DIRECTION OF ARRIVAL ESTIMATION USING ANTENNA ARRAYS

Project Report submitted in partial fulfilment of the requirement

for the degree of

Bachelor of Technology

in

Electronics and Communication Engineering

Under the Supervision of

Prof. SUNIL V BHOOSHAN

By

PARWINDERJIT SINGH (091074)

PIYUSH KASHYAP (091077)

RAJAT SHARMA (091078)

to

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY

Jaypee University of Information and Technology

Waknaghat, Solan – 173234, Himachal Pradesh

CERTIFICATE

This is to certify that project report entitled, "**DIRECTION OF ARRIVAL ESTIMATION USING ANTENNA ARRAYS**" submitted by **Parwinderjit Singh, Piyush Kashyap and Rajat Sharma** in partial fulfilment for the award of degree of Bachelor of Technology in Electronics and Communication Engineering to Jaypee University of Information Technology, Waknaghat, Solan has been carried out under my supervision.

This work has not been submitted partially or fully to any other University or Institute for the award of this or any other degree or diploma.

Date: 30th May, 2013

Supervisor's Name - Sunil V Bhooshan

Designation – Professor and Head of Department

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We are deeply indebted to Professor Sunil V Bhooshan for his able guidance, expert advice and showing us the right way of carrying out the research project. We are also thankful to him for giving us the special attention and cooperation in between his very busy hours. We would like to take this opportunity to express our heart-felt indebtedness for his support and enlightenment. We are also grateful to our entire electronics faculty for their efforts to strengthen the fundamental concepts so that one could take up any assignment of serious gravity with confidence. At this moment we would also like to express our gratitude to the technical staff of our laboratories. They have always helped us in every-way they could during our project work.

Date: 30th May, 2013

Parwinderjit Singh (091074)

Piyush Kashyap (091077)

Rajat Sharma (091078)

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Abstract

Direction of Arrival Estimation (DOA) using Antenna arrays is becoming popular for use in various applications which previously worked on Single Non Isotropic Antennas. New applications to the antenna have emerged which have highlighted some inherent disadvantages of a single Non Isotropic Antenna. First problem: there has been a consistent demand to increase the Radiation gain of an antenna to cover large geographical regions. Second problem: A single Non isotropic antenna is directional in nature. The only way to make High Gain Antennas cover every possible direction is by mechanically rotating a Large Sized Antenna over all directions. This has its own economic and physical limitations.

As part of our Final year project, the problem of Direction of Arrival Estimation using Antenna Arrays is considered. The first part of the project includes Plotting and studying Field pattern plots of Linear, Rectangular and Circular Arrays. Array Factor plots of two antenna arrays have been analysed for different values of Intensity/excitation currents and angles of maxima.

In the Second part, Desired Pattern synthesis problem has been dealt with. We do this by finding weights (phases) to be assigned to every element of the array when interference signals of different intensities are introduced at required angles.

In the third part, DOA Estimation for a 2-element array using lookup tables has been studied. In addition, DOA estimation algorithm by electronic beam steering has been implemented.

Chapter 1: Introduction

1.1 Antenna Arrays

An antenna array (often called a phased array) is a set of two or more antennas. The signals from the antennas are combined or processed in order to achieve improved performance over that of a single antenna. The antenna array can be used to:

- Increase the overall gain
- Provide diversity reception
- Cancel out interference from a particular set of directions
- "Steer" the array so that it is more sensitive in particular direction
- Determine the direction of arrival of incoming signal
- To maximize signal to interference plus noise ratio (SINR)

In general, the performance of an antenna array (for whatever application it is being used), increases with the number of antennas (elements) in the array; the drawback of course in the increased cost, size and complexity.

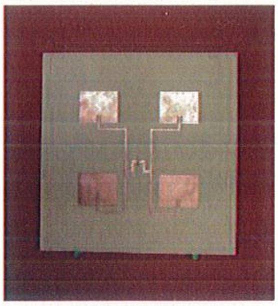

Figure 1.1 - Antenna Array

1.2 Direction of Arrival estimation

In signal processing literature, **direction of arrival** denotes the direction from which usually a propagating wave arrives at a point, where usually a set of sensors are located. This set of sensors forms what is called a sensor array. Often there is the associated technique of beam forming which is estimating the signal from a given direction. Various engineering problems addressed in the associated literature are:

- Find the direction relative to the array where the underwater sound source is located.
- Directions of different sound sources around you are also located by you using a process similar to those used by the algorithms in the literature.
- Radio telescopes use these techniques to look at a certain location in the sky.
- Various techniques for calculating the direction of arrival, such as Angle of Arrival (AOA), Time Difference of Arrival (TDOA), Frequency Difference of Arrival (FDOA), or other similar associated techniques.

1.3 Plan of Action

Step 1

Study of Linear, Planar (Circular, Rectangular) Antenna Arrays and their Radiation Patterns.

Step 2

Plotting of the Radiation patterns Using MATLAB.

Step 3

Desired Pattern Synthesis using MATLAB.

- Introducing a Null at a particular direction.
- Introducing a Null for a range of Angles.

- Developing an Algorithm to get a desired Radiation Pattern from a Linear Antenna Array: - This MATLAB program takes in Desired Field Pattern and gives corresponding array specifications.

Step 4

Studying DOA (Direction of Arrival) estimation problem.

- Why not single Non-Isotropic Antenna?
- A simple method of DOA estimation using a look-up table for a 2-element Linear Array and Disadvantages of using this method.
- DOA estimation using 2-element array assuming radiation pattern is on z-y plane.
- DOA estimation using 2-element array assuming radiation pattern is in 3-Dimensional space : The problem involves steering the beam and scanning in all directions.

Chapter 2: Antenna Basics

2.1 Radiation Pattern

An antenna radiation pattern or antenna pattern is defined as "a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates. In most cases, the radiation pattern is determined in the far field region and is represented as a function of the directional coordinates.

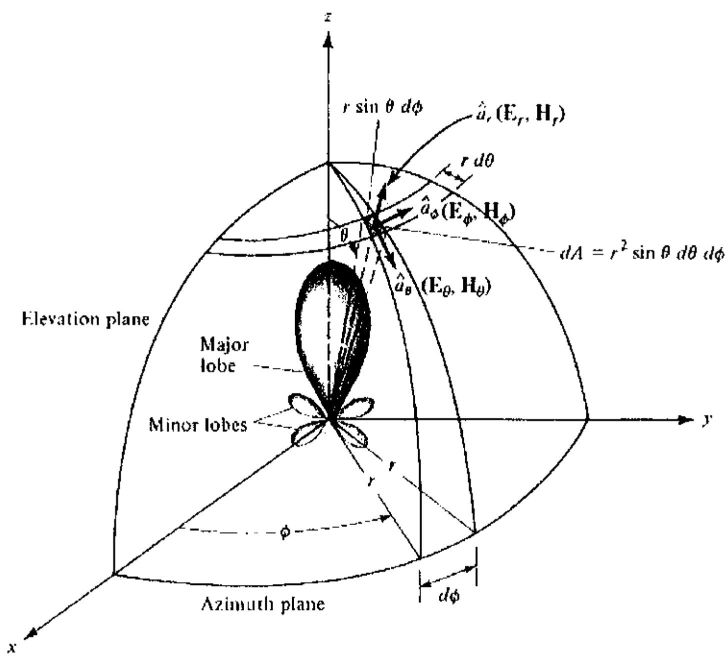

Figure 2.1 – Coordinate System for Antenna Analysis.

In the figure above, a Radiation Pattern and the reference coordinate system for Antenna Analysis has been shown.

A trace of the received electric (magnetic) field at a constant radius is called the **amplitude field pattern**. A graph of the spatial variation of the power density along a constant radius is called an **amplitude power pattern**. Often the field and power patterns are normalized with respect to their maximum value, yielding **normalized field and power patterns**.

2.2 Radiation Pattern Lobes

Various parts of a radiation pattern are referred to as lobes, which may be sub classified into major or main, minor, side, and back lobes.

- A **radiation lobe** is a "portion of the radiation pattern bounded by regions of relatively weak radiation intensity."
- A **major lobe** (also called main beam) is defined as "the radiation lobe containing the direction of maximum radiation."
- A **minor lobe** is any lobe except a major lobe.
- A **side lobe** is "a radiation lobe in any direction other than the intended lobe." (Usually a side lobe is adjacent to the main lobe and occupies the hemisphere in the direction of the main beam.)
- A **back lobe** is "a radiation lobe whose axis makes an angle of approximately 180° with respect to the beam of an antenna." Usually it refers to a minor lobe that occupies the hemisphere in a direction opposite to that of the major (main) lobe.
- The (+) and (-) signs in the lobe (figure 2.2) indicate the relative polarization of amplitude between the various lobes, which changes (alternates) as the nulls are crossed.

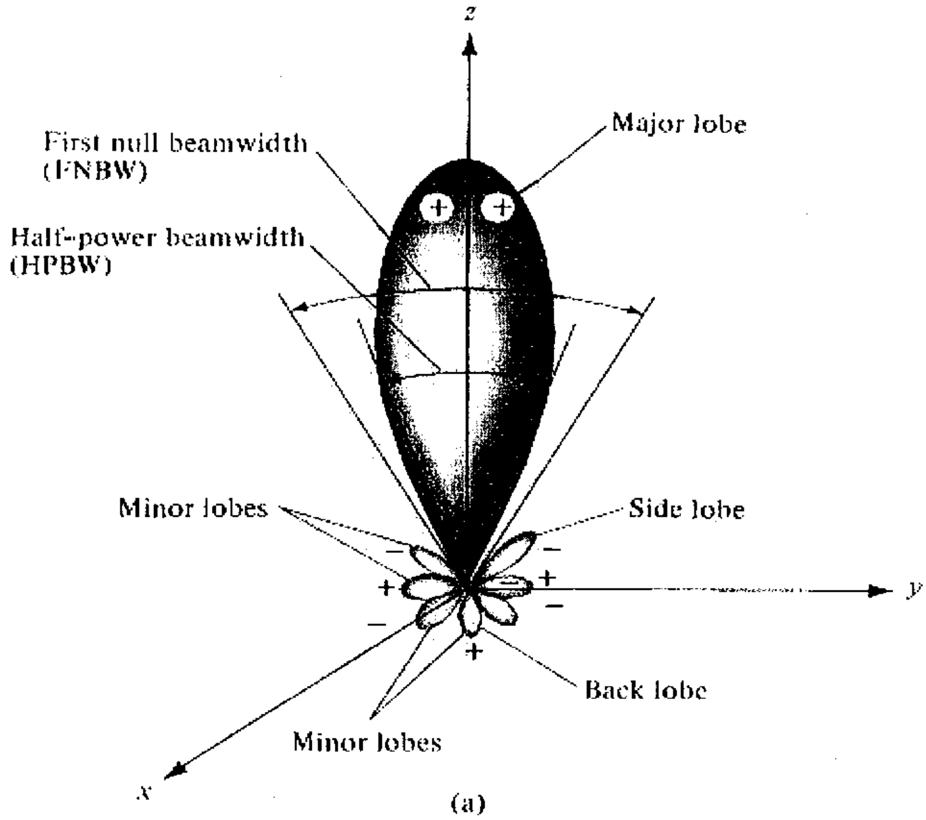

Figure 2.2 – Radiation Lobes and Beam width of an Antenna Pattern

2.3 Directivity

Directivity is a fundamental antenna parameter. It is a measure of how directional an antenna's radiation pattern is directivity of an antenna

It is defined as "the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. The average radiation intensity is equal to the total power radiated by the antenna divided by 4π. If the direction is not specified, the direction of maximum radiation intensity is implied."

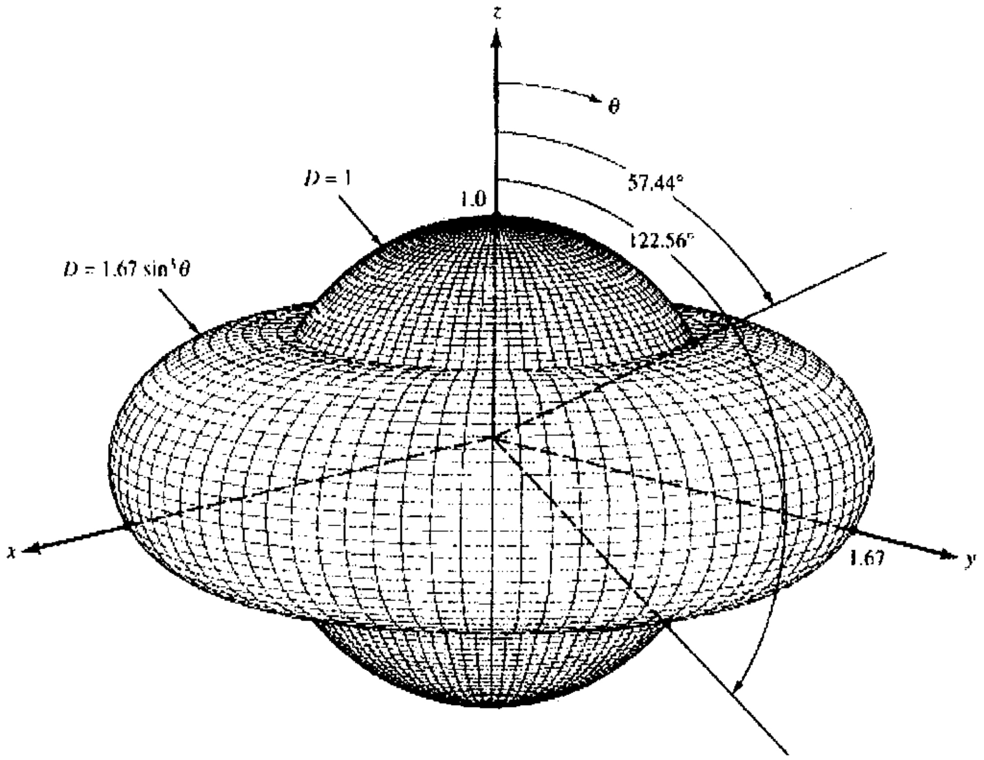

Figure 2.3 – Three Dimensional directivity Pattern of a λ/2 dipole and Isotropic Antenna.

2.4 Gain

Gain of an antenna (in a given direction) is defined as "the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically. It is a measure that takes into account the efficiency of the antenna as well as its directional capabilities.

2.5 Radiation Pattern of Single Isotropic Antenna

An isotropic antenna is a theoretical point source of electromagnetic waves which radiates the same intensity of radiation in all directions. It has no preferred direction of radiation. It radiates the same intensity of radiations in all directions uniformly over a sphere centred on the source.

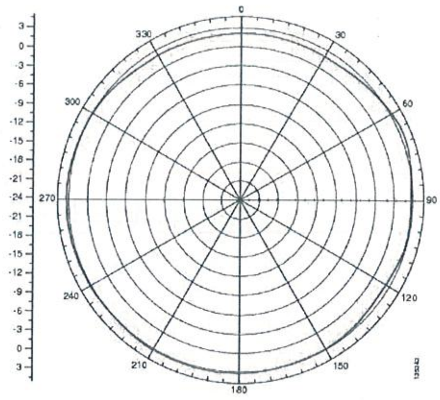

Figure 2.4 - Radiation Pattern of Single Isotropic Antenna

2.6 Radiation Pattern for Single Non- Isotropic Antenna:

A Non-Isotropic antenna is a directional antenna i.e. it has a preferred direction of radiation. The Power level is not the same in all directions. This allows increased performance on transmit and receive and reduced interference from unwanted sources. All practical antennas are at least somewhat directional.

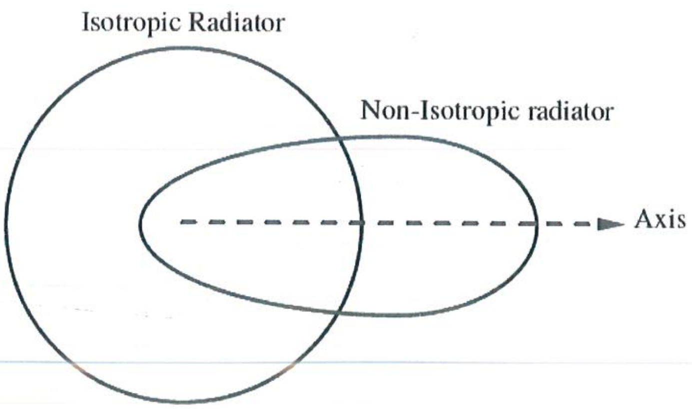

Figure 2.5 - Radiation Pattern for Single Non-Isotropic Antenna

Chapter 3: Antenna Array

An Antenna array is a set of 2 or more antennas. The signals from these antennas are combined or processed in order to achieve improved performance over that of a Single Antenna.

3.1 Advantages of Antenna Array:

- They can provide the capability of Change in Radiation direction.
- They can provide High Gain (Array gain) by using simple antenna elements.
- They can be used to cancel out interference from a particular set of directions.
- They can be used to steer the array so that it is most sensitive in a particular direction.
- They can be used to determine direction of arrival of incoming signal.
- They can be used to maximize the Signal to Interference plus Noise Ratio (SNR).

3.2 Array factor: Radiation Pattern of Antenna Array

Due to the physical arrangement of the array elements, a path difference **dcosθ** is introduced and these results in a phase difference in signal received at both the array elements.

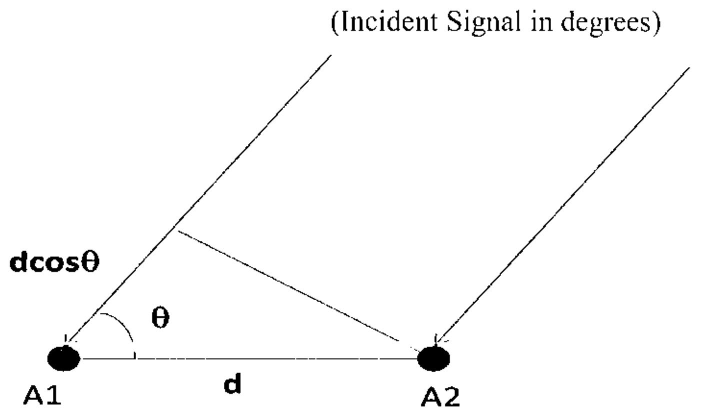

Figure 3.1-2 Element Linear Array.

- **Path difference** of λ → 2π **(Phase difference)**
- dcosθ →(2π/λ) dcosθ = βdcosθ

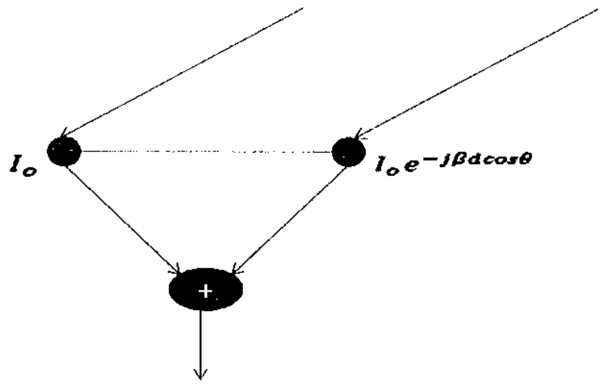

Figure 3.2- Obtaining Array Factor for 2-Element Linear Array

For 2-element array: $AF = I_0 + I_0 e^{-j\beta d\cos\theta}$

3.3 Radiation Pattern: N-element Linear Array

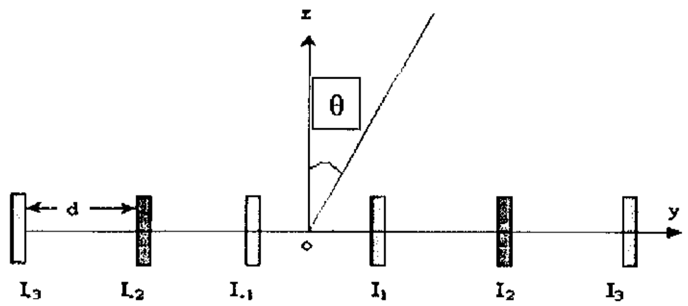

Figure 3.3 - n-Element Linear Array

Array Factor for N-element linear array:

$$\sum_{k=0}^{n} I_k\, e^{-j\beta dk\cos\theta}$$

Array Factor plot for a 6-element arrays is given below:

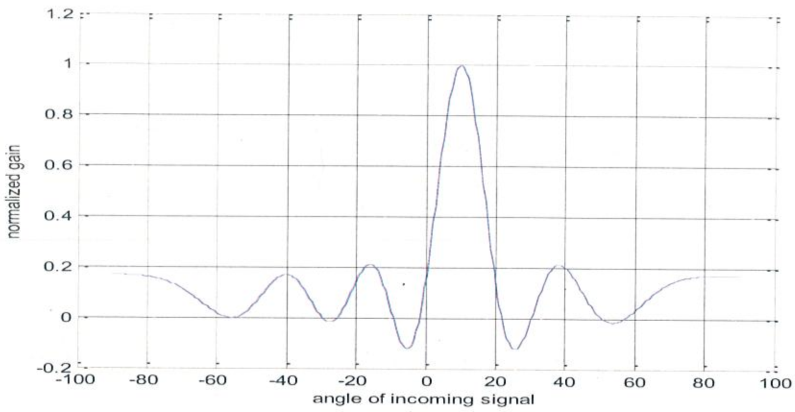

Figure 3.4 – Array Factor Plot for Maxima at θ=10° (6- Element Array)

3.3.1 Flowchart: Plotting Array factor plots of linear array

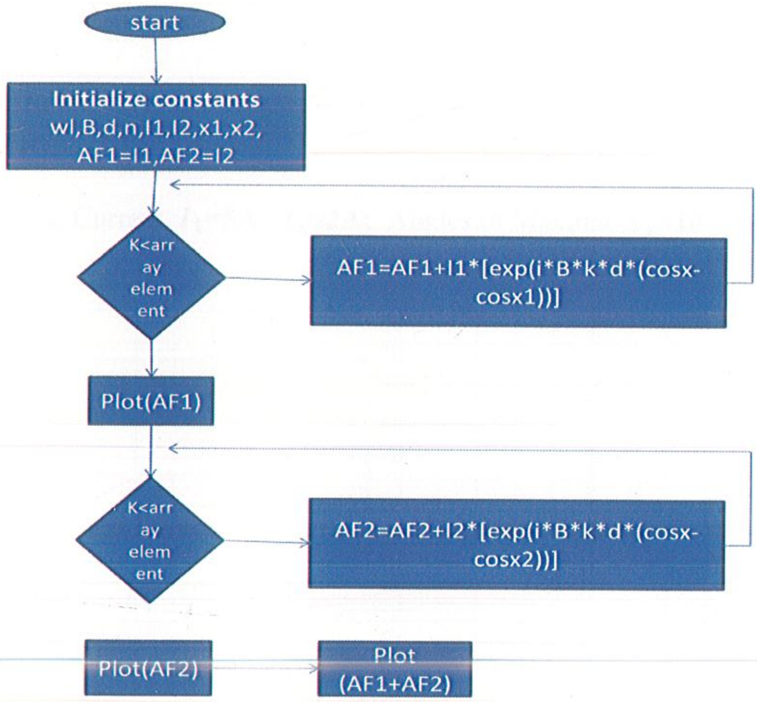

Figure 3.5 - Flowchart

- wl= Wavelength of incoming signal.
- B= Propagation Constant.
- N= Number of Elements.
- I_1, I_2= Common Amplitude coefficients for Antenna Array 1 and Antenna Array 2 respectively.
- x_1, x_2= Desired angles of Maxima for Antenna Array 1 and Antenna Array 2 respectively.
- AF_1, AF_2= Array Factor for Antenna Array 1 and Antenna Array 2 respectively.
- AF_1+AF_2= Effective Radiation Pattern for Antenna Array 1 and Antenna Array 2 respectively.

To obtain Array Factor, Signal Incident at every element multiplied with corresponding phase coefficient are added up. This is done for 2 linear arrays with patterns AF_1 and AF_2. Results of adding up the two Array Factor are analysed for different values of Intensities (I_1,I_2) and angles of Maxima (x_1,x_2).

3.3.2 Radiation Pattern comparison

Excitation Current I_1=5A , I_2=2A; Angles of Maxima x_1=10°, x_2=30°

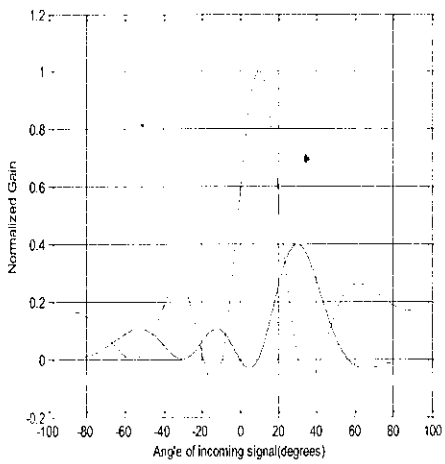

Figure 3.6 AF_1 and AF_2

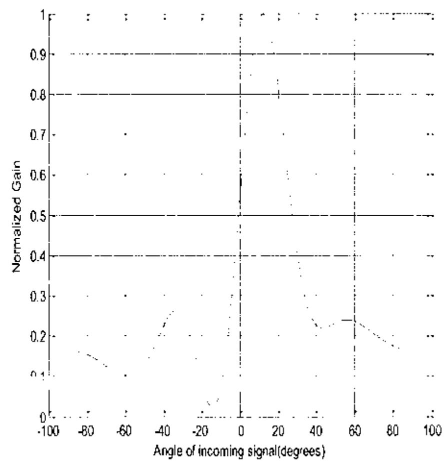

Figure 3.7 AF_1 + AF_2

(For Figure 3.6 and Figure 3.7) Since $I_1 > I_2$, the maxima of Array Factor ($AF_1 + AF_2$) is directed towards more towards Maxima of AF_1.

Excitation Current $\boldsymbol{I_1}$**=5A,** $\boldsymbol{I_2}$**=5A;** Angles of Maxima $\boldsymbol{x_1}$**=45°,** $\boldsymbol{x_2}$**=45°**

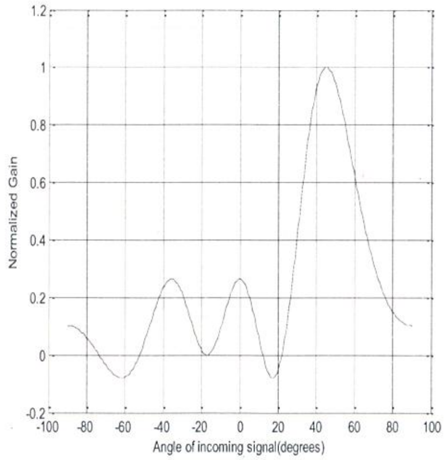

Figure 3.8 – AF_1 and AF_2

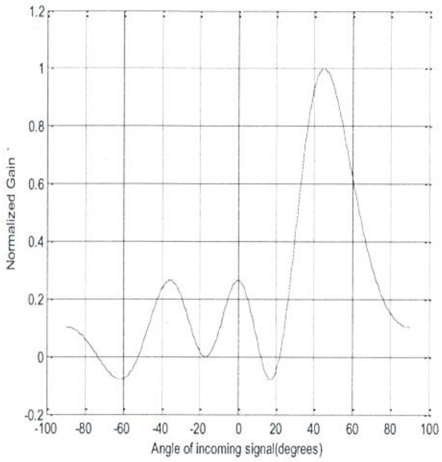

Figure 3.9 – $AF_1 + AF_2$

Since both AF_1 and AF_2 have identical Intensities and angles of Maxima. Effective Array Factor (AF_1+AF_2) is similar with greater gain.

(Here $AF_1 + AF_2$ is normalised.)

Excitation Current I_1=**5A**, I_2=**5A**; Angles of Maxima x_1=**10°**, x_2=**30°**

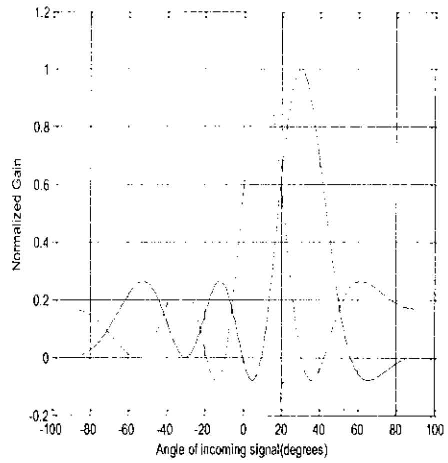

Figure 3.10 – $AF_1$and AF_2

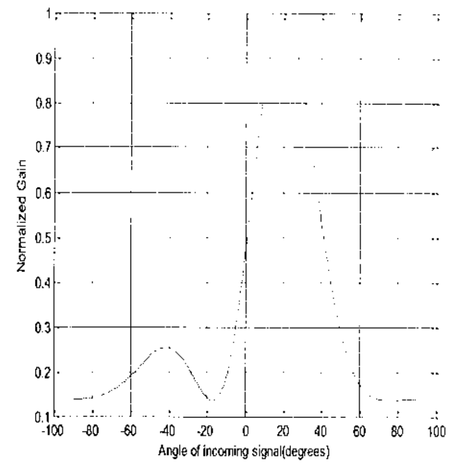

Figure 3.11 – AF_1 + AF_2

Here $AF_1$and AF_2 have identical intensities and different angle of maxima. Therefore angle of maxima of resultant Array Factor (AF_1 + AF_2) is in between x_1 and x_2.

Excitation Current I_1=**40A**, I_2=**1A**; Angles of Maxima x_1=**10°**, x_2=**30°**

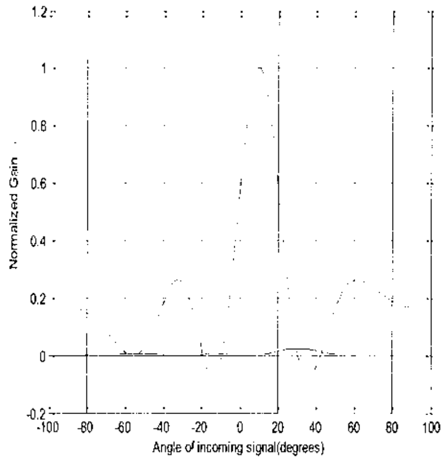

Figure 3.12 – $AF_1$and AF_2

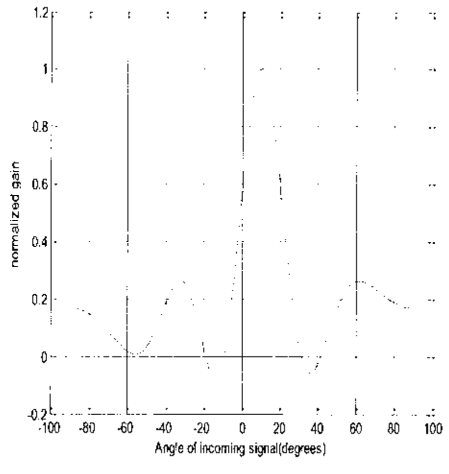

Figure 3.13– AF_1+ AF_2

Since Intensity of I_1>>>I_2, resultant Array Factor (AF_1 +AF_2) remain approximately identical.

3.4 Radiation Pattern: Rectangular (Planar) Array

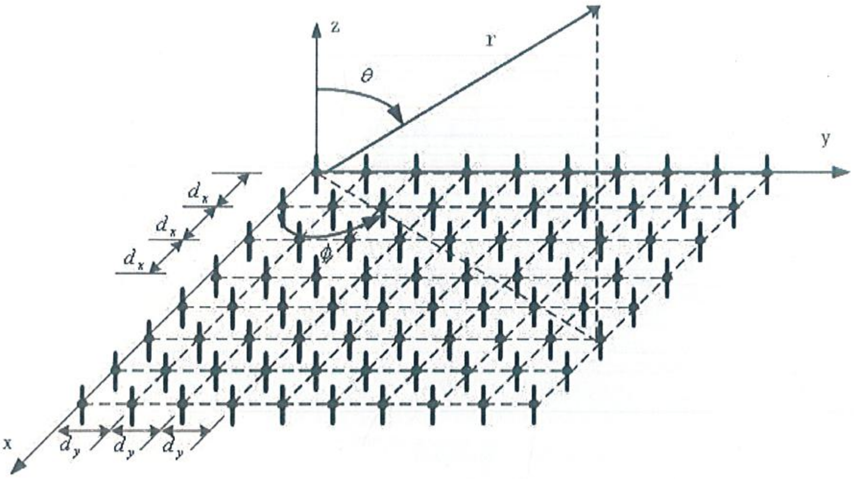

Figure 3.14- Planar Rectangular Antenna Array

Array Factor for N*N Rectangular array:

Consider one side of Rectangular array to be a linear array of N elements. Let its array factor be: $\boldsymbol{AF_x}$. Similarly, let array factor of other side of N elements be: $\boldsymbol{AF_y}$. Array Factor of N*N Rectangular array = $\boldsymbol{AF_x}$* $\boldsymbol{AF_y}$. This is also called **Pattern multiplication.**

$$\mathbf{AF}_x = \sum_{k=1}^{N} \mathbf{AF}_{x_{K-1}} \, e^{\beta kd \cos\Phi \sin\theta}$$

$$\mathbf{AF}_y = \sum_{k=1}^{N} \mathbf{AF}_{y_{K-1}} \, e^{\beta kd \sin\Phi \sin\theta}$$

Using the above equations, we get the results shown in Figure 3.15. It can be observed that a maxima is achieved for θ= 0° and 180°, i.e., the region is perpendicularly above and below the plane of array. For this case, the signals incident on array elements have a zero phase difference, and therefore they constructively interfere to give maxima.

3.4.1 Array Factor with both θ and φ varying

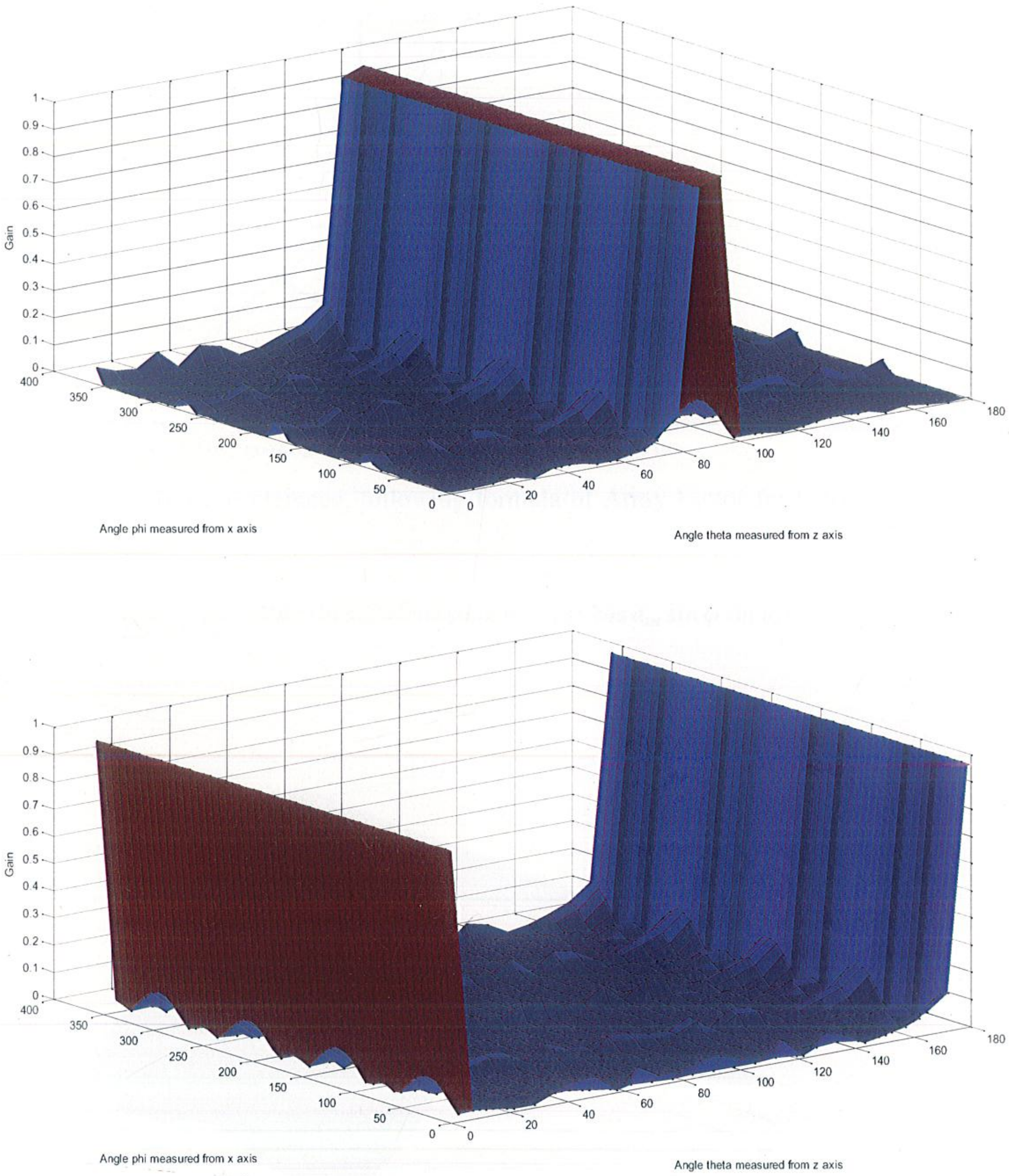

Figure 3.15 – 3 Dimensional representation of Array Factor with both θ and φ varying for a 13x13 array

3.5 Radiation Pattern for Circular array

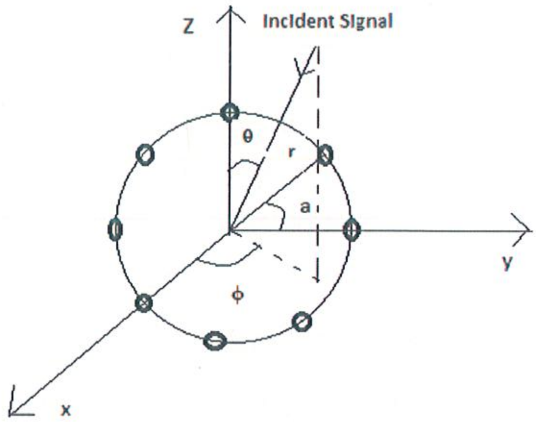

Figure 3.16 – Arrangement of Elements for circular Array

Using above Figure 3.18 as a reference, following formula of Array Factor for Circular Array is obtained:

$$\textbf{Array Factor} = \sum_{m=1}^{n} I_0 \, e^{((j\beta r \sin a_m)*(\cos\varphi \sin\theta)+(j\beta r \cos a_m \sin\varphi \sin\theta))}$$

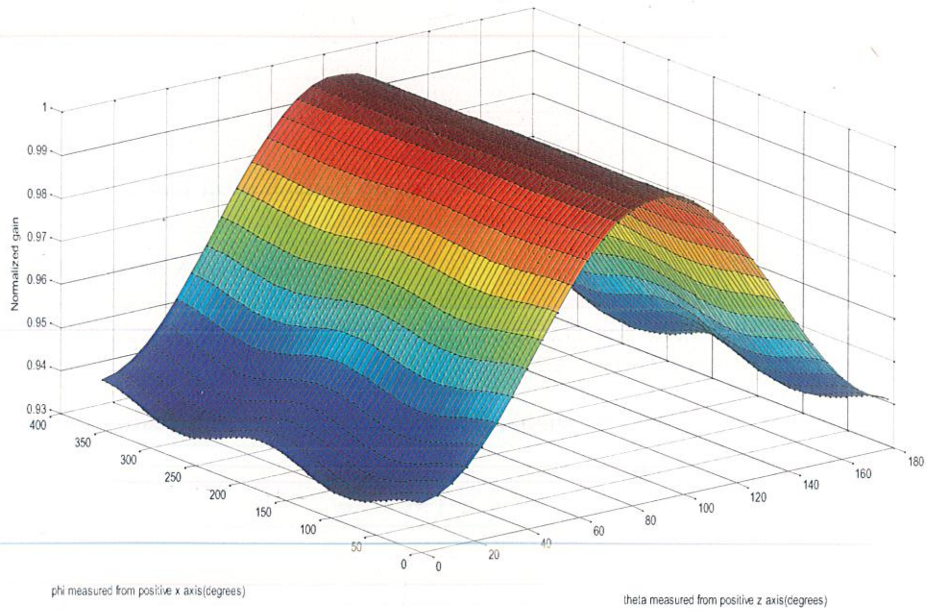

Figure 3.17 – 3 Dimensional Radiation Pattern for 20 Elements Circular Array.

Chapter 4: Pattern Synthesis

Beam forming allows the placement of single or multiple nulls in the antenna pattern at specific interference directions. Prescribed nulls in the radiation pattern need to be formed to suppress interferences from specific directions. For broadband interference, nulls in the pattern should be wide and deep enough to suppress peak side lobe levels at the angular sector of arrival of interference. Wide nulls are formed by placing multiple adjacent nulls in the radiation pattern.

The nulling methods are based on controlling complex weights: amplitude only, phase only and amplitude and phase control. Interference suppression with complex weights is the most efficient as it has greater degree of freedom in any 3-dimensional space.

We have focussed on optimising element coefficients by using phase control. The arrays possess even symmetry about the center of linear array as the number of phase shifters and computational time are halved.

The technique involves creating a Weight matrix (W) that contains phase coefficients to be multiplied with each element of the array to get the desired pattern.

$$W = \mu\phi_u^{-1}U_d^* \quad \text{--- [3]}$$

Where,

U_d is Matrix of signal from desired direction

μ is an arbitrary non zero scalar.

W is the Weight Matrix

ϕ_u is covariance matrix of undesired signal defined as

$$\phi_u = \sigma^2 I + A_i^2 U_i^* U_i^T$$

U_i is the matrix specifying angles where interfernece signals are introduced.

4.1 Introducing a null at one particular angle and Range of angles

The Algorithm

To find a weight vector giving an acceptable pattern with the given set of elements; first the main beam is steered in required direction by choosing the vector U_d .Then to force the sidelobes down in other directions one or large number of closely spaced interfering signals are assumed to be incident on the array from side lobe region. The powers of these interfering signals are then adjusted iteratively, until null is obtained.

Original Pattern

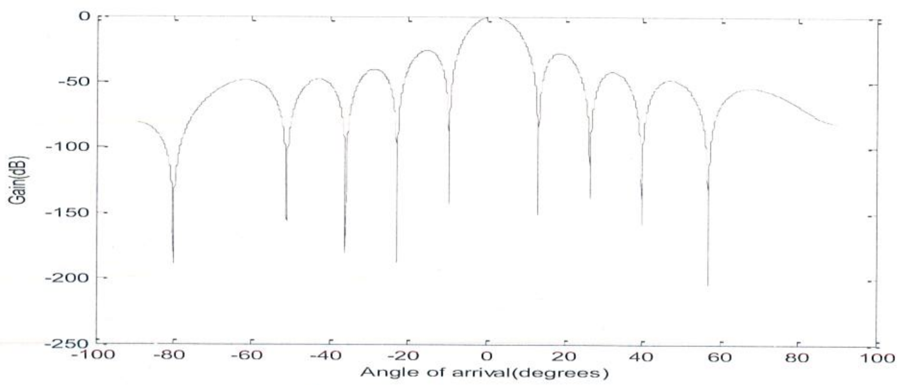

Figure4.1 – Original Pattern

(a) Pattern obtained – Null at single angle

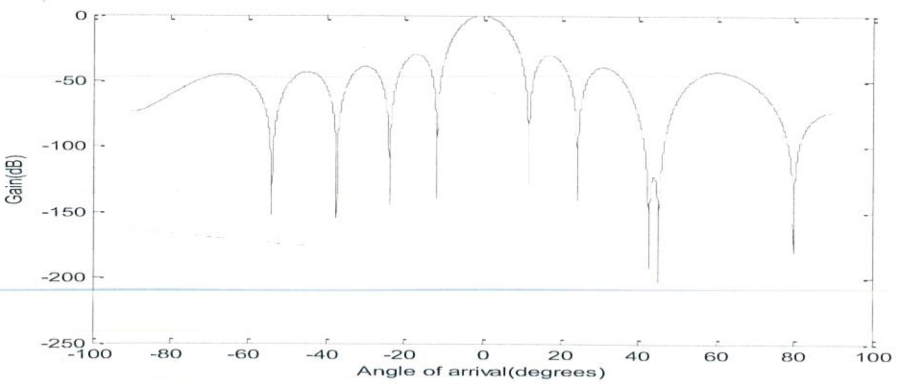

Figure 4.2 – Pattern Obtained: Null at 45 °

(b) Pattern obtained: Null at range of angles

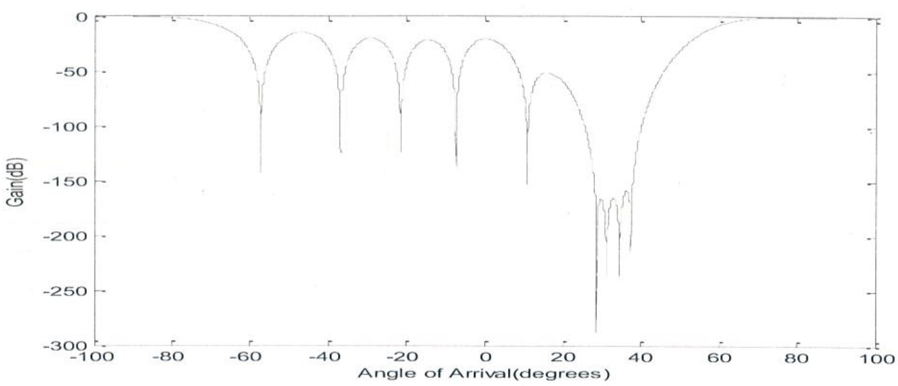

Figure 4.3- Pattern Obtained: Null from 28° - 38°

4.2 Desired Pattern Synthesis

The underlying approach is to assume that the given array elements are used as elements of an adaptive array. The main beam is steered in the desired direction by choosing the steering vector for that direction. To reduce side lobes a large number of interference signals are assumed to be incident on the array from side lobe region. On a computer, one solves for the resulting adapted weights. The adapted pattern is then computed and compared with the design objective. At any angle where side lobes are too high or too low, the interference power is increased or decreased accordingly and the weights are recalculated. This process is repeated iteratively until a suitable final pattern is obtained. The final adapted weights are then used as design weights for actual (non adaptive) array.

The problem we consider is the following:

We assume that number of array elements (n), the element patterns, and element spacing d are all given. We want to find a Weight vector W for which the pattern has beam maximum at some angle θ_d and meets a given sidelobe specification at other angles.

The method we use to change interference amplitudes for desired pattern synthesis is described below in the Algorithm.

$$W = \begin{bmatrix} n_1 & n_2 & n_3 & \ldots n_n \end{bmatrix}_{1xn}$$

$$U = \begin{matrix} \\ n_1 \\ n_2 \\ \cdot \\ \cdot \\ \cdot \end{matrix} \begin{bmatrix} 1^o & 2^o & 3^o \ldots\ldots 180^o \\ & & \\ & & \\ & & \\ & & \\ & & \end{bmatrix}_{nx180}$$

$$P = \begin{bmatrix} & & & & \end{bmatrix}_{1x180}$$
$$\begin{matrix} \uparrow & \uparrow & \uparrow & & \uparrow \\ 1^O & 2^O & 3^O & \ldots\ldots\ldots\ldots & 180^O \end{matrix}$$

4.2.1 Algorithm

1. U matrix is formed: U is a vector containing inter element phase shifts and element pattern.
2. Matrix A_i with 32 different combinations of random numbers is formed: A_i is the interference amplitude.
3. Matrix A_i is used to form a pattern P.
4. This pattern is compared with the ideal pattern element to get the Error Matrix.
5. For the next iteration, the combination corresponding to the 16 least valued errors are used. In addition, other 16 combinations are randomly generated. These 32 new combinations now form newA_i.
6. Process is repeated recursively, depending on number of iterations specified.
7. Finally, combination with least error is used to form a pattern P close to the ideal pattern.

4.2.2 Pattern Synthesis Results

The original pattern of the 10 element array without multiplying its elements with weight vector and with the excitation current of 1 amp is given by:

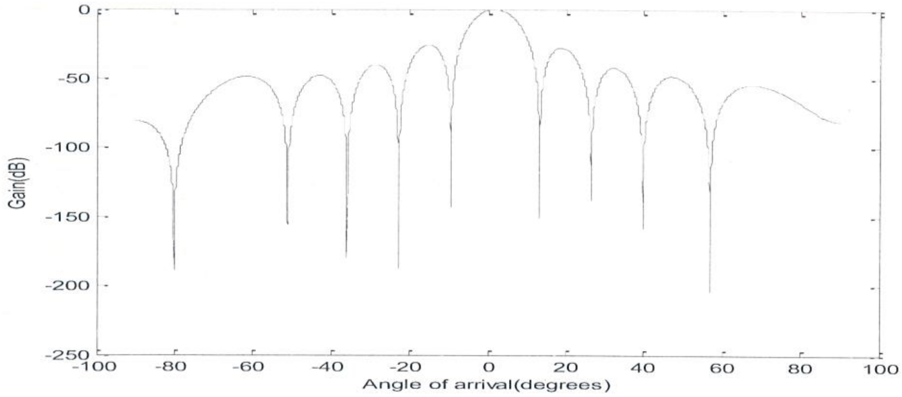

Figure 4.4 – Original Pattern

The pattern which we ideally desire is given as follows. As we can observe the main lobe should be at -50° .The algorithm we have followed, aims at reconstructing a pattern as close as possible to this pattern.

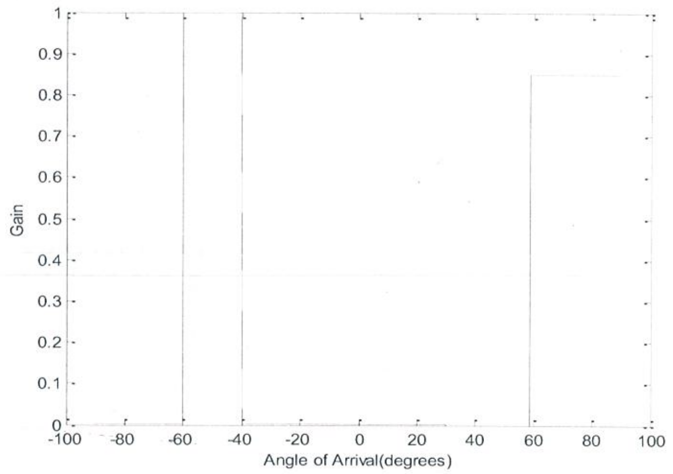

Figure 4.5 – Ideal Desired Pattern

Using the Algorithm (Section 4.2.1), the desired pattern and pattern obtained are compared, and the difference between them is reduced by following an iteration technique in which the Interference Amplitude matrix A_i is repeatedly updated to give a pattern relatively closer to the desired pattern. The number of iterations decide how close we get our pattern to the one desired. More iteration clearly indicates a much better matrix A_i , and therefore a much better pattern.

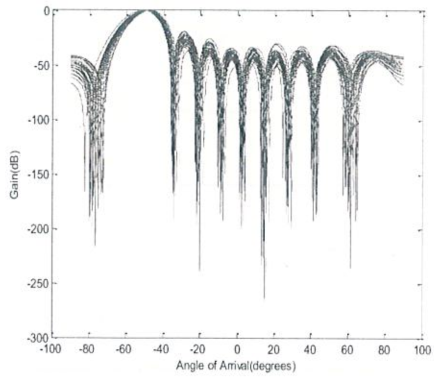

Figure 4.6 - 1st Iteration

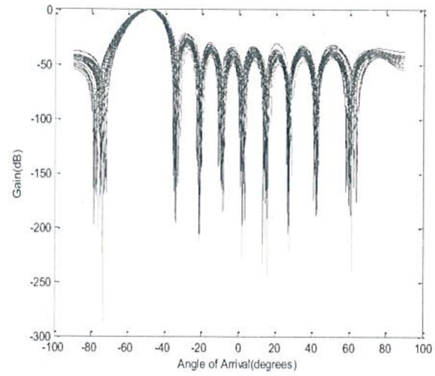

Figure 4.7 - 2nd Iteration

The Figures 4.6 shows 32 different patterns obtained each with a different Interference amplitude matrix A_i . Of these 16 patterns with least error (i.e. least deviation from desired pattern) are carried forward to the next iteration shown in figure 4.7; and 16 more random patterns are obtained by introducing 16 randomly generated values in matrix A_i.

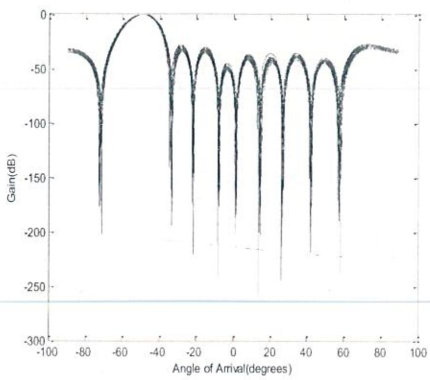

Figure 4.8- 10th Iteration

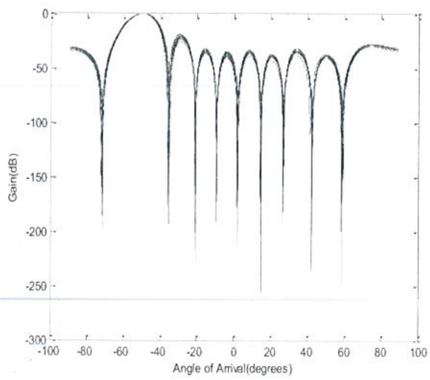

Figure 4.9 - 15th Iteration

Similarly for Figure 4.8 and 4.9, 4.10 number of iterations have increased. It can be seen that for the 10th, 15th and 25th iterations the 32 different patterns obtained have relatively lessor errors (deviations) and they overlap more. This results in a finer plot .Clearly as iterations are increased, the plot becomes finer and closer to the desired pattern.

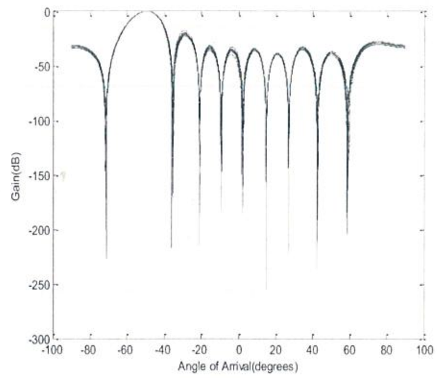

Figure 4.10 - 25th Iteration

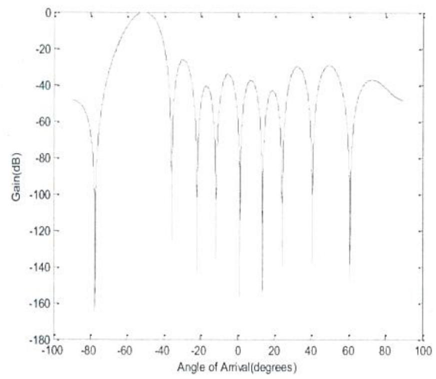

Figure 4.11 -Final Pattern Obtained

After the desired no of iterations, of the 32 patterns obtained, the one with least deviation is selected and plotted. The weight matrix and Interference amplitude matrix A_i for this final pattern will be used for the desired pattern synthesis problem.

Chapter 5: Direction of Arrival Estimation

5.1 The DOA Estimation problem:

Since the power radiated by an isotropic antenna is same in every direction, signals coming from different directions can't be distinguished.

A single non isotropic antenna has to be mechanically steered to get the angle at which the gain is maximum. The other method is to use a look up table where the one to one relation between the angle and the signal received at the array can be used to get the direction of arrival. But the results obtained here can be misleading because the signal received depends on both the angle and the signal strength.

A simple method to find DOA is to use antenna array of 2 elements with the look up table. Here the two array factors used vary according to cosθ and sinθ. When the two are divided, the signal strength cancels out and we get a one to one relation between tanθ and signal received. The limitations of this method are that the resolution or the accuracy depends on the no. of predefined values of the look up table and sharp change in value of close to $\pi/2$.

5.1.1 Why not Single Non Isotropic Antenna?

Angle (degrees)	s*g
0	10mW
20	8.7mW
40	7.5mW
60	6mW

Table 1

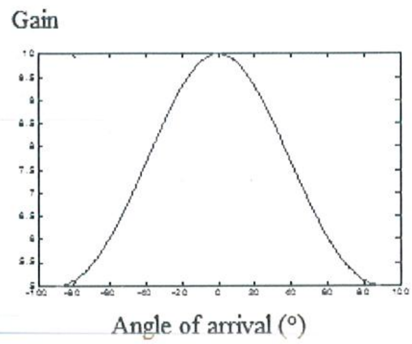

Figure 5. 1 – Gain Pattern of 2-element array

Let us assume a receiving antenna with radiation pattern as shown in figure .The antenna has a gain "g" and a signal of strength "s" is incident on it. Finally signal power P=s*g is received by the antenna.

- Signal strength = **s**
- Gain = **g**
- Signal power received **P= s*g**
- For **DOA estimation** ,Look up table used

The power received **P** is looked up in the Look Up table and corresponding value of angle is found. But the results can be misleading, because the power received depends on both signal strength "s" and gain "g" ; i.e. Same value of P can be expected for different value of gain and therefore different angles of arrival.

5.1.2 DOA Estimation using 2 element Array

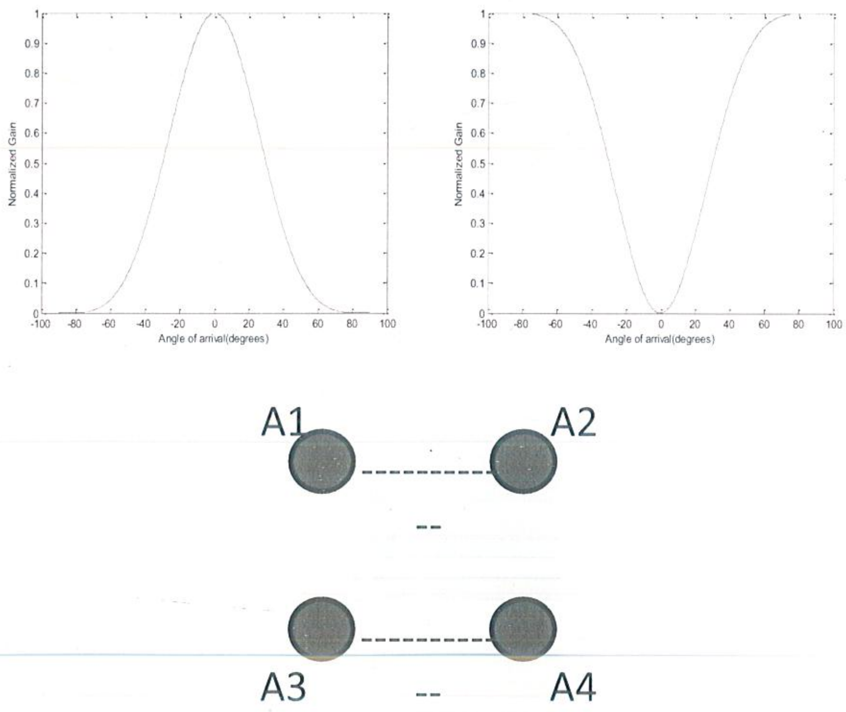

Figure 5.2 – 2-element linear arrays placed adjacent with above patterns.

$$s_1 g_1 = s_1 |\sin \theta|$$

$$s_1 g_2 = s_1 |\cos \theta|$$

$$c = \frac{s_1 |\sin \theta|}{s_1 |\cos \theta|}$$

$$c = |\tan \theta|$$

Angle(in degrees)	C
0	0
20	0.364
40	0.839
60	1.732
80	5.671
85	11.43

Table 2

2-element linear arrays are placed adjacent to each other, and their patterns can be represented by sine and cos curves repectively.In the previous section, the power term was both "s" and "g" dependant; thus the misleading result. Here this has been rectified. The new look up table parameter "c" depends only on the angle θ .

$$\theta = \tan^{-1} c$$

Limitations:

- Use of finite discrete values of angles in **Look-Up table**. Results are not very accurate.
- As angle tends to -π/2 and π/2, c=tanθ varies largely and tends to infinity.

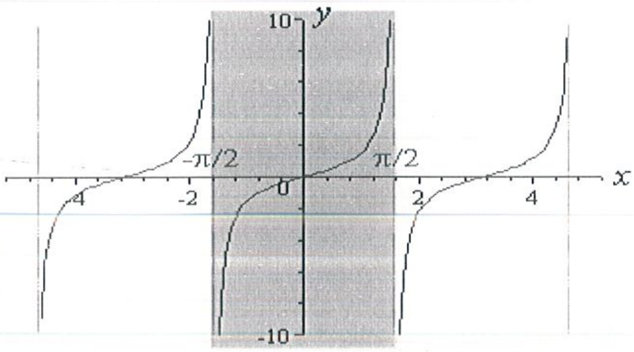

Figure 5. 3 –curve of y= $\tan^{-1} x$

5.2 DOA Estimation in 2-Dimension

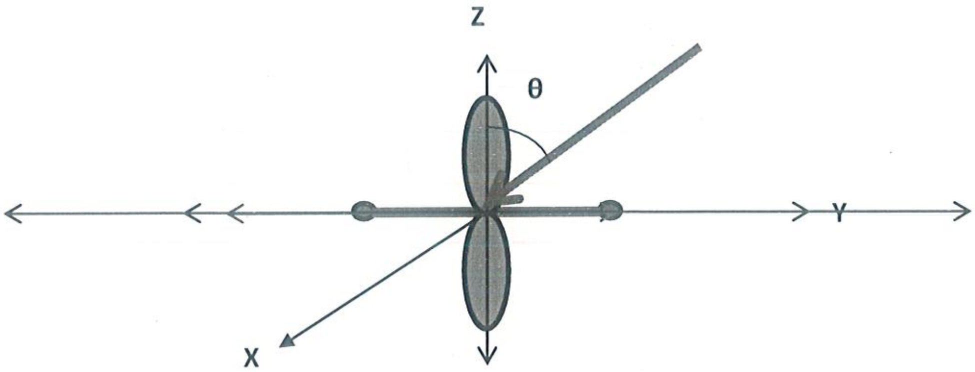

Figure 5.4 – 2-element linear array placed on y- axis, pattern on z-y plane.

Figure 5.4 shows the 2 dimensional radiation pattern (aligned along the z axis) of a 2 element array (placed on the y axis around the origin).

Assuming the signal to be arriving in Z-Y plane, here we change the phases of every element of the array so that the pattern associated with it rotates in Z-Y plane. This scans every θ in Z-Y plane and stores the values (Gain*Signal Strength) obtained. Among all the values stored the maximum value is the one corresponding to the direction of arrival.

5.2.1 Array factors for different positions (θ)

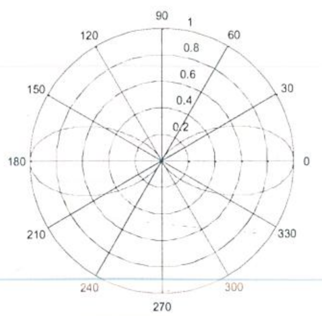

Figure 5. 5 – Array Factor for θ=0°

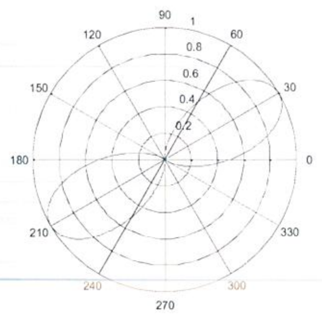

Figure 5. 6- Array Factor for θ=30°

The figures in the section represent polar plots of Array factor. The radius represents the amplitude of array factor and is plotted against angle θ (angle measured from the z axis).

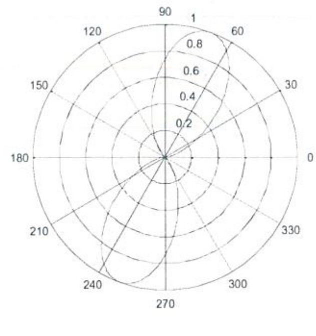

Figure 5. 7- Array Factor for θ=70°

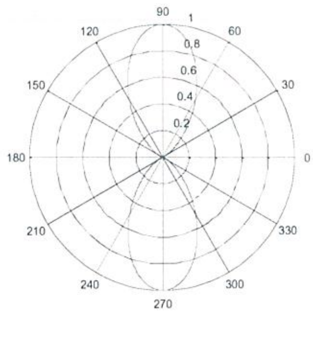

Figure 5. 8- Array Factor for θ=90°

In these figures ,we rotate the beam along θ such that angles of maxima of the beams occur at θ= 0°,30°,70°,90° .This way we scan for different values of θ and a signal coming from any direction on the ZY plane will be detected, It goes on without saying that by increasing the resolution of better estimation can be achieved.

5.3 DOA Estimation in 3-Dimension

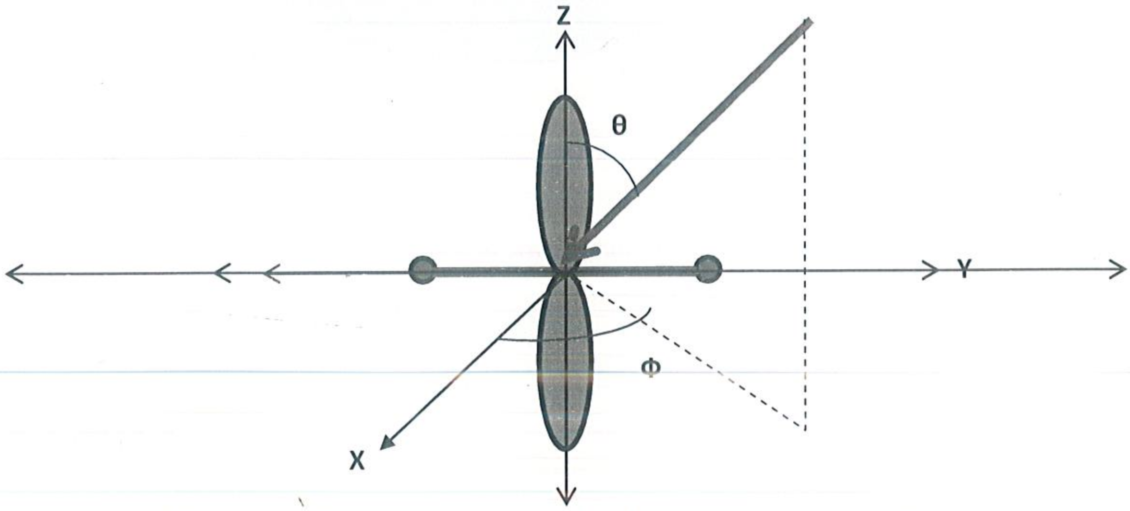

Figure 5. 9 - 2-element linear array placed on y- axis, pattern in 3-D space.

The above figure shows the layout of a 2-element array and its radiation pattern on the coordinate system.We will be referring to thise axis and angle representations ahead.

Here a 3 dimensional array factor (varying with both θ and φ) is obtained. The phases of the elements are canged so that the pattern rotates in 3-D space.Keeping the value of φ fixed all the θs are scanned.Then the same procedure is followed for next value of φ.For every orientation of the lobe, a maximum value of (Gain*Signal Strength) for that orientation is stored.Among all the stored values the maximum value corresponds to the direction of arrival.

5.3.1 3-Dimensional Array Factor

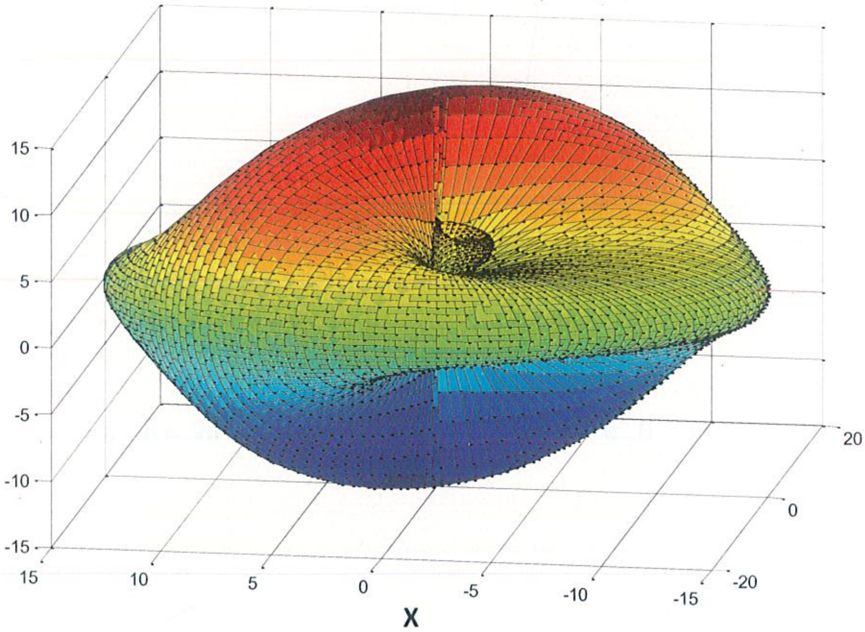

Figure 5. 10- 3-dimensional array factor for 2-element array

The figure 5.10 shows a 3-dimensional plot of array factor of a 2 element array placed on the y axis. It is plotted for the rectangular coordinate system. This is the actual radiation pattern obtained in 3-D space.

5.3.2 Array Factors for different positions (θ and φ)

Representing the rotation of plots to scan the 3d space is much more convenient if we take a slice off the 3-d plot. The resulting 2-d plot can then shown to be rotated for each value of φ and θ. In the figures below, we have fixed φ=0 and rotated the plot for different θ. This way all angles are scanned on the φ=0 plane. Similarly we increment φ and scan it for all θ, again. This is repeated till all possible φ and θ are covered in the 3d space. Again, it can be reemphasised that increasing the resolution of both φ and θ in this case, will improve DOA estimation.

φ = 0°,θ = 0°

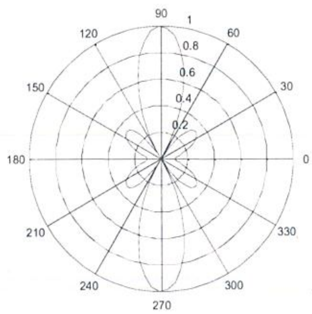

Figure 5. 11-Array Factor for φ = 0°,θ = 0°

φ=0°,θ = 45°

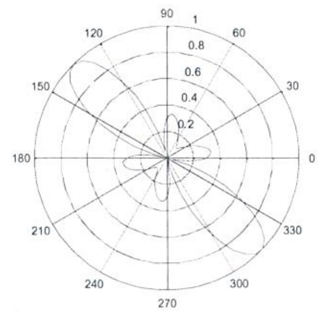

Figure 5. 12-Array Factor φ=0°,θ = 45°

φ = 0°, θ = 90°

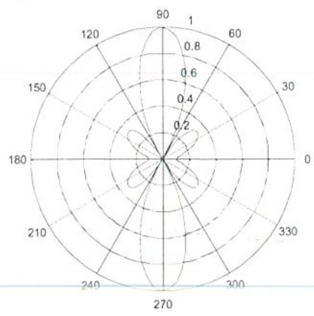

Figure 5. 13-Array Factor φ = 0°, θ = 90°

φ = 0°,θ = 135°

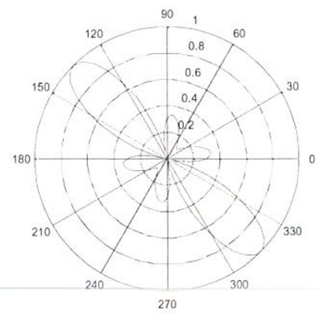

Figure 5. 14-Array Factor φ = 0°,θ = 135°

Chapter 6: Conclusion

The aim of the project was to arrive at a working algorithm to estimate the direction of arrival using antenna array. A comprehensive knowledge of array factors or radiation patterns of linear and planar arrays was required. This was achieved by plotting and comparing array factor plots for different values of intensity/excitation current and desired angles of maxima. Further, the desired pattern synthesis problem was dealt with. Pattern synthesis was achieved by multiplying every element with a phase coefficient. This was done by creating and multiplying a weight matrix. Signal interferences were introduced to create nulls in the pattern. An iteration technique was used to bring the pattern obtained as close to the desired pattern as possible. This was done by reducing the errors(i.e. the difference in the pattern obtained and desired pattern) by changing the value of interference signals. With every iteration, it was observed, we successfully managed to achieve a better pattern. Finally, an algorithm for DOA estimation was developed. This essentially involved, rotating the radiation pattern by introducing phase weights and scanning around the 3-D space. The resolution of angles with which we rotate the pattern will directly affect the efficiency of DOA estimation in our algorithm.

Equipped with a technique to synthesise a desired pattern and a working algorithm to scan the space around for signal detection and positioning, DOA estimation can be further improved by synthesising different patterns, and rotating these patterns around using the algorithm developed.

References

[1] Antenna Theory: Analysis and Design by Constantine A. Balanis.

[2] Antenna Theory and Design by R.S. Elliott

[3] "A Numerical Pattern Synthesis Algorithm for Arrays"

CARL A OLEN, Member, IEEE, AND R.T. COMPTON, JR., Fellow, IEEE

[4] Direction of Arrival estimation Introduction

www.comm.utoronto.ca/~rsadve/Notes/**DOA**.pdf

[5] Fundamentals of Engineering Electromagnetics by Sunil V Bhooshan.

Appendix A: Source code

Circular array radiation pattern

```
clc;
clear all;
close all;
n=20;
x=0:0.1:pi;
a=0:0.1:pi*2;
t=0:2*pi/n:2*pi;
b=1;
r=0.5;
Ix=50;
AF=zeros(length(x),length(a));
for k=1:length(x)
    for l=1:length(a)
        for m=1:length(t)

AF(k,l)=AF(k,l)+(Ix*(exp(i*b*r*sin(t(m))*cos(a(l))*sin(x(k)
)+(i*b*r*cos(t(m))* sin (a(l))*sin(x(k))))));
        end
    end

end
figure;
meshgrid(x*180/pi,a*180/pi);
surf(x*180/pi,a*180/pi,fftshift(abs(AF')));
```

Rectangular array radiation pattern

```
%array factor for a rectangular array varying with both phi
and theta%
clc;
clear all;
close all;
x=0:0.125:pi;                    %theta : angle with z axis
assuming spherical coordinates%
a=0:0.125:pi*2;                 %phi :   angle with x axis
dx=4;                                 %distance between array
elements along 1 axis
dy=4;                                 %distance between array
elements along 2nd axis
b=1;                              %2pi/lemda :constant value
depends on wavelength of radiation
Imn=5;                          %intensity of reference array
element

%in following program we implement Array factor plot for a
planar (square)
%array of size 169 element array(13*13)

%AFx is Array factor of row elemnts(along x axis) passing
through the
%refernece element
%AFy is Array factor of column elemnts(along y axis)
passing through the
%refernece element
%Effective Array factor of the recatngular array=Afx*AFy

AFx=zeros(length(x),length(a));
```

```
AFy=AFx;
for k=1:length(x)
    for l=1:length(a)
        for m=-6:6          %2m+1 reflects array size

AFx(k,l)=AFx(k,l)+(Imn*exp(i*b*m*dx*cos(a(l))*sin(x(k))));
        end
    end

end
%plot(abs(AFx'))
%meshgrid(x*180/pi,a*180/pi);
%surf(x*180/pi,a*180/pi,abs((AFx')));

for k=1:length(x)
    for l=1:length(a)
        for m=-6:6

AFy(k,l)=AFy(k,l)+(Imn*exp(i*b*m*dy*sin(a(l))*sin(x(k))));
        end
    end
end

%figure(2)
%plot(abs((AFy')))
%meshgrid(x*180/pi,a*180/pi);
%surf(x*180/pi,a*180/pi,abs(fftshift(AFy')));


AF=(AFx.*AFy);

figure(3)
```

```
plot(x*180/pi, abs(AF'));
xlabel('Angle theta measured from z axis');
ylabel('Gain');

figure();
plot(a*180/pi,abs(AF));
xlabel('Angle phi measured from x axis');
ylabel('Gain');


figure();
meshgrid(x*180/pi,a*180/pi);
surf(x*180/pi,a*180/pi,abs(fftshift(AF')));
xlabel('Angle theta measured from z axis');
ylabel('Angle phi measured from x axis');
zlabel('Gain');


figure();
meshgrid(x*180/pi,a*180/pi);
surf(x*180/pi,a*180/pi,abs((AF')));
xlabel('Angle theta measured from z axis');
ylabel('Angle phi measured from x axis');
zlabel('Gain');
```

Array factor comparison

```
clc;
clear all;
close all;
wl=1;                  %wavelength of signal
B=(2*pi)/wl;              %constant depending on wavelength of
radiation(beta)
d=wl/2;                %distance between 2 array elements
n=4;                   %no. of array elements

%In the following program we implement Array Factor plots
for 2 different Linear arrays for different values of
Intensity,Angles of desired Maximas
%We implement Effective Array factor plots for two
different linear arrays
%for differnet values of Intensities(I1 &I2) and Angles of
desired
%maxima(x1 and y1)
%differnet AF plots are compared and the effects on
Intensities and Angle of
%Maxima in the resulting Field pattern are analysed.


%Array Factor plot for 2 different Arrays,2 different
intensities,2
%different direction of maxima(x1 and y1)
x=-pi/2:0.01:pi/2;
I1=5;
x1=pi/18;
AF1=I1;
y1=pi/6;
```

```
I2=2;
AF2=I2;
%AF1=zeros(1,length(x));
for k=1:(n-1)
    AF1=AF1+(I1* exp(i*B*k*d*(sin(x)-sin(x1))));
end

plot(x*180/pi,(AF1/abs(max(AF1))));
hold on
for l=1:n-1
    AF2=AF2+(I2* exp(i*B*l*d*(sin(x)-sin(y1))));
end
grid on;
plot(x*180/pi,(AF2/abs(max(AF1))),'green');
ylabel('normalized gain');
xlabel('angle of incoming signal');


%Effective Array Factor plot for 2 different arrays when
I1=I2; x1=y1
I1=5;
I2=5;
x1=pi/4;
x2=pi/4;
AF1=I1;
for k=1:n-1
    AF1=AF1+(I1* exp(i*B*k*d*(sin(x)-sin(x1))));
end
AF2=I2;
for l=1:n-1
    AF2=AF2+(I2* exp(i*B*l*d*(sin(x)-sin(x2))));
end
```

```
figure();
 plot(x*180/pi,((AF1+AF2)/abs(max(AF1+AF2))),'black')
 ylabel('normalized gain');
xlabel('angle of incoming signal');
grid on;



%Effective Array Factor plot for 2 different arrays when
I1=I2 ; x1 NOT EQUAL to x2%
I1=5;
I2=5;
x1=pi/18;
x2=pi/6;
AF1=I1;
for k=1:n-1
    AF1=AF1+(I1* exp(i*B*k*d*(sin(x)-sin(x1))));
end
AF2=I2;
for l=1:n-1
    AF2=AF2+(I2* exp(i*B*l*d*(sin(x)-sin(x2))));
end
figure();
 plot(x*180/pi,(AF1+AF2)/abs(max(AF1+AF2)),'red');
 ylabel('normalized gain');
xlabel('angle of incoming signal');
grid on;


%Effective Array Factor plot for 2 different arrays when I1
NOT EQUAL I2; x1 NOT EQUAL x2%
I1=5;
```

```
I2=2;
x1=pi/18;
x2=pi/6;
AF1=I1;
for k=1:n-1
    AF1=AF1+(I1* exp(i*B*k*d*(sin(x)-sin(x1))));
end
AF2=I2;
for l=1:n-1
    AF2=AF2+(I2* exp(i*B*l*d*(sin(x)-sin(x2))));
end
figure();
 plot(x*180/pi,(AF1+AF2)/abs(max(AF1+AF2)),'green')
ylabel('normalized gain');
xlabel('angle of incoming signal');
grid on;


%Effective Array Factor plot for 2 different arrays when
I1>>I2
I1=40;
I2=1;
x1=pi/18;
x2=pi/6;
AF1=I1;
for k=1:n-1
    AF1=AF1+(I1* exp(i*B*k*d*(sin(x)-sin(x1))));
end
AF2=I2;
for l=1:n-1
    AF2=AF2+(I2* exp(i*B*l*d*(sin(x)-sin(x2))));
end
```

```
figure();
 plot(x*180/pi,(AF1+AF2)/abs(max(AF1+AF2)),'blue')
 ylabel('normalized gain');
xlabel('angle of incoming signal');
grid on;



%Effective Array Factor plot for 2 different arrays when
x2>>x1
I1=5;
I2=3;
x1=pi/18;
x2=pi/2;
AF1=I1;
for k=1:n-1
    AF1=AF1+(I1* exp(i*B*k*d*(sin(x)-sin(x1))));
end
AF2=I2;
for l=1:n-1
    AF2=AF2+(I2* exp(i*B*l*d*(sin(x)-sin(y1))));
end
figure();
plot(x*180/pi,(AF1+AF2)/abs(max(AF1+AF2)),'magenta')
ylabel('normalized gain');
xlabel('angle of incoming signal');
grid on;
```

Introducing a null at one particular angle:

```
clc;
clear all;
close all;
wl=1;
B=(2*pi)/wl;
d=wl/2;
y=-pi/2:0.001:pi/2;
n=10;
sigma=0.01;
Ai=3.16;
mu=1;
U=zeros(length(y),n);
for k=1:length(y);
    for j=1:n
        U(k,j)=exp(-i*2*pi*sin(y(k))*(j-1)*d);
    end
end
U=U.';
for j=1:n
    Ui(j)=exp(-i*2*pi*sin(pi/4)*(j-1)*d);%interference
angle%
end
Ui=Ui.';
for j=1:n
    Ud(j)=exp(-i*2*pi*sin(0)*(j-1)*d);
end
Ud=Ud.';
I=eye(n,n);
x=(sigma)^2*I+(Ai^2)*(conj(Ui)*(Ui).');
```

```
W=(mu)*inv(x)*conj(Ud);
P=W.'*U;
P=P/max(P);
P=20*log(P);
plot(y*180/pi,(P));
```

Introducing a null at a range of angles:

```
%pattern synthesis for any array with multiple angles of
interference;the
%desired direction and null angles are known%
clc;
clear all;
close all;
wl=1;
B=(2*pi)/wl;
d=wl/2;
y=-pi/2:0.001:pi/2;
n=10;
sigma=0.01;
Ai=3.16;
mu=1;
U=zeros(length(y),n);
for k=1:length(y);
    for j=1:n
        U(k,j)=exp(-i*2*pi*sin(y(k))*(j-1)*d);
    end
end
U=U.';
```

```
l=28*(pi/180):pi/720:38*(pi/180);%angle range%
Ui=zeros(n,length(l));

for j=1:n
    for k=1:length(l)
        Ui(k,j)=exp(-i*2*pi*sin(l(k))*(j-1)*d);
    end
end
Ui=Ui';

s=zeros(n,n);
for k=1:length(l)

s=s+(Ai^(2))/((sigma)^(2))*(conj(Ui(1:n,k)))*(Ui(1:n,k).');
end

I=eye(n,n);
x=(sigma)^2*(I+s);


for j=1:n
    Ud(j)=exp(-i*2*pi*sin(0)*(j-1)*d);
end
Ud=Ud.';


W=(mu)*inv(x)*conj(Ud);
P=W.'*U;
P=P/max(P);
P=20*log(P);
plot(y*180/pi,(P));
```

Desired pattern synthesis

```
clc;
clear all;
close all;
wl=1;
B=(2*pi)/wl;
d=wl/2;
y=-pi/2:0.001:pi/2;
n=10;
sigma=0.01;
mu=1;

drp=zeros(1,length(y));%desired radiation pattern
drp(1,1:523)=0.0001;  %-80dB
drp(1,524:872)=1;   %0dB
drp(1,873:2095)=0.0001;  %-80d
drp(1,2600:3142)=0.7; %-4dB
figure(100);
plot(y*180/pi,drp);

U=zeros(length(y),n);
for k=1:length(y);
    for j=1:n
        U(k,j)=exp(-i*2*pi*sin(y(k))*(j-1)*d);
    end
end
U=U.';


for j=1:n
```

```
    Ud(j)=exp(-i*2*pi*sin(-50*(pi/180))*(j-1)*d);
end
Ud=Ud.';


 l=-90*(pi/180):pi/180:90*(pi/180);
m=32;
Ai=1+(200-1).*rand(m,length(l));


 for w=1:8


   l=-90*(pi/180):pi/180:90*(pi/180);
Ui=zeros(n,length(l));

for j=1:n
    for k=1:length(l)
         Ui(k,j)=exp(-i*2*pi*sin(l(k))*(j-1)*d);
    end
end
Ui=Ui.';
s=zeros(n,n);
for j=1:m
   for k=1:length(l)

s=s+(Ai(j,k)^(2))/((sigma)^(2))*(conj(Ui(1:n,k)))*(Ui(1:n,k
).');
   end
       I=eye(n,n);
       x=((sigma)^2)*(I+s);
```

```
W=(mu)*inv(x)*conj(Ud);
P=W.'*U;
P=abs(P)/max(abs(P));
PdB=20*log(P);
figure(w);
plot(y*180/pi,(PdB));

hold on;
s=0;
for c=1:length(P)
        s=s+(P(c)-drp(c))^(2);
end
erp(j)=sqrt((1/length(P))*(s));
end
E=sort(erp);


Ainew=zeros(m,length(l));
for j=1:m/2
    for k=1:m
        if E(j)==erp(k)
            Ainew(j,1:length(l))=Ai(k,1:length(l));
            break
        else
        continue
        end
    end
end
```

```
c=[0.9 0.75 0.6 0.55 0.5 0.45 0.4 0.35 0.3 0.25 0.2 0.15
0.1 0.8 0.85 0.95];
```

```
a=1;
for k=(m/2)+1:m
    Ainew(k,1:length(l))=Ainew(1,1:length(l))+
c(a)*(Ainew(a,1:length(l))-(1+(200-1).*rand(1,length(l))));
    a=a+1;
end

Ai=Ainew;

end

Ai=Ainew(1,1:length(l));

U=zeros(length(y),n);
for k=1:length(y);
    for j=1:n
        U(k,j)=exp(-i*2*pi*sin(y(k))*(j-1)*d);
    end
end
U=U.';


l=-90*(pi/180):pi/180:90*(pi/180);
Ui=zeros(n,length(l));

for j=1:n
    for k=1:length(l)
         Ui(k,j)=exp(-i*2*pi*sin(l(k))*(j-1)*d);
    end
end
Ui=Ui';
```

```
s=zeros(n,n);
for k=1:length(l)

s=s+((Ai(k))^(2))/((sigma)^(2))*(conj(Ui(1:n,k)))*(Ui(1:n,k
).');
end

I=eye(n,n);
x=(sigma)^2*(I+s);

W=(mu)*inv(x)*conj(Ud);
P=W.'*U;
P=abs(P)/max(abs(P));
figure(200)
PdB=20*log(P);
plot(y*180/pi,(PdB));
```

DOA Estimation using 2-element Antenna array(θ, Φ varying)

```
% linear array pattern varying with theta and phi
clc;
clear all;
close all;
x=linspace(0,2*pi);                    %theta : angle with z
axis assuming spherical coordinates%
a=linspace(0,2*pi);                    %phi  :  angle with x
axis
dx=4;                                  %distance between array
elements
b=1;                                   %2pi/lemda :constant value
depends on wavelength of radiation
Imn=5;                                 %intensity of reference array
element

phi1=0;
theta1=0;

AFx=zeros(length(x),length(a));
sig=zeros(length(x),length(a));
sig(13,13)=1;
mem=zeros(6,9);

v=1;
for c=1:6

    theta1=0;
 for t=1:9
```

```
AFx=zeros(length(x),length(a));

for k=1:length(x)
    for l=1:length(a)
        for m=0:2             %  array size
            AFx(k,l)=AFx(k,l)+(Imn*exp(i*b*m*dx*cos(a(l)-
phi1)*sin(x(k)-theta1)));
        end
    end

end

AF2=AFx.*sig;
mem(c,t)=max(max(abs(AF2)));



figure();
polar(x,abs(AFx(1:100,v)')/max(AFx(1:100,v)'));
theta1=theta1+pi/4;
 end
 v=v+13;
 phi1=phi1+pi/4;
end

[c,i]=max(abs(mem));
[c1,i1]=max(c);
row=i(i1);
col=i1;
thetafinal=(col*(pi/4)-(pi/4))*(180/pi);
phifinal=(row*(pi/4)-(pi/4))*(180/pi);

% figure();
```

```
% meshgrid(x,a);
% surf(x*180/pi,a*180/pi,abs(AFx));

  figure();
  [x,a]=meshgrid(x,a);
  [X,Y,Z]=sph2cart(x,a,abs(AFx));
  surf(X,Y,Z);
```

DOA Estimation using 2-element Antenna array(θ varying)

```
clc;

clear all;

close all;

wl=1;
B=(2*pi)/wl;
d=wl/2;
n=2;
x=0:0.01:2*pi;
I1=5;
x1=0;
AF1=I1;

sig=zeros(1,length(x));
sig(108)=1;
mem=zeros(1,37);

for l=1:37
    AF1=5;
for k=1:(n-1)
```

```
    AF1=AF1+(I1* exp(i*B*k*d*(sin(x-x1))));
end
AF2=AF1.*sig;
mem(l)=max(AF2);
figure();

%  plot(x*180/pi,(AF1/abs(max(AF1))));
 polar(x,(AF1/abs(max(AF1))))
x1=x1+(pi/180)*10;
end

%  for s=1:37
%       if(mem(s)==max(mem))
%            angle=s*10-10;
%       else
%            continue
%       end
%  end

[c,i]=max(mem);
angle=(i*10)-10;
```