting band pass output.

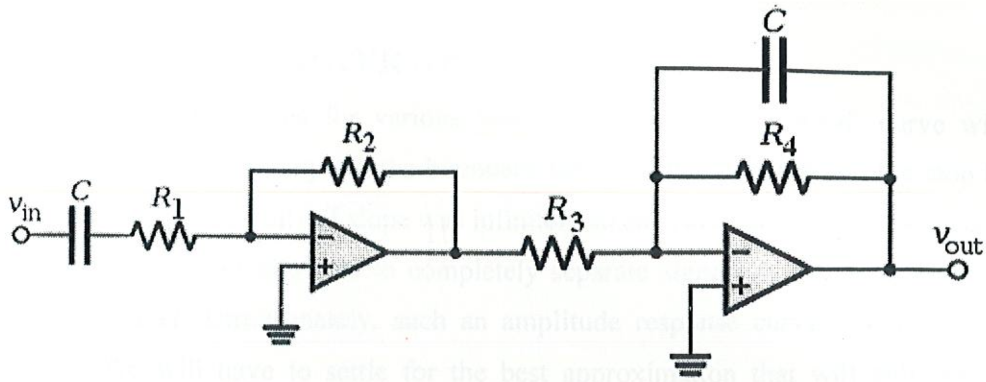


FIGURE 3.4

The gain of the band pass filter is equal to the multiplication of the gain of the high pass and the low pass filters cascaded.

CHAPTER4

FILTER APPROXIMATIONS

Amplitude response curves for various filter types include an ``ideal" curve with a rectangular shape, indicating that the boundary between the passband and the stop band was abrupt and that the roll-off slope was infinitely steep. This type of response would be ideal because it would allow us to completely separate signals at different frequencies from one another. Unfortunately, such an amplitude response curve is not physically realizable. We will have to settle for the best approximation that will still meet our requirements for a given application. Deciding on the best approximation Involves making a compromise between various properties of the filter's transfer function.

4.1 PROPERTIES

The important properties are listed below.

4.1.1 Filter Order

The order of a filter is important for several reasons. It is directly related to the number of components in the filter, and therefore to its cost, its physical size, and the complexity of the design task. Therefore, higher-order filters are more expensive, take up more space, and are more difficult to design. The primary advantage of a higher order filter is that it will have a steeper roll-off slope than a similar lower-order filter.

4.1.2 Roll- off Rate

Usually expressed as the amount of attenuation in dB for a given ratio of frequencies. The most common units are ``dB/octave" and ``dB/decade". While the ultimate roll-off rate will be 20 dB/decade for every filter pole in the case of a low-pass or high-pass filter and 20 dB/decade for every pair of poles for a band pass filter, some filters will have steeper attenuation slopes near the cutoff frequency than others of the same order.

4.1.3 Attenuation Rate Near the Cutoff Frequency

If a filter is intended to reject a signal very close in frequency to a signal that must be passed, a sharp cutoff characteristic is desirable between those two frequencies. Note that this steep slope may not continue to frequency extremes.

4.1.4 Transient Response

Curves of amplitude response show how a filter reacts to steady-state sinusoidal input signals. Since a real filter will have far more complex signals applied to its input terminals, it is often of interest to know how it will behave under transient conditions. An input signal consisting of a step function provides a good indication of this. Figure 1 shows the responses of two low-pass filters to a step input. Curve (b) has a smooth reaction to the input step, while curve (a) exhibits some ringing. As a rule of thumb, filters will sharper cutoff characteristics or higher Q will have more pronounced ringing. FIGURE 1. Step response of two different filters. Curve (a) shows significant ``ringing", while curve (b) shows none. The input signal is shown in curve (c).

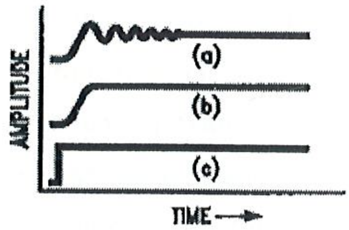


FIGURE 4.1. Step response of two different filters. Curve (a) shows significant ``ringing", while curve (b) shows none. The input signal is shown in curve (c).

4.1.5 Monotonicity

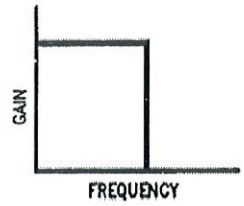
A filter has a monotonic amplitude response if its gain slope never changes sign in other words, if the gain always increases with increasing frequency or always decreases with increasing frequency. Obviously, this can happen only in the case of a low-pass or high-pass filter. A band pass or notch filter can be monotonic on either side of the center frequency.

4.1.6 Passband Ripple

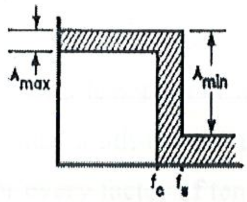
If a filter is not monotonic within its passband, the transfer function within the passband will exhibit one or more ``bumps''. These bumps are known as ``ripple''. Some systems don't necessarily require monotonicity, but do require that the passband ripple be limited to some maximum value (usually 1 dB or less) although band pass and notch filters do not have monotonic transfer functions, they can be free of ripple within their passbands.

4.1.7 Stopband Ripple

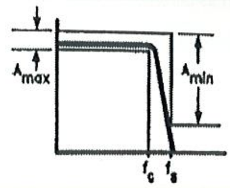
Some filter responses also have ripple in the stopbands. We are normally unconcerned about the amount of ripple in the stop band, as long as the signal to be rejected is sufficiently attenuated. Given that the ideal filter amplitude response curves are not physically realizable, we must choose an acceptable approximation to the ideal response. The word acceptable may have different meanings in different situations. The acceptability of a filter design will depend on many interrelated factors, including the amplitude response characteristics, transient response, and the physical size of the circuit and the cost of implementing the design. The ideal low pass amplitude response is shown again in Figure 4.2(a) .If we are willing to accept some deviations from this ideal in order to build a practical filter, we might end up with a curve like the one in Figure 4.2(b), which allows ripple in the pass-Amax is the maximum allowable change in gain within the passband. This quantity is also often called the maximum passband ripple, but the word ripple implies non-monotonic behavior, while



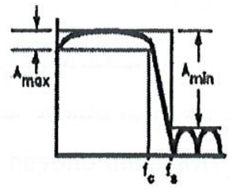
(a) "ideal" Low-Pass Filter Response



(b) Amplitude Response Limits for a Practical Low-Pass Filter



(c) Example of an Amplitude Response Curve Falling with the Limits Set by f_c, f_s, A_{min}, and A_{max}



(d) Another Amplitude Response Falling within the Desired Limits

FIGURE 4.2

A_{max} can obviously apply to monotonic response curves as well.

A_{min} is the minimum allowable attenuation (referred to the maximum passband gain) within the stopband. fc is the cutoff frequency or passband limit. fs is the frequency at which the stop band begins. If we can define our filter requirements in terms of these parameters, we will be able to design an acceptable filter using standard cookbook design methods. It should be apparent that an unlimited number of different amplitude response curves could fit within the boundaries determined by these parameters, as illustrated in Figure 4.2(c) and4.2 (d) .

Filters with acceptable amplitude response curves may differ in terms of such characteristics as transient response, passband and stop band flatness, and complexity. How does one choose the best filter from the infinity of possible transfer functions? Fortunately for the circuit designer, a great deal of work has already been done in this area, and a number of standard filter characteristics have already been defined. These usually provide sufficient flexibility to solve the majority of filtering problems. The classic filter functions were developed by mathematicians (most bear their inventors names), and each was designed to optimize some filter property. The most widely used of these are discussed below. No attempt is made here to show the mathematical derivations of these functions, as they are covered in detail in numerous texts on filter theory.

4.2 Butterworth Filters

The first and probably best-known filter approximation is the Butterworth or maximally-flat response. It exhibits a nearly flat passband with no ripple. The roll-off is smooth and monotonic, with a low-pass or high-pass roll off rate of 20 dB/decade (6 dB/octave) for every pole. Thus, a 5th-order Butterworth low-pass filter would have an attenuation rate of 100 dB for every factor of ten increases in frequency beyond the cutoff frequency. The general equation for a Butterworth filter's amplitude response is

$$H(\omega) = \frac{1}{1 + \left(\frac{\omega}{\omega_0}\right)^{2n}} \qquad (4.1)$$

Where n is the order of the filter, and can be any positive whole number (1, 2, 3,), and 0 is the b3 dB frequency of the filter.

Figure 4.3 shows the amplitude response curves for Butterworth low-pass filters of various orders. The frequency scale is normalized to f/fb3 dB so that all of the curves show 3 dB attenuation for f/fc e 1.0.

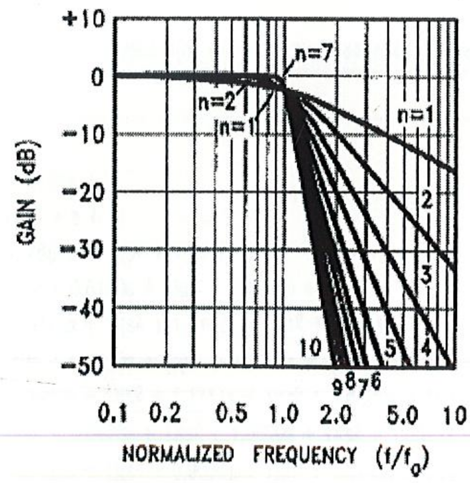


FIGURE 4.3 Amplitude Response Curves for Butterworth Filters of Various Orders

The coefficients for the denominators of Butterworth filters of various orders are shown in Table 1(a). Table 1(b) shows
the denominators factored in terms of second-order polynomials.
Again, all of the coefficients correspond to a corner
frequency of 1 radian/s (finding the coefficients for a different
cutoff frequency will be covered later). As an example,
the tables show that a fifth-order

TABLE 1(a). Butterworth Polynomials

Denominator coefficients for polynomials of the form $s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_1 s + a_0$.

n	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
1	1									
2	1	1.414								
3	1	2.000	2.000							
4	1	2.613	3.414	2.613						
5	1	3.236	5.236	5.236	3.236					
6	1	3.864	7.464	9.142	7.464	3.864				
7	1	4.494	10.098	14.592	14.592	10.098	4.494			
8	1	5.126	13.137	21.846	25.688	21.846	13.137	5.126		
9	1	5.759	16.582	31.163	41.986	41.986	31.163	16.582	5.759	
10	1	6.392	20.432	42.802	64.882	74.233	64.882	42.802	20.432	6.392

TABLE 1(b). Butterworth Quadratic Factors

n	
1	$(s + 1)$
2	$(s^2 + 1.4142s + 1)$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)$
5	$(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)$
6	$(s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.9319)$
7	$(s + 1)(s^2 + 0.4450s + 1)(s^2 + 1.2470s + 1)(s^2 + 1.8019s + 1)$
8	$(s^2 + 0.3902s + 1)(s^2 + 1.1111s + 1)(s^2 + 1.6629s + 1)(s^2 + 1.9616s + 1)$
9	$(s + 1)(s^2 + 0.3473s + 1)(s^2 + 1.0000s + 1)(s^2 + 1.5321s + 1)(s^2 + 1.8794s + 1)$
10	$(s^2 + 0.3129s + 1)(s^2 + 0.9080s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.7820s + 1)(s^2 + 1.9754s + 1)$

Butterworth low-pass filter's transfer function can be written:

$$H(s) = \frac{1}{s^5 + 3.236s^4 + 5.236s^3 + 5.236s^2 + 3.236s + 1}$$

$$= \frac{1}{(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)} \tag{4.2}$$

This is the product of one first-order and two second-order transfer functions. Note that neither of the second-order transfer functions alone is a Butterworth transfer function, but that they both have the same center frequency.

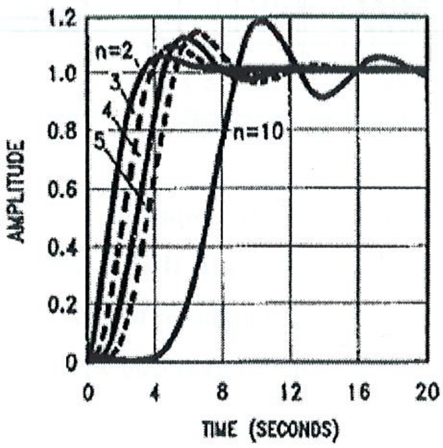


FIGURE 4.4

Figure 4.4 shows the step response of Butterworth low-pass filters of various orders. Note that the amplitude and duration of the ringing increases as n increases.

4.3 Chebyshev Filters

Another approximation to the ideal filter is the Chebyshev or equal ripple response. As the latter name implies, this sort of filter will have ripple in the passband amplitude response. The amount of passband ripple is one of the parameters used in specifying a

Chebyshev filter. The Chebyschev characteristic has a steeper roll off near the cutoff frequency when compared to the Butterworth, but at the expense of monotonicity in the passband and poorer transient response. A few different Chebyshev filter responses are shown in Figure 4.5 . The filter responses in the figure have 0.1 dB and 0.5 dB ripple in the passband, which is small compared to the amplitude scale in Figure 4.5(a) and 4.5(b) , so it is shown expanded in Figure 5(c) .

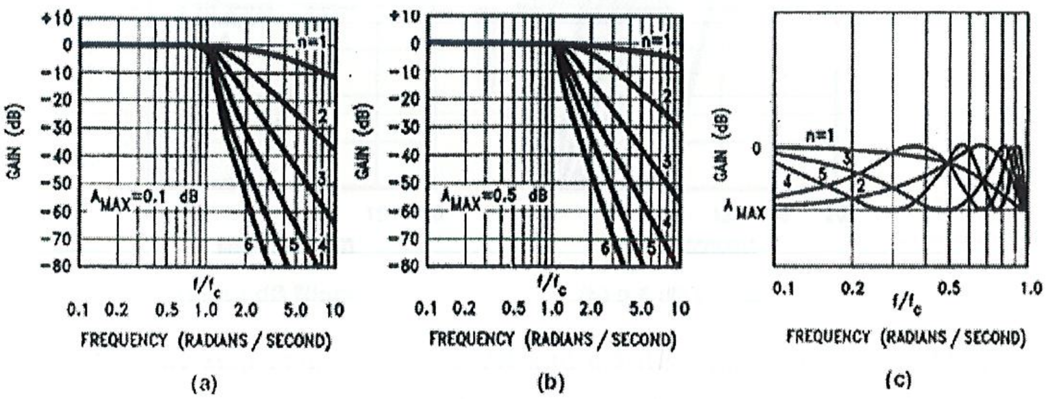


FIGURE 4.5. Examples of Chebyshev amplitude responses. (a) 0.1 dB ripple (b) 0.5 dB ripple. (c) Expanded view of passband region showing form of response below cutoff frequency.

Note that a Chebyshev filter of order n will have nb1 peaks or dips in its passband response. Note also that the nominal gain of the filter (unity in the case of the responses inFigure 5) is equal to The filter's maximum passband gain. An odd order Chebyshev will have a dc gain (in the low-pass case) equal to the nominal gain, with ``dips" in the amplitude response curve equal to the ripple value. An even-order Chebyshev low-pass will have its dc gain equal to he nominal filter gain minus the ripple value; the nominal gain for an even-order Chebyshev occurs at the peaks of the passband ripple. Therefore, if you're designing a fourth-order Chebyshev low-pass filter with 0.5 dB ripple and you want it to have unity gain at dc, you'll have to design for a nominal gain of 0.5 dB. The cutoff frequency of a Chebyshev filter is not assumed to be the b3 dB frequency as in the case of a Butterworth filter. Instead, the Chebyshev's cutoff frequency is normally the frequency at which the ripple (or A_{max}) specification is exceeded. The addition of passband ripple as a parameter makes the specification process for a Chebyshev filter a bit more complicated than for a Butterworth filter, but also increases flexibility.

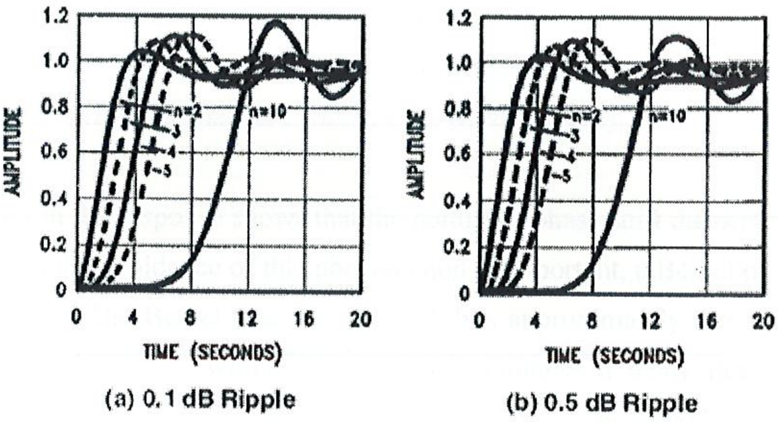


FIGURE 4.6 The step response of 0.1 dB and 0.5 dB ripple Chebyshev filters of various orders. As with the Butterworth filters, the higher order filters ring more.

4.4 Bessel Filters

All filters exhibit phase shift that varies with frequency. This is an expected and normal characteristic of filters, but in certain instances it can present problems. If the phase increases linearly with frequency, its effect is simply to delay the output signal by a constant time period. However, if the phase shift is not directly proportional to frequency, components of the input signal at one frequency will appear at the output shifted in phase (or time) with respect to other frequencies.

The overall effect is to distort non-sinusoidal wave shapes, as illustrated in Figure 4.7 for a square wave passed through a Butterworth low-pass filter. The resulting waveform exhibits ringing and overshoot because the square wave's component frequencies are shifted in time with respect to each other so that the resulting waveform is very different from the input square wave.

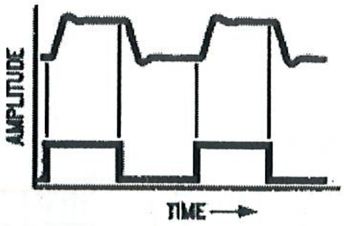


FIGURE 4.7 Response of a 4th-order Butterworth lowpass (upper curve) to a square wave input (lower curve).

The ringing in the response shows that the nonlinear phase shift distorts the filtered wave shape. When the avoidance of this phenomenon is important, a Bessel or Thompson filter may be useful. The Bessel characteristic exhibits approximately linear phase shift with frequency, so its action within the passband simulates a delay line with a low-pass characteristic. The higher the filter order, the more linear the Bessel's phase response.

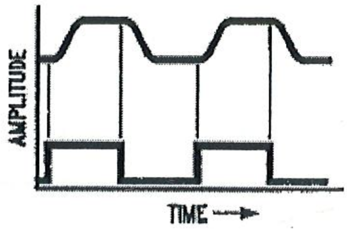


Figure 4.8 shows the square-wave response of a Bessel low-pass filter.

Note the lack of ringing and overshoot. Except for the rounding off of the square wave due to the attenuation of high-frequency harmonics, the wave shape is preserved.

Note the lack of ringing in the response. Except for the rounding of the corners due to the reduction of high frequency components, the response is a relatively undistorted version of the input square wave.

The amplitude response of the Bessel filter is monotonic and smooth, but the Bessel filter's cutoff characteristic is quite gradual compared to either the Butterworth or Chebyshev as can be seen from the Bessel low-pass amplitude response curves in

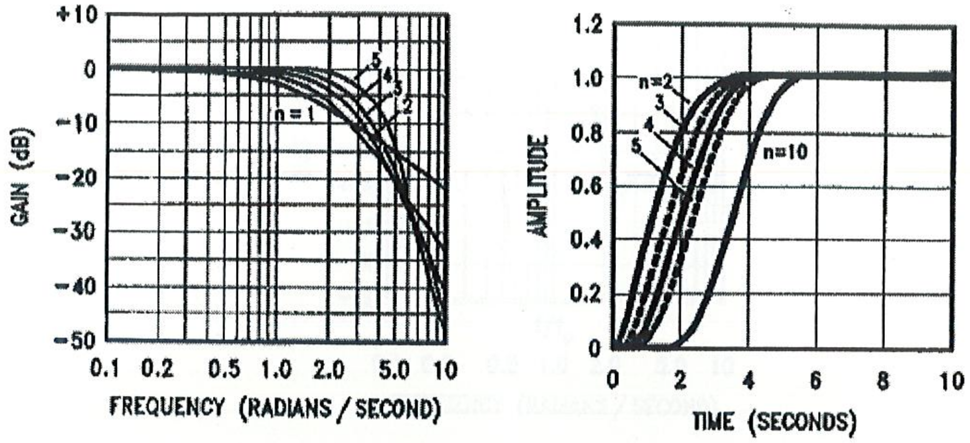


FIGURE 4.9: Amplitude response curves for Bessel filters of various orders. The nominal delay of each filter is 1 second.

4.5 Elliptic Filters

The cutoff slope of an elliptic filter is steeper than that of a Butterworth, Chebyshev, or Bessel, but the amplitude response has ripple in both the passband and the stopband, and the phase response is very non-linear. However, if the primary concern is to pass frequencies falling within a certain frequency band and reject frequencies outside that band, regardless of phase shifts or ringing, the elliptic response will perform that function with the lowest-order filter. The elliptic function gives a sharp cutoff by adding notches in the stopband. These cause the transfer function to drop to zero at one or more frequencies in the stopband. Ripple is also introduced in the passband (see Figure 11). An elliptic filter function can be specified by three parameters (again excluding gain and cutoff frequency): passband ripple, stopband attenuation, and filter order n. Because of the greater complexity of the elliptic filter, determination of coefficients is normally done with the aid of a computer.

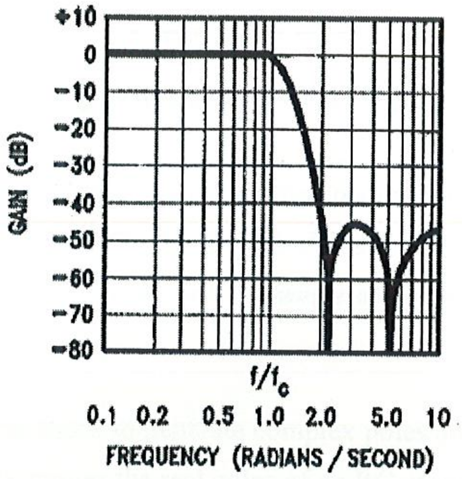


FIGURE 4.10 Example of a elliptic low-pass amplitude response.

This particular filter is 4th-order with A_{max} = 0.5 dB and $\frac{f_s}{f_c} = 2$.

CHAPTER 5

The Biquad Active Filter

A **biquad filter** is a type of linear filter that implements a transfer function that is the ratio of two quadratic functions. The name *biquad* is short for *biquadratic*.Biquad filters are typically active and implemented with a **single-amplifier biquad (SAB)** or **two-integrator-loop** topology.

- The SAB topology uses feedback to generate complex poles and possibly complex zeros. In particular, the feedback moves the real poles of an RC circuit in order to generate the proper filter characteristics.
- The two-integrator-loop topology is derived from rearranging a biquadratic transfer function. The rearrangement will equate one signal with the sum of another signal, its integral, and the integral's integral. In other words, the rearrangement reveals a state variable filter structure. By using different states as outputs, any kind of second-order filter can be implemented.

The SAB topology is sensitive to component choice and can be more difficult to adjust. Hence, usually the term **biquad** refers to the two-integrator-loop state variable filter topology.

5.1 Tow-Thomas Biquad Example

For example, the basic configuration in Figure 5.1 can be used as either a low- pass or bandpass filter depending on where the output signal is taken from the second-order low-pass transfer function is given by

$$H(s) = \frac{H\omega^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

where low-pass gain $H = R_2 / R_1$. The second-order bandpass transfer function is given by

$$H(s) = \frac{H\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{s} + \omega_0^2}$$

with bandpass gain $H = -R_4 / R_2$.

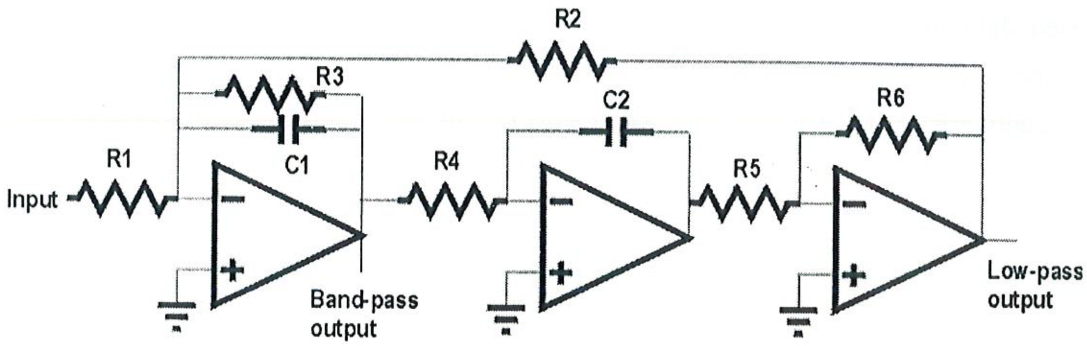


Figure 5.1: The common Tow-Thomas biquad filter topology.

In both cases, the

Natural frequency is

$$\omega_0 = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}$$

Quality factor is

$$Q = \sqrt{\frac{R_3^2 C1}{R_2 R_4 C_2}}$$

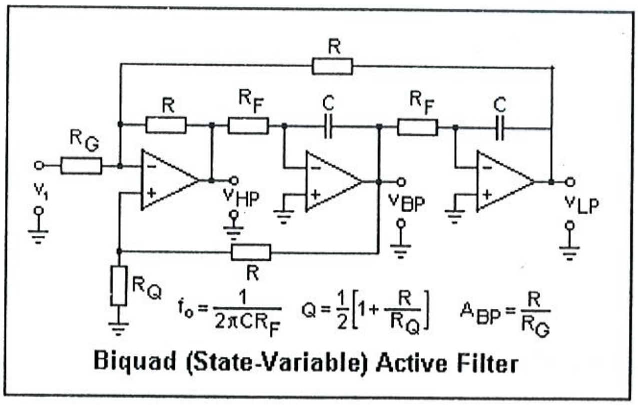
The bandwidth is approximated by $B = \omega_0 / Q$, and Q is sometimes expressed as a damping constant $\zeta = 1 / 2Q$. If a noninverting low-pass filter is required, the output can be taken at the output of the second operational amplifier. If a noninverting bandpass filter is required, the order of the second integrator and the inverter can be switched, and the output taken at the output of the inverter's operational amplifier.

5.2 USE OF INTEGRATOR IN BIQUAD

Operational amplifiers can be used to realize a linear system with an arbitrary biquadratic transfer function, as shown below. The complex variable $s = j\omega$, where $\omega = 2\pi f$. This function is the ratio of two quadratic expressions in s. The denominator specifies the characteristic frequency ω_0 and the Q-factor Q. The three arbitrary complex constants a_0, a_1 and $a_2$specify the filter properties in terms of the basic filter types of high pass, bandpass and lowpass, which occur when only one of the constants is nonzero. Combinations of these types create further filter types, such as notch and allpass filters.

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

The parameter Q has an easily appreciated meaning in the case of the bandpass filter. The gain is maximum at the angular frequency ω_0 and falls off at lower and higher frequencies. Near the centre frequency, the denominator can be written as $(\omega_0 + \omega)(\omega_0 - \omega) + j\omega\omega_0/Q$, or approximately as $2\omega_0\Delta\omega + j\omega_0\omega/Q$. The magnitude of the transfer function will be down by $1/\sqrt{2}$ when these two terms are equal in magnitude, or $\Delta\omega = \omega_0/2Q$. Therefore, the total width of the peak at half maximum power (the square of this) is ω_0/Q.



Biquad (State-Variable) Active Filter

FIGURE 5.2

A biquad transfer function can be realized straightforwardly with a pair of integrators in series, each introducing a factor 1/s, with their outputs added to the input signal in a summer, as illustrated in the circuit at above. This circuit looks complex at first, but if you carry out a circuit analysis its operation will become clear. Assume that the signal at the output of the summer, the leftmost operational amplifier, is x. Then find the signals that are added at the summer in terms of x, and finally solve for x in terms of the input signal v_1. You will find that the result is in the form of a high pass transfer function. The outputs of the following two amplifiers will be a bandpass function and a lowpass function. This amplifier realizes the three basic amplifier types, which can then be combined as desired.

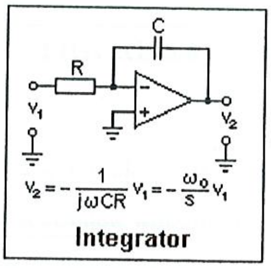
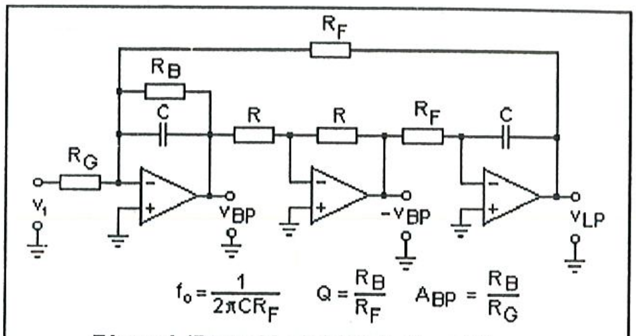


FIGURE 5.3

The effect of an integrator stage is illustrated at the above, to help in the analysis. The parameters f_0, Q and the bandpass gain at the centre frequency are given in terms of the circuit parameters in the figure. These expressions are sufficient for the design of a filter. In any case, resistances should be 5k or greater to avoid overloading the operatonal amplifiers. The frequency-determining values of R_F and C must be carefully matched if a high Q is to be realized.

5.3 ANOTHER BIQUAD FILTER

A different biquad filter is shown below. In this circuit, the first integrator is also used as the summer. One might think that in this case only two op-amps would be required, but unfortunately a signal sign change is necessary, so a unity-gain inverting amplifier is included. The inverting amplifier and the second integrator may be in either order. A drawback is that the highpass response is not available, only the bandpass and lowpass functions.



Biquad (State-Variable) Active Filter

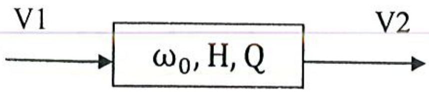
FIGURE 5.4

It is instructive to analyze this circuit, which is straightforward since there is a virtual ground node at each amplifier. The feedback impedance of the first integrator is $R_B/(1 + j\omega R_B C)$, which can be used in the usual expressions for the gain of an inverting amplifier. Some algebra is necessary, but the appropriate biquad transfer function is obtained.

R_F and C determine the centre frequency ω_0. The value of R is unimportant; 10k is a usual value. R_B determines $Q = R_B/R_F$ and the bandpass gain $A_{BP} = R_B/R_G$. The bandwith $\Delta f = f_o/Q = 1/2\pi C R_B$ depends only on R_B, which can be said to set the bandwidth instead of Q, independent of frequency. R_F can be varied to change the centre frequency, perhaps using ganged rheostats. However, to maintain a high Q the resistances must track very accurately.

5.4 A 2^{nd} ORDER FILTER

$$T(s) = \frac{H\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \longrightarrow \quad (1)$$



Scale Frequency (divide s by ω_0) , s_n=s in subscript

$$T(s) = \frac{H\omega_0^2}{(\frac{s}{\omega_0})^2 + (\frac{1}{Q})(\frac{s}{\omega_0}) + 1} = \frac{H}{s_n^2 + \frac{1}{Q}s_n + 1}$$

Or (1) divided by ω_0^2

RESULT :- same as setting ω_0=1,so sometimes refer to frequency scaling as setting ω_0 to 1.

To make a simpler equation the frequency is scaled. This is a standard technique in filter design.

Dropping s_n subscript and assuming an invertor

$$T(s) = \frac{V_L}{V_1} = \frac{-H}{s^2 + \frac{1}{Q}s + 1} \qquad (2)$$

V_L= lowpass filtered version of V_1

Rewriting as $(s^2 + \frac{1}{Q} s + 1) V_L = -H V_1$ (3)

Go to time domain in Laplace tranform, s becomes differentiation and equation becomes 2^{nd} order differential equation.

$$\frac{d^2V_L(t)}{dt^2} + \frac{1}{Q}\frac{dV_L(t)}{dt} + V_L(t) = -H V_1(t)$$

Need to perform two 2 integrations to get V_L from V_1.

Rewriting (3) again to identify circuit elements easier

$$s^2V_L + \frac{1}{Q}V_L s + V_L = -H V_1$$

$$s (s + \frac{1}{Q}) V_L = -(H V_1 + V_1)$$

Rewrite the equation to get it into a form where we can identify circuit elements in s plane remembering that in the La Place transform 1/s is like a capacitor further develop equation below

$$sV_L = -\frac{1}{s+\frac{1}{Q}}(H V_1 + V_L) = V_B \qquad (4)$$

V_L Is obtained by integrating voltage on right side identified here as V_B.

$$V_L = \frac{1}{s} V_B = \frac{1}{s}[-\frac{1}{s+\frac{1}{Q}} (HV_1 + V_L)]$$

Rewrite equation (4),

$$V_B = -1 + V_1 - \frac{V_L}{s + \frac{1}{Q}}$$

identifying V_B and V_L and how to get them

$$V_B = -\frac{1}{s}(HV_1 + V_L + \frac{1}{Q}V_B) = -\frac{1}{s}V_H$$

V_B obtained by integrating with a sign inversion to voltage V_H .

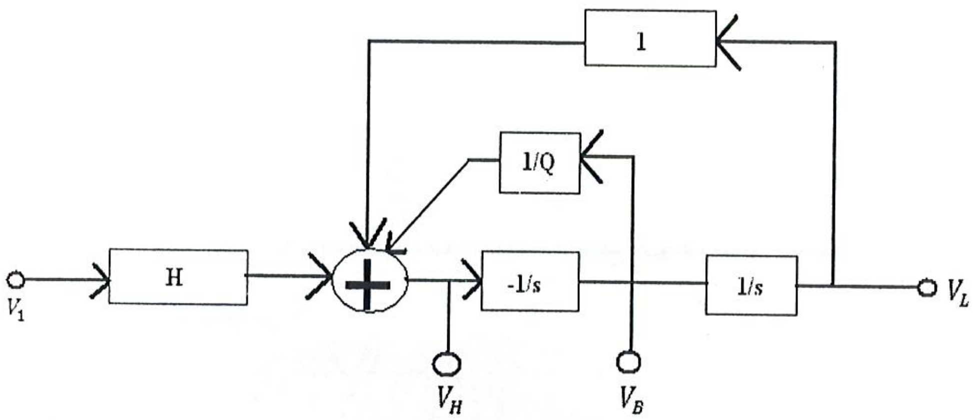
V_H is obtained by summing three scaled voltages

$$V_H = HV_1 + V_L + \frac{1}{Q}V_B$$

This is the double integration to obtain

$$V_L = (\frac{1}{s}) \text{ X } [-(\frac{1}{s}) \text{ X } V_H]$$

Basic definitions of voltages needed.



Block Diagram of a Two Integrator loop (using an inverting and a non-inverting integrator)

FIGURE 5.5

Summing Node realizes the weighted of 3 voltagesV_1, V_L, V_B .

$$V_H = HV_1 + V_L + \frac{1}{Q}V_B$$

Note : V_H and V_B outputs

V_B obtained by multiplying V_L by s

V_H obtained by multiplying V_B by –s

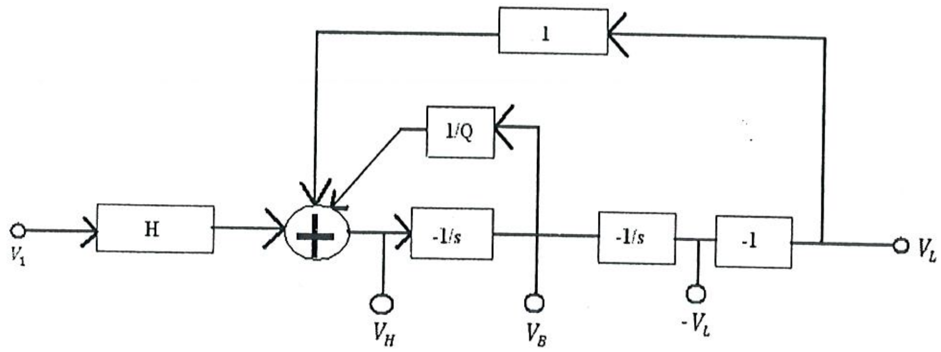
$$V_L = \frac{-H}{s^2 + \frac{1}{Q}s + 1} V_1 = T_L V_1 \qquad \text{LOWPASS FILTER}$$

$$V_B = \frac{-Hs}{s^2 + \frac{1}{Q}s + 1} V_1 = T_B V_1 \qquad \text{BANDPASS FILTER}$$

$$V_H = \frac{Hs^2}{s^2 + \frac{1}{Q}s + 1} V_1 = T_H V_1 \qquad \text{HIGHPASS FILTER}$$

by rewriting the equation to isolate certain terms it can be shown that the same equation can produce other filter types.

For stability using only negative feedback,use 2 negative integrators,as shown in block diagram below.



Block diagram of a Two integrator loop (using two inverting integrator and an invertor)

FIGURE 5.6

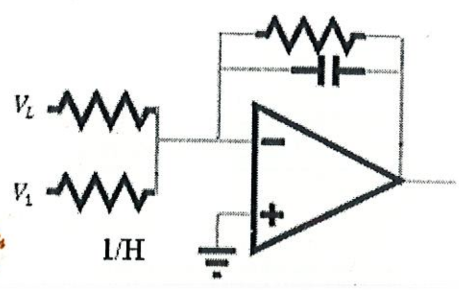


FIGURE 5.7 SUMMER

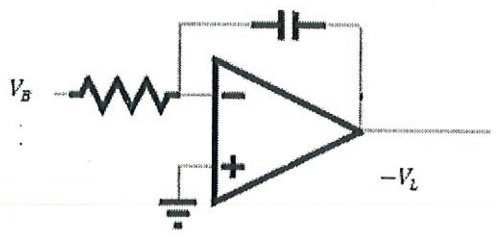


FIGURE 5.8 INVERTING INTEGRATOR

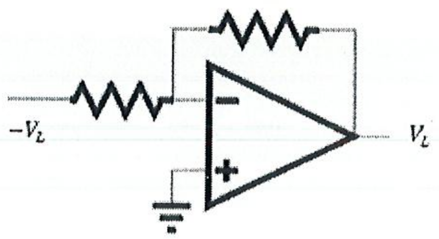


FIGURE 5.9 INVERTOR

On combing above three with resistor and capacitor in parallel we get the following circuit diagram.

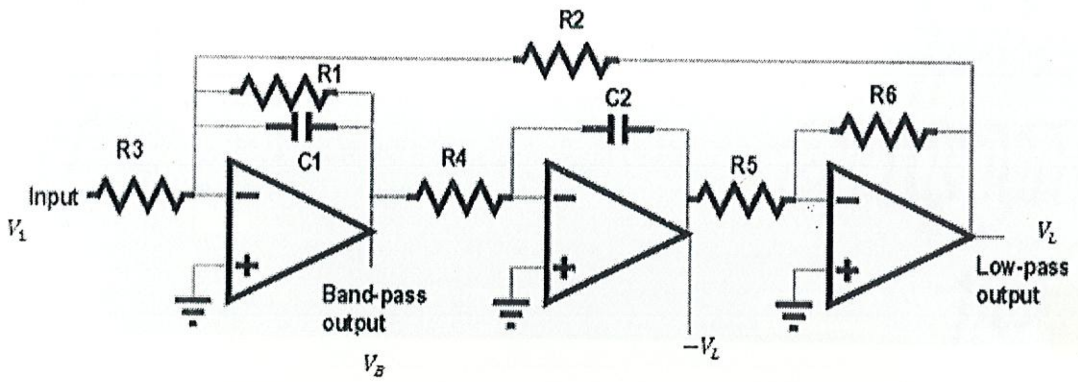


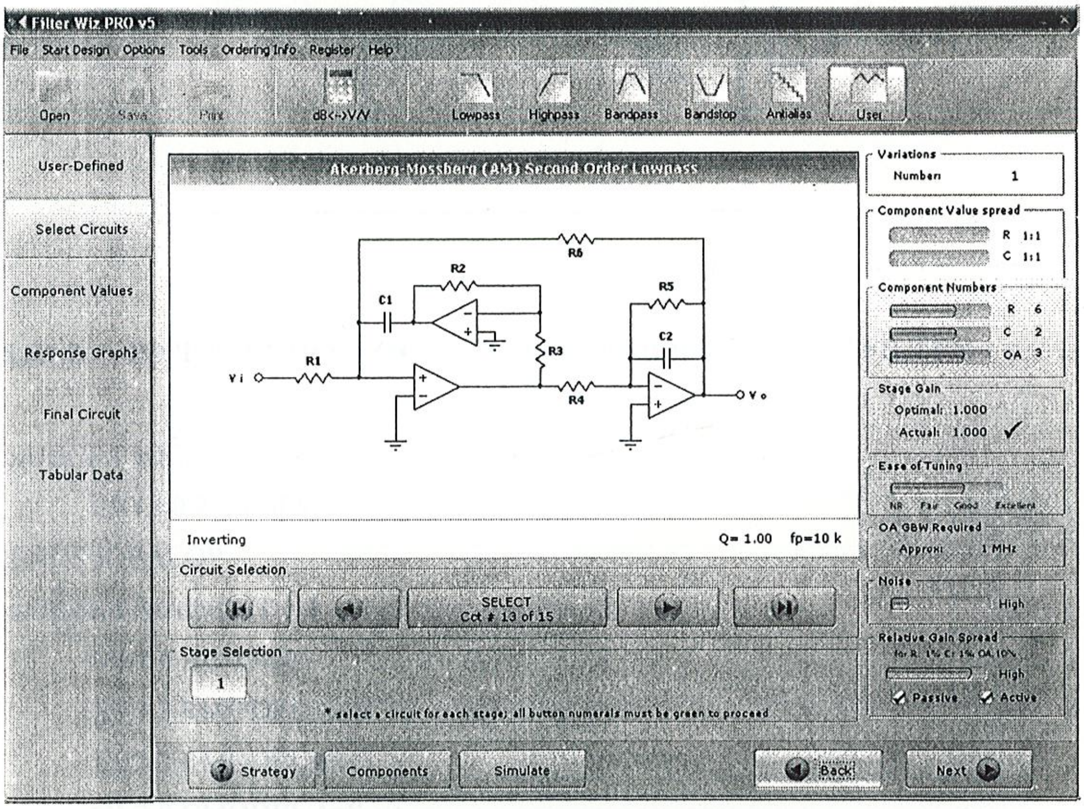
FIGURE 5.10 Circuit Diagram of a Biquad Filter

5.5 SIMULATION RESULTS OF THE BIQUAD FILTERS

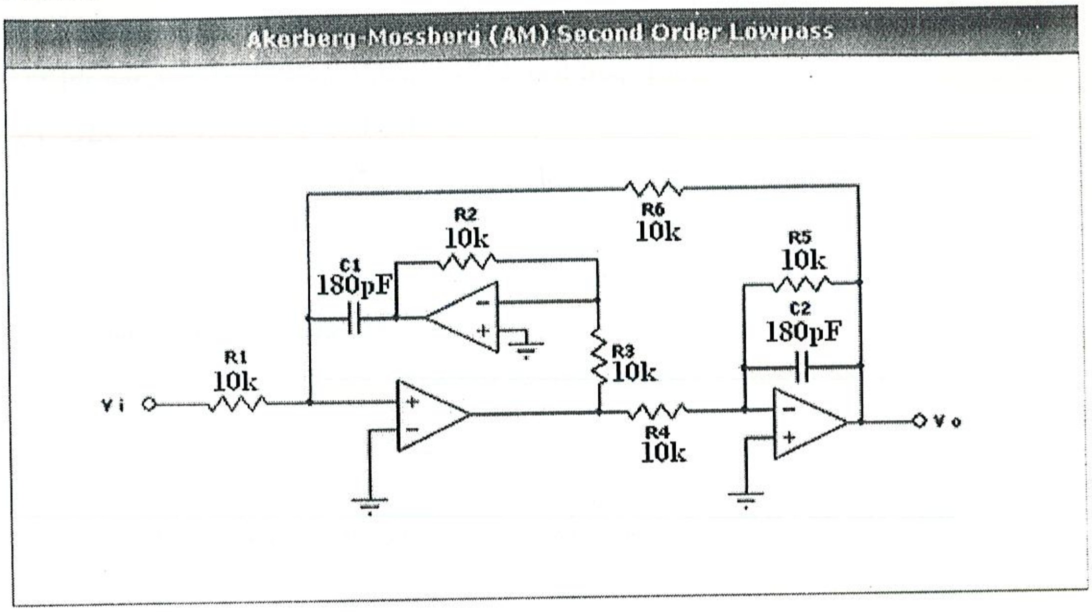
- **LOW PASS FILTER**
- **HIGH PASS FILTER**
- **BAND PASS FILTER**

5.5.1 LOW PASS FILTER

5.5.1.1 The circuit diagram of the second order Low Pass Filter.



5.5.1.2 The circuit diagram of the second order low pass filter with values of resistors and capacitors.

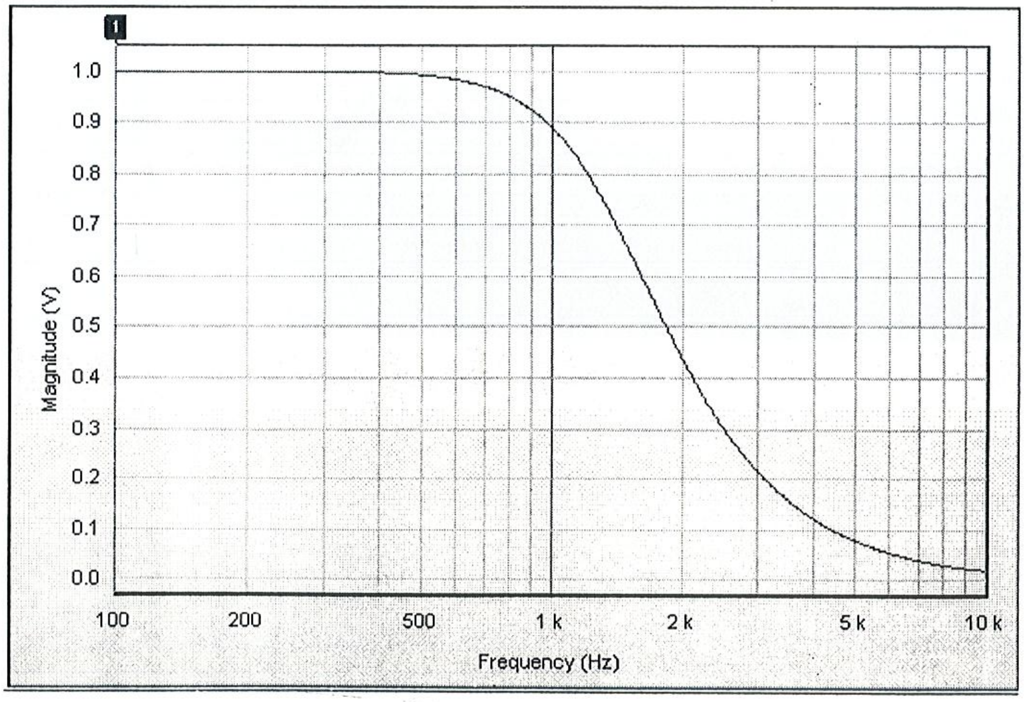


5.5.1.3 PSPICE CODING OF A SECOND ORDER LOW PASS FILTER

```
.SUBCKT IDEALOP 2 1 3
        EIO 3 0 2 1 1.0E6
.ENDS IDEALOP
.SUBCKT MAINCCT 1 4
        V1 1 0 AC 1.000
        R1 1 2 RMODEL xxxx
        R2 2 3 RMODEL xxxx
        C1 3 0 CMODEL 1nF
        C2 2 4 CMODEL 10nF
        XA1 3 4 4 IDEALOP
.ENDS MAINCCT

.MODEL RMODEL RES(R=1 DEV=1.00%)
```

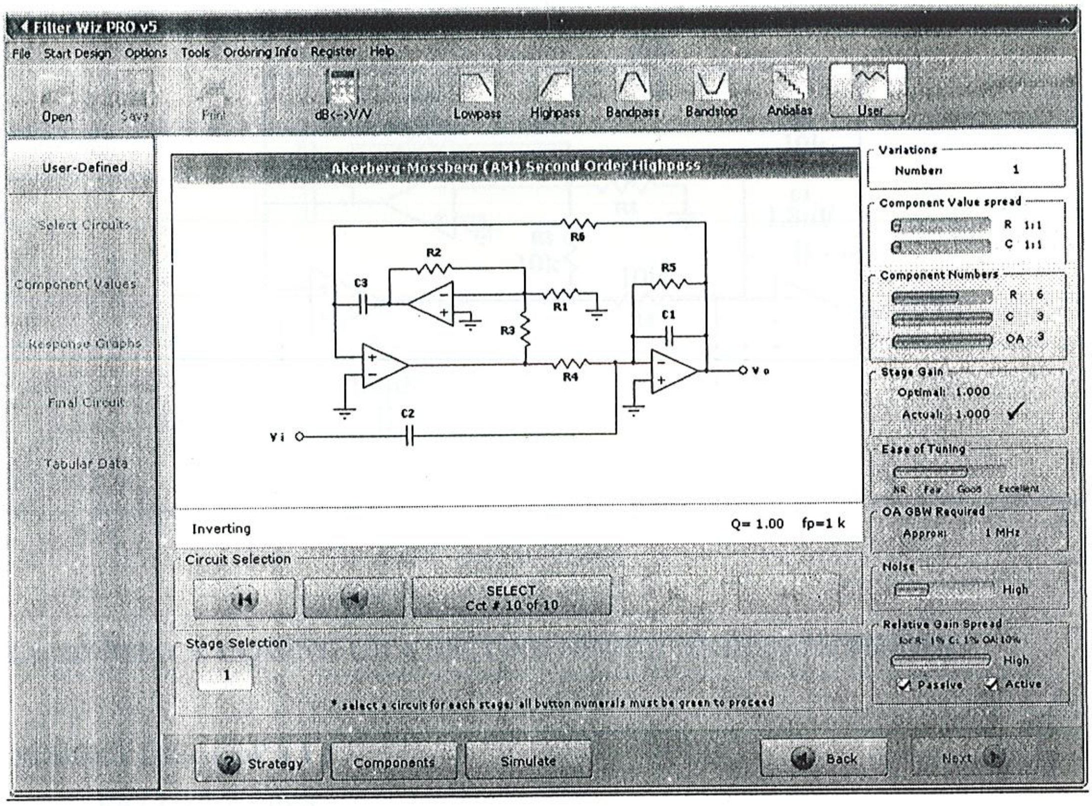
```
.MODEL CMODEL CAP(C=1 DEV=1.00%)

X1 1 2 MAINCCT
*** Pts per decade: 100  start freq: 10.233  end freq: 100 k ***
.AC DEC 100  1.023E+1  10.000E+4
***.NOISE V(2) X1.V1 100***
.PROBE V(2)

*** Monte Carlo runs: 100 ***
***.MC 100 AC V(2) YMAX   Remove asterisk and this comment for Monte Carlo***
.END
```

5.5.1.4 BODE PLOT OF THE SECOND ORDER LOW PASS FILTER

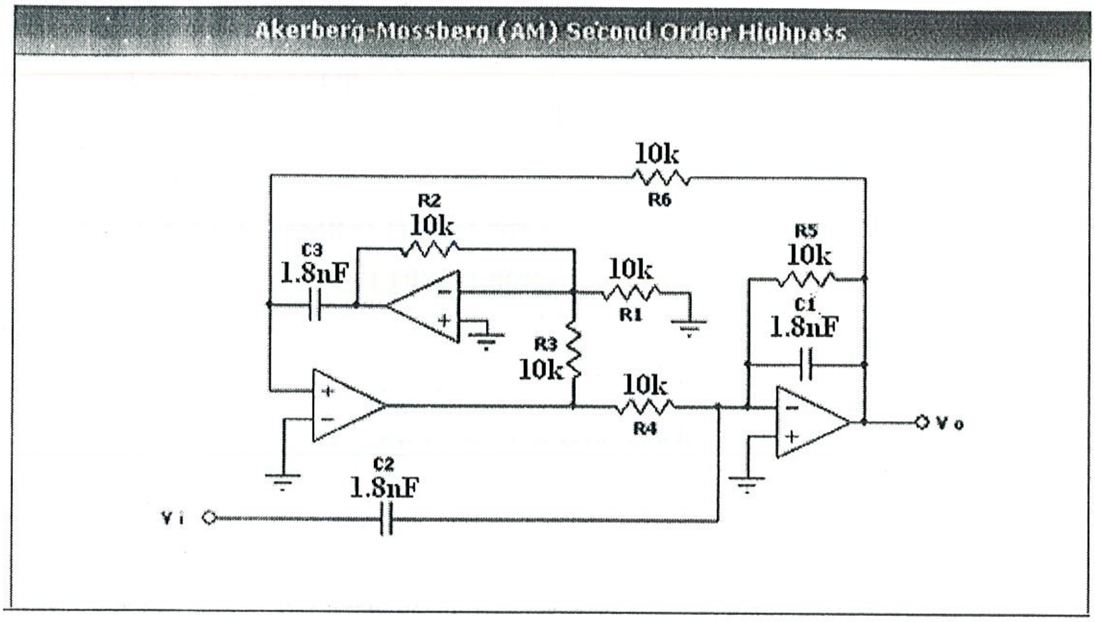


5.5.2 HIGH PASS FILTER

5.5.2.1 The circuit diagram of the second order High Pass Filter.



5.5..2.2 The circuit diagram of the second order high pass filter with values of resistors and capacitors

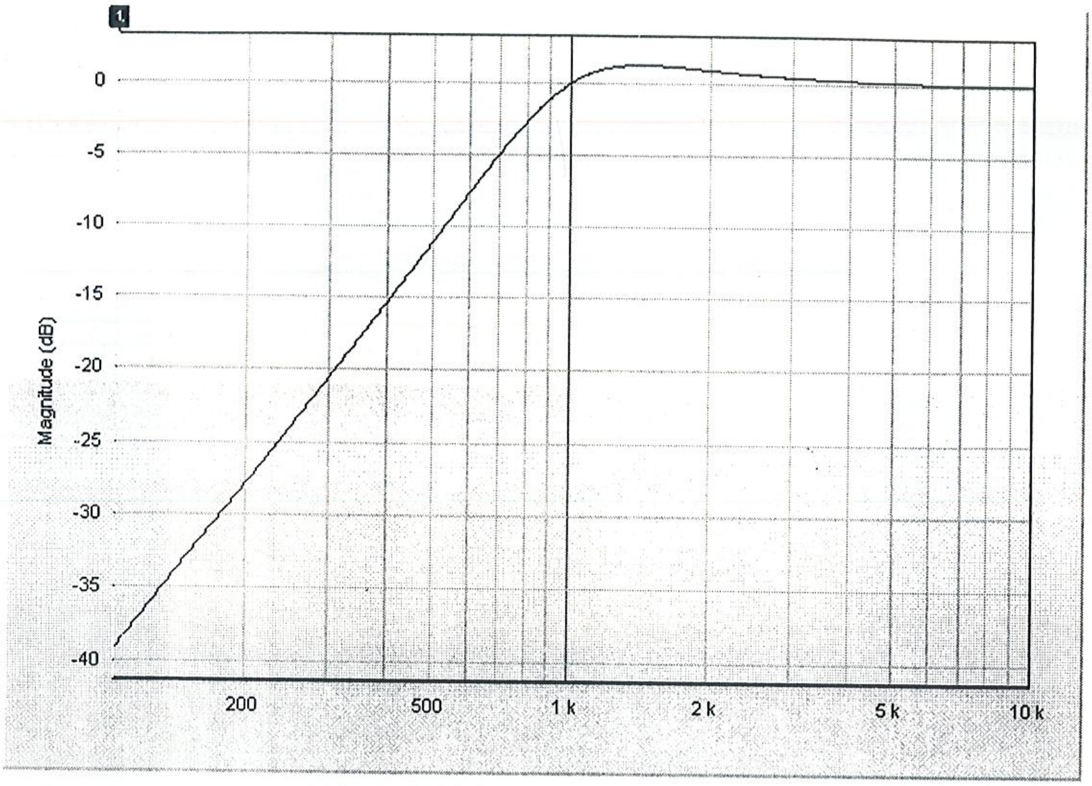


5.5..2.3 PSPICE CODING OF A SECOND ORDER HIGH PASS FILTER

```
.SUBCKT IDEALOP 2 1 3
        EIO 3 0 2 1 1.0E6
.ENDS IDEALOP
.SUBCKT MAINCCT 1 6
        V1 1 0 AC 1.000
        R1 2 0 RMODEL xxxx
        R2 3 2 RMODEL xxxx
        R3 2 4 RMODEL xxxx
        R4 4 5 RMODEL xxxx
        R5 5 6 RMODEL xxxx
        R6 7 6 RMODEL xxxx
        C1 5 6 CMODEL 1.8nF
```

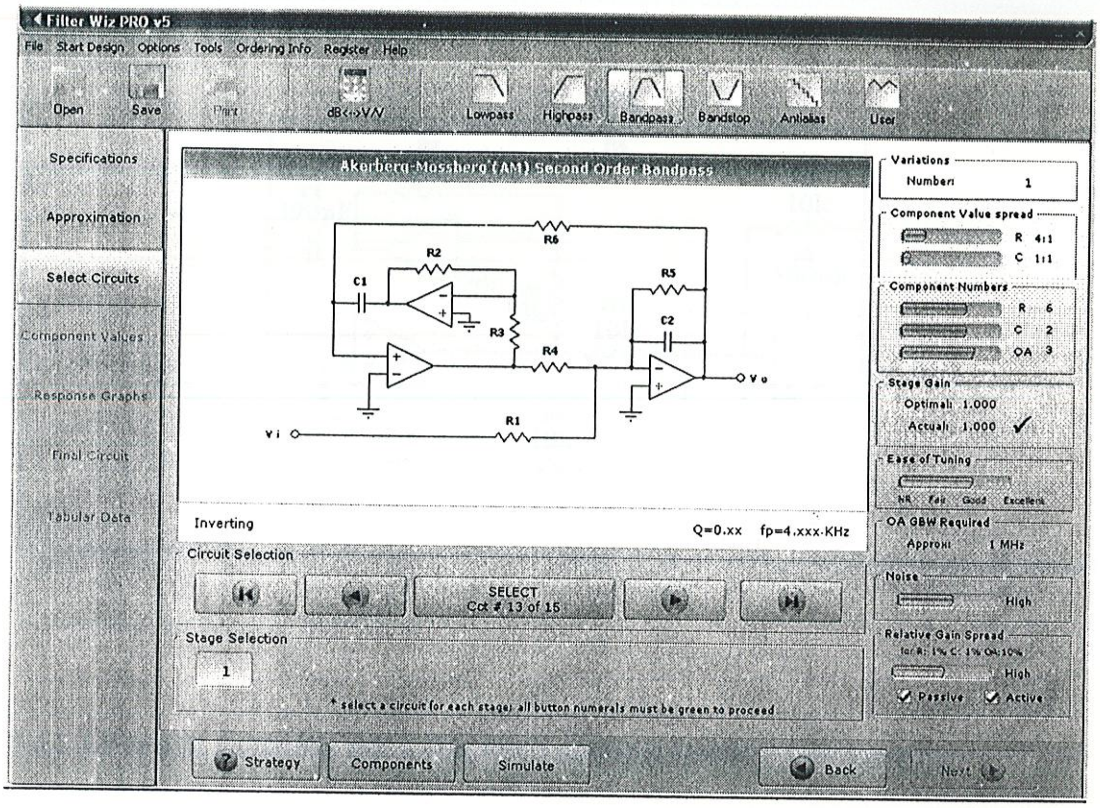
```
    C2 1 5 CMODEL 1.8nF
    C3 7 3 CMODEL 1.8nF
    XA1 0 2 3 IDEALOP
    XA2 0 5 6 IDEALOP
    XA3 7 0 4 IDEALOP
.ENDS MAINCCT

.MODEL RMODEL RES(R=1 DEV=1.00%)
.MODEL CMODEL CAP(C=1 DEV=1.00%)

X1 1 2 MAINCCT
*** Pts per decade: 100 start freq: 10.233 end freq: 100 k ***
.AC DEC 100 1.023E+1 10.000E+4
***.NOISE V (2) X1.V1 100***
.PROBE V (2)
```

5.5.2.4 BODE PLOT OF THE SECOND ORDER HIGH PASS FILTER

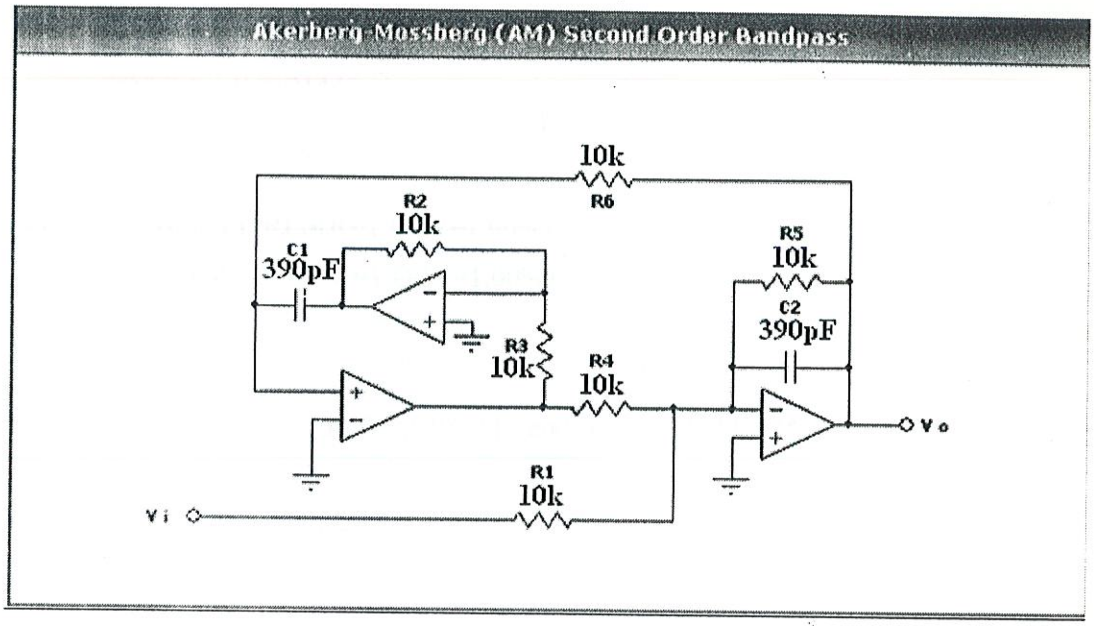


5.5.3 BAND PASS FILTER

5.5.3.1 The circuit diagram of the second order Band Pass Filter.



5.5.3.2 The circuit diagram of the second order band pass filter with values of resistors and capacitors.

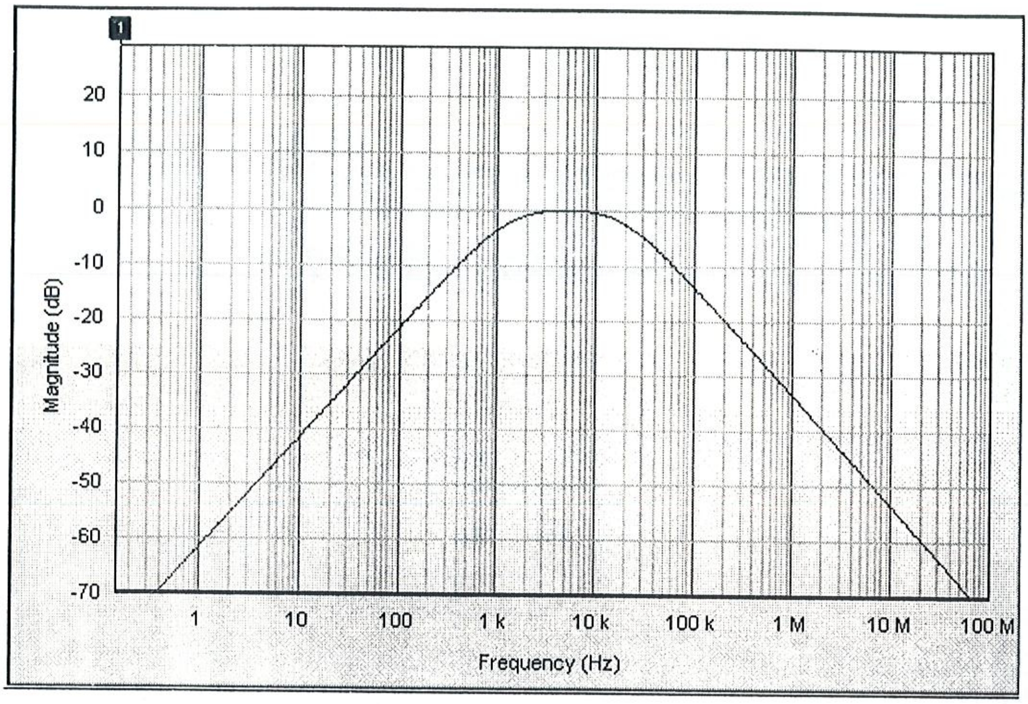


5.5.3.3 PSPICE CODING OF A SECOND ORDER HIGH PASS FILTER

```
.SUBCKT IDEALOP 2 1 3
        EIO 3 0 2 1 1.0E6
.ENDS IDEALOP
.SUBCKT MAINCCT 1 7
        V1 1 0 AC 1.000
        R1 1 2 RMODEL 1000
        R2 5 6 RMODEL 1000
        R3 6 4 RMODEL 1000
        R4 4 2 RMODEL 1000
        R5 2 7 RMODEL 1000
        R6 3 7 RMODEL 1000
```

```
        C1 3 5 CMODEL 390pF
        C2 2 7 CMODEL 390pF
        XA1 3 0 4 IDEALOP
        XA2 0 6 5 IDEALOP
        XA3 0 2 7 IDEALOP
.ENDS MAINCCT

.MODEL RMODEL RES(R=1 DEV=1.00%)
.MODEL CMODEL CAP(C=1 DEV=1.00%)

X1 1 2 MAINCCT
*** Pts per decade: 100 start freq: 401.112 end freq: 69.749 k ***
.AC DEC 100 4.011E+2 6.975E+4
***.NOISE V (2) X1.V1 100***
.PROBE V (2)

*** Monte Carlo runs: 100 ***
```

5.5.3.4 BODE PLOT OF THE SECOND ORDER BAND PASS FILTER

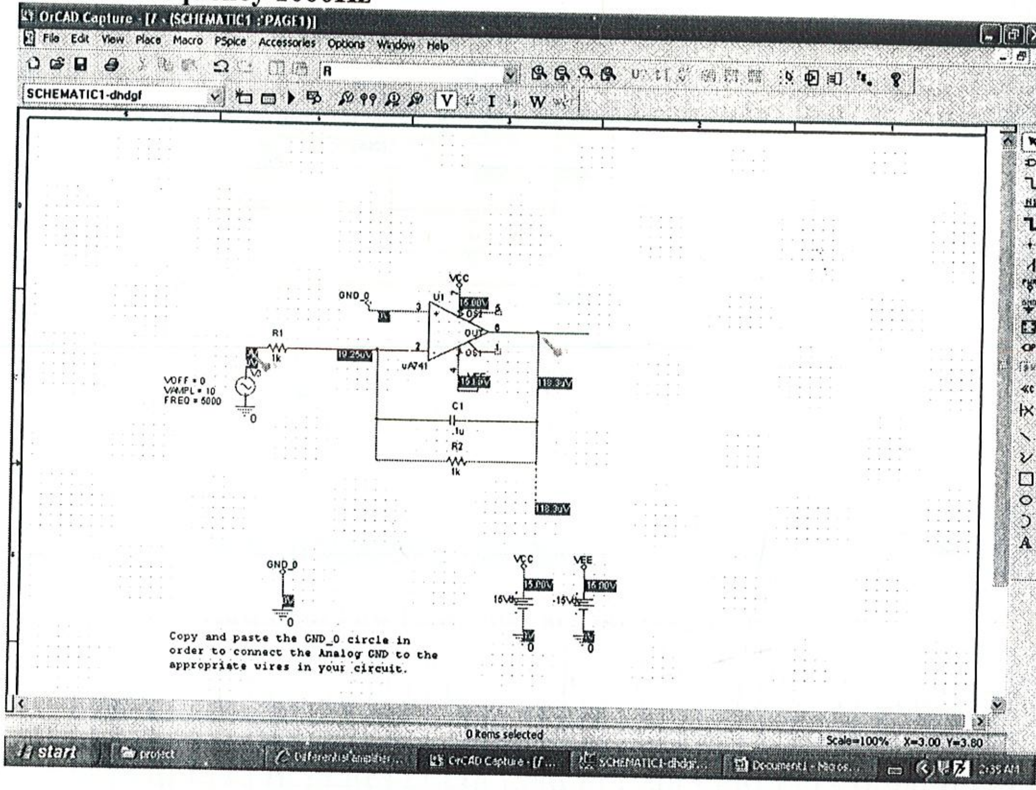


CHAPTER 6

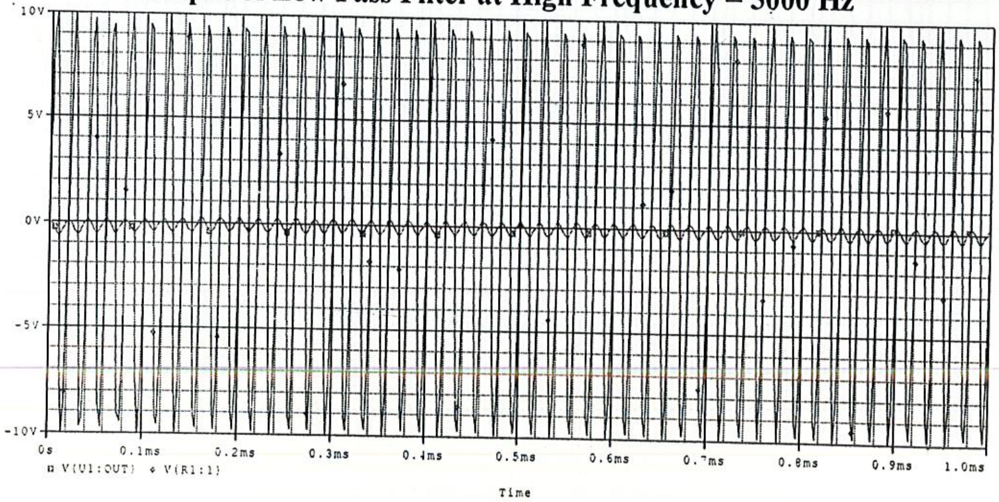
PSPICE SIMULATIONS

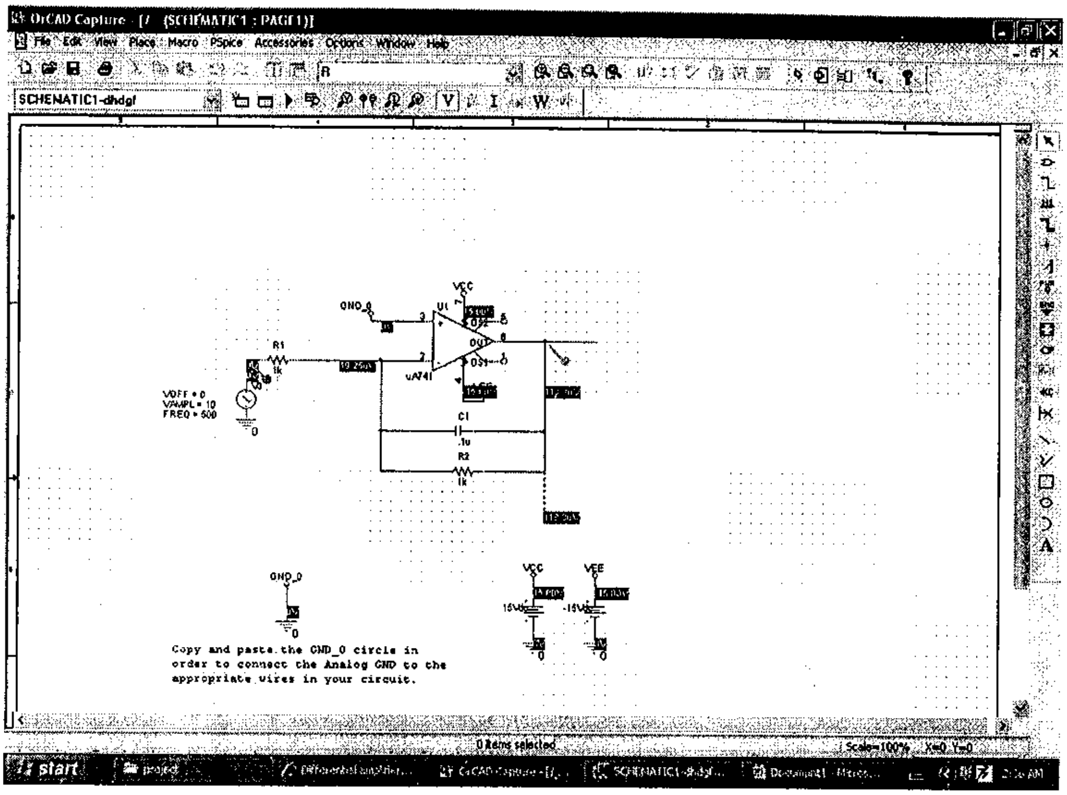
6.1 Low Pass Filter.

Cut Off Frequency 1000Hz

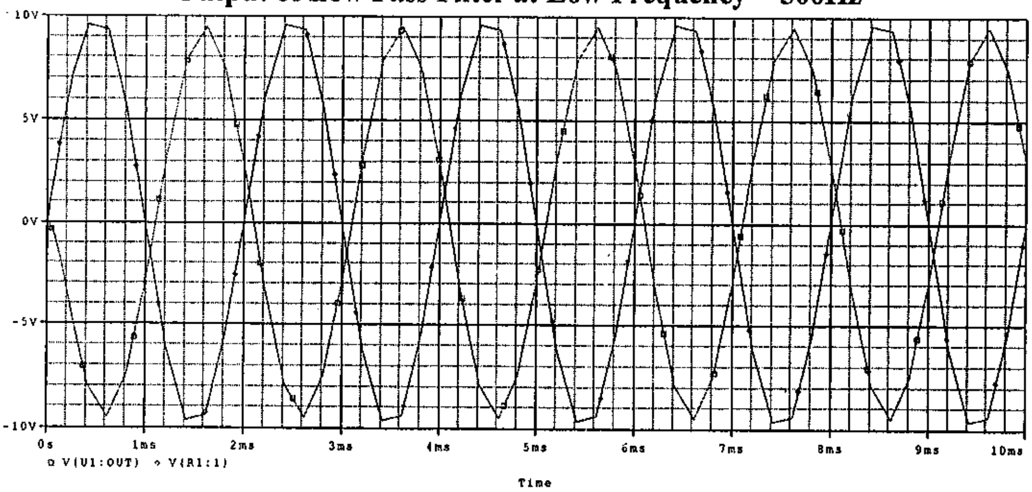


Output of Low Pass Filter at High Frequency = 5000 Hz

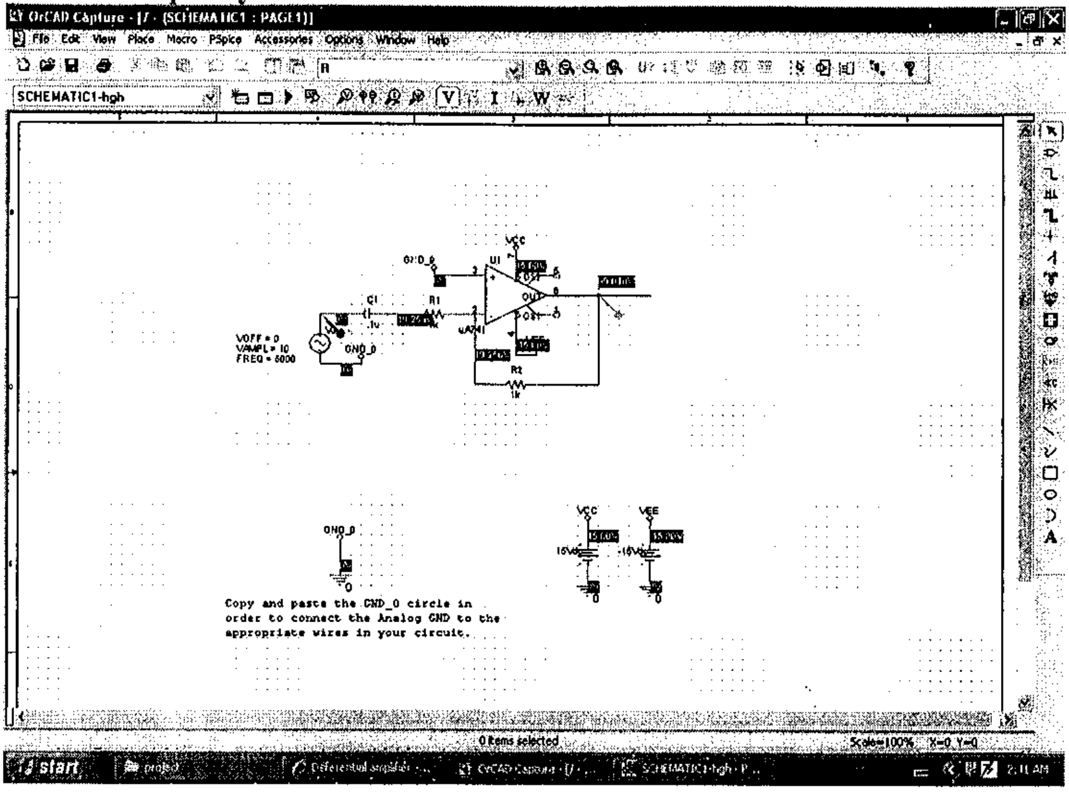




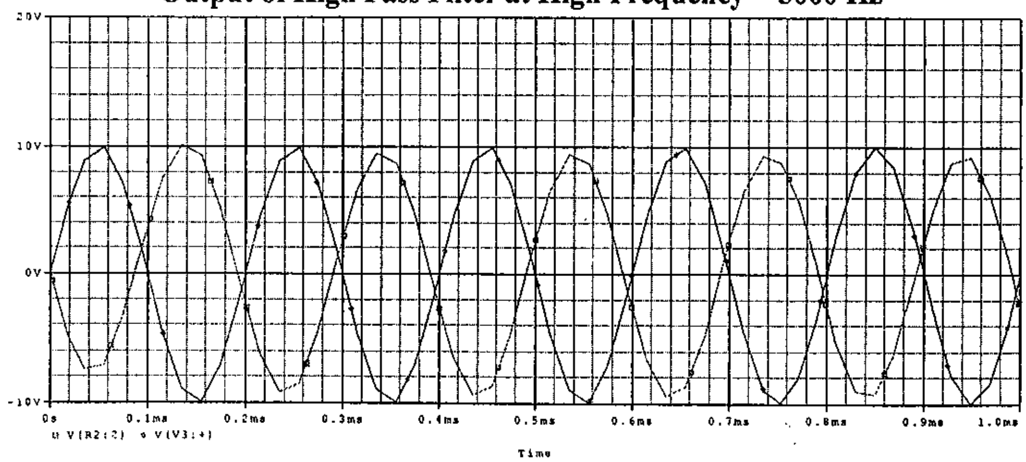
Output of Low Pass Filter at Low Frequency = 500Hz

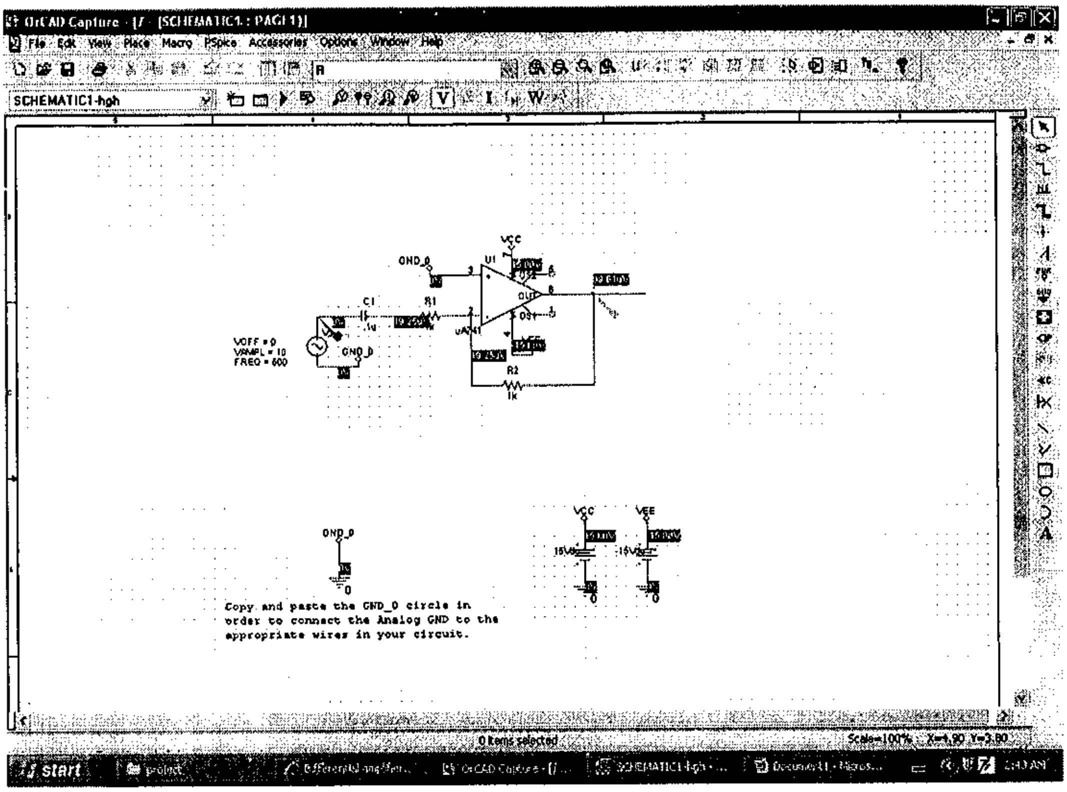


6.2 High Pass Filter
Cut Off Frequency 1000Hz

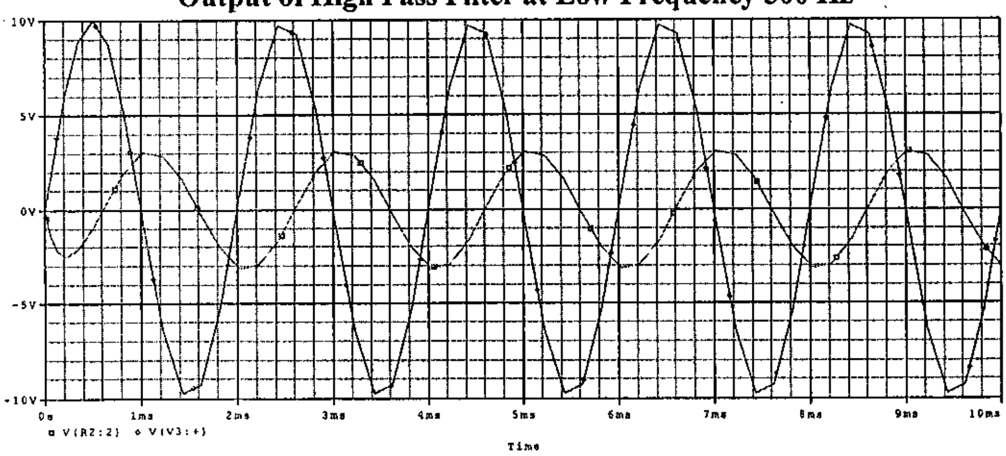


Output of High Pass Filter at High Frequency = 5000 Hz



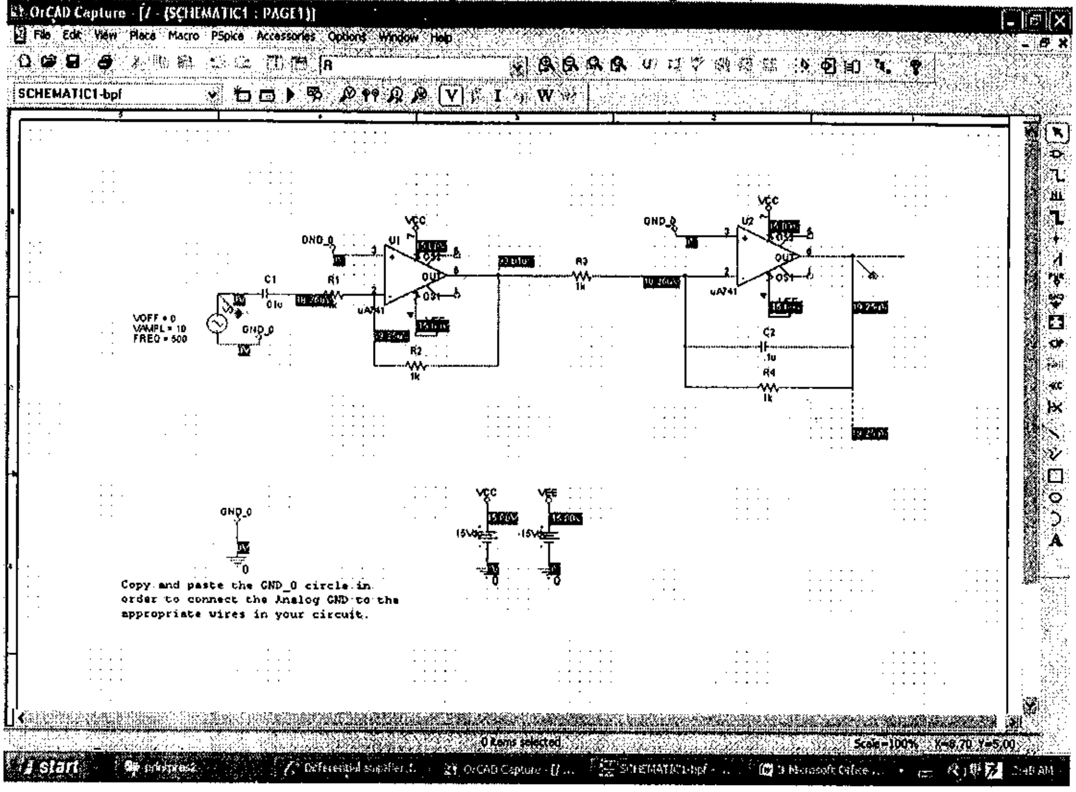


Output of High Pass Filter at Low Frequency 500 Hz

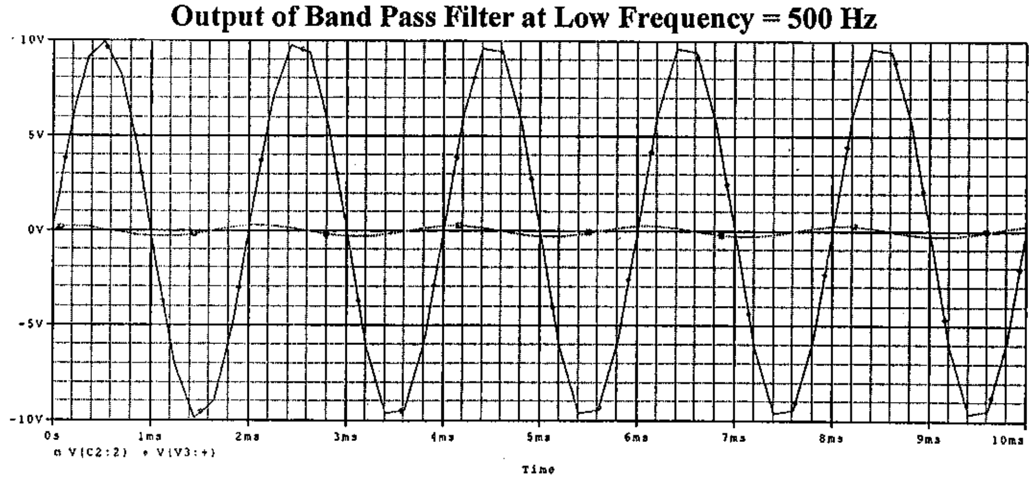


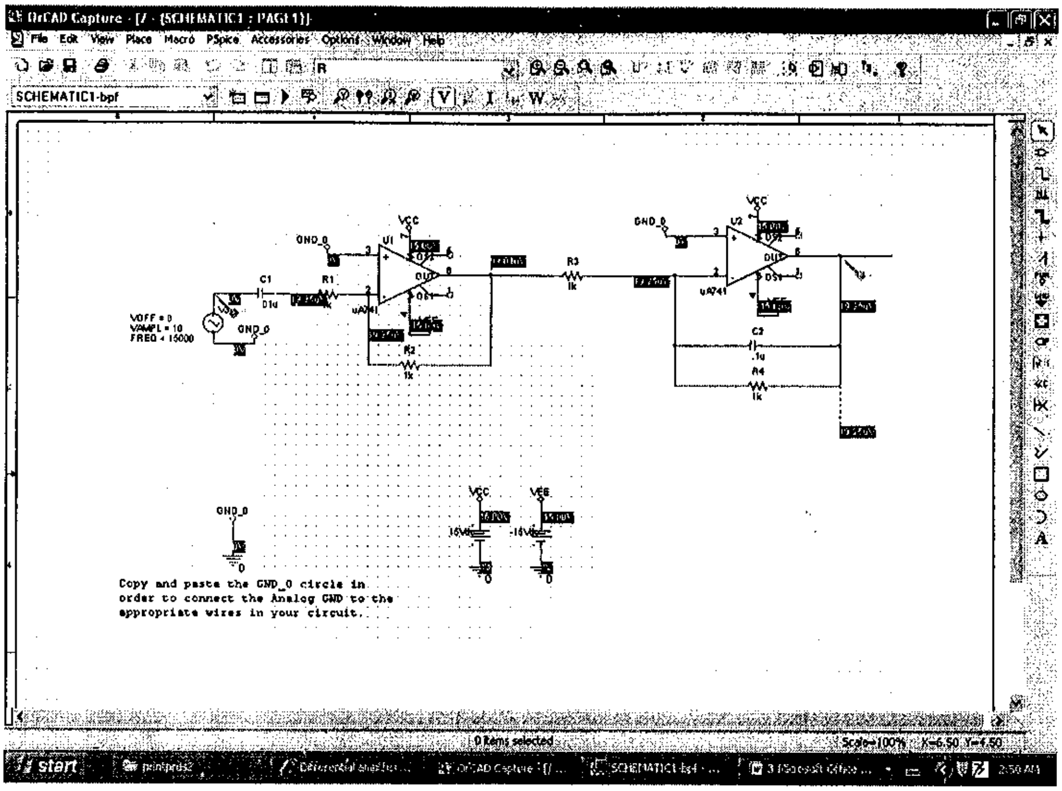
6.3 Band Pass Filter

Lower Cut Off Freq. = 1000Hz, Higher Cut Off Freq. = 10,000Hz

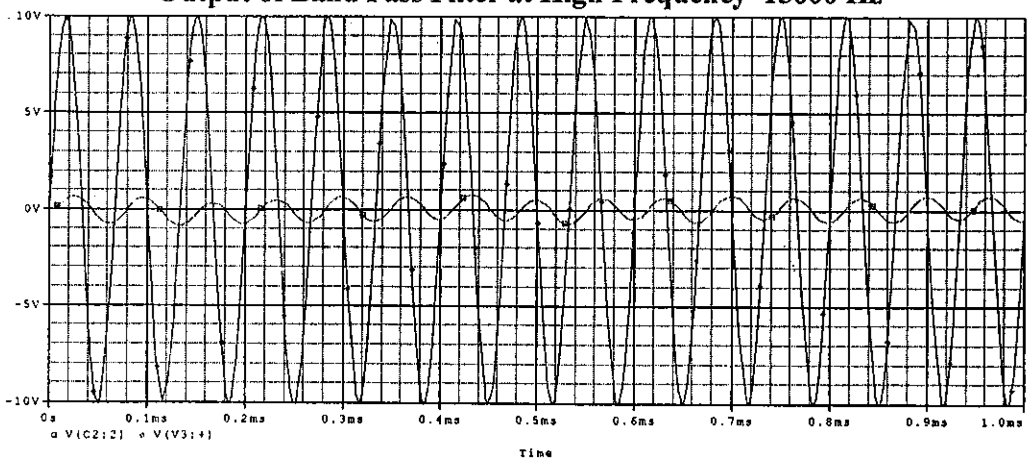


Output of Band Pass Filter at Low Frequency = 500 Hz





Output of Band Pass Filter at High Frequency 15000 Hz



CHAPTER 7

CONCLUSION AND FUTURE WORK

7.1 Results

The design of the three basic types of biquad filters namely, the low pass filter, the high pass filter, the band pass filter was successfully completed.The simulations of these designs were carried out using PSPICE and FILTERWIZ softwares.

7.2 Design

The development of the entire project can be broken into three parts:

1. The Design of first order passive filters .
2. The Design of first order active filters.
3. The Design of Biquad (second order) filters .

7.3 Difficulties

Although literature we found was very specific and was about a very particular kind of filter.Another problem we faced was the complex mathematics involved in the designing of the filters(complex coefficients were involved in the calculations).

7.4 Future Work

A lot of work has already been done by various people in this field,but a lot needs to be done.Biquad filters can be implemented by using OTA's (operational transconductance amplfiers) as they offer certain advantages over operational amplifiers such as the output of an operational tranconductance amplifier is current in contrast to voltage in operational amplifiers,biquad filters may be implemented by using less number of components.

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4. **Realizations of Voltage Mode Biquad Filters**

   Abhirup Lahiri, Abhinav Misra, Kinshuk Gupta
   Netaji Subhas Institute of Technology, Department of Electronics and Communications, Delhi, India
5. **A Basic Introduction to Filters**
   National Semiconductor Application Note 779 Kerry Lacanette April 1991

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9. **www.analog.com**

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APPENDICES

Appendix A

Matlab code for design of filters

```
n=input('enter the order of filter');
type = input('enter the type of filter lp hp or bp', 's');
if (type == 'bp')
   freql=input('enter the lower cut off frequency');
   frequ=input('enter the upper cut off frequency');
   w=[freql,frequ];
else
   w=input('enter the cut off frequency');
end
if(type == 'bp')
   type = 'bandpass';
elseif(type== 'lp')
   type = 'low';
elseif(type == 'hp')
   type='high';
end
a=input('enter b for buttorworth filter\nc1 for chebyshev type1 filter\nc2 for chebyshev
type2 filter\ne for elliptic filter\nbe for bessel filter','s');
if(a == 'b')
   [z,p,k]=butter(n,w,type,'s');
   zeros=z
   poles=p
   gain=k
elseif(a=='c1')
   Rp=input('enter the passband ripple factor');
   [z,p,k] = cheby1(n,Rp,w,type,'s');
   zeros=z
```

```
    poles=p
    gain=k
elseif(a=='c2')
    Rs=input('enter the stopband ripple factor');
    [z,p,k] = cheby2(n,Rs,w,type,'s');
    zeros=z
    poles=p
    gain=k
elseif(a=='e')
      Rp=input('enter the passband ripple factor');
      Rs=input('enter the stopband ripple factor');
    [z,p,k] = ellip(n,Rp,Rs,w,type,'s');
    zeros=z
    poles=p
    gain=k
elseif(a=='be')
    [z,p,k] = besself(n,w,type,'s');
    zeros=z
    poles=p
    gain=k
end
d=poly(p);
n=poly(z);
H=tf(n,d)
figure
grid on
bode(n,d)
title('Frequency response of the system')
clear
```