JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -1 EXAMINATION- 2024

B. Tech. -III Semester (CSE-AI&ML, AI&DS)

COURSE CODE (CREDITS): 24B11CI311(3) MAX. MARKS: 15

COURSE NAME: Computational Fundamentals for Optimization

COURSE INSTRUCTORS: SST MAX. TIME: 1 Hour

Note: *(a) All questions are compulsory.*

(b) Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems

(d) Use of scientific calculator is allowed.

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1. For a weather forecasting Markov model, the transition matrix is given by:

$$M_{3\times3} = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.4 & 0.2 & 0.4 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}.$$

a) How is energy of a matrix related to the loss function in recommender systems?

b) Obtain the Frobenius norm for $M_{3\times3}$.

c) Find the energy of $M_{3\times3}$.

d) Write the relation between energy and trace of any matrix A. **(CO 1)[1+0.5+0.5+1]**

2. Answer the following:

a) Are the vectors $u = (2,-2,0), v = (6,1,4), w = (2,0,-4)$ linearly independent or not?

b) What is the role of linear independence of vectors in data redundancy?

c) Show that the vectors $w_1 = (0,2,0)$, $w_2 = (3,0,3), w_3 = (-4,0,4)$ form an orthogonal basis for R^3 with the Euclidean inner product, and use this basis to find an orthonormal basis. **(CO 1)[1+1+1]**

3. Use the following *LU-decomposition*:

$$\begin{bmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 4 & 0 \\ -4 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$, i.e., $A = LU$, to solve the following system of linear equations:

$$3x - 6y - 3z = -3$$
$$2x \qquad + 6z = -22$$
$$-4x + 7y + 4z = 3$$

(CO 2)[3]

4. Obtain the final image of the input vector $\overline{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ after implementing the following sequence of transformations: *Rotation of* $\frac{\pi}{6}$ *radians in the counter clockwise direction →Reflection about the y-axis→ Contraction with the factor 0.5.* **(CO 2)[1+1+1]**

5. Verify that the mapping $T: (R^2, +, .) \rightarrow (R^3, +, .)$ defined as $T(a, b) = (a + b, a - b, b)$ is a linear transformation or not. **(CO 3)[3]**