JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATION- 2024

Ph.D.-II Semester (MATHEMATICS)

COURSE CODE (CREDITS): 17P1WMA111 (3) MAX. MARKS: 35

COURSE NAME: DIFFERENTIAL GEOMETRY

COURSE INSTRUCTOR: P K Pandey MAX. TIME: 2 Hours

***Note: (a)** All questions are compulsory.*

(b)Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

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1. For the sphere S given by $x^2 + y^2 + z^2 = a^2$ find: [7M] [CO5]
   (i) Unit normal vector field.
   (ii) Shape operator.
   (iii) Principal curvatures.
   (iv) Gaussian curvature.
2. Define a topological surface, and give an example of it. [4M] [CO4]
3. Consider the mapping $\boldsymbol{X}: \mathbb{R}^2 \to \mathbb{R}^3$ given by: [7M] [CO4]
   $$\boldsymbol{X}(u,v) = (u+v,\ u-v,\ uv)$$
   Show that $\boldsymbol{X}$ is a proper patch and that the image of $\boldsymbol{X}$ is the entire surface $M: z = \frac{(x^2-y^2)}{4}$.
4. Define the normal curvature, and compute the normal curvature for the cylinder:
   $$\boldsymbol{X}(u,v) = (\cos u,\quad \sin u,\ v)$$
   at the point $\boldsymbol{X}(0,0) = (1,0,0)$. [5M] [CO5]
5. Prove or disprove that for any function f, the vector fields: [4M] [CO2]
   $E_1 = (\sin f\, U_1 + U_2 - \cos f\, U_3)/\sqrt{2}$, $E_2 = (\sin f\ U_1 - U_2 - \cos f\ U_3)/\sqrt{2}$, and $E_3 = (\cos f\ U_1 + \sin f\ U_3)$;
   constitute a frame field.
6. Compute the *Frenet apparatus* $\kappa,\ \tau,\ T,\ N,\ B$ of the unit-speed curve: [5M] [CO3]
   $$\beta(s) = \left(\tfrac{4}{5}\cos s, 1 - \sin s, -\tfrac{3}{5}\cos s\right).$$
7. Given that $\boldsymbol{v} = (1,\ 2, -3)$ and $p = (0, -2,\ 1)$ compute the differential of $f = xe^{yz}$ and find the directional derivative $\boldsymbol{v}_p[f]$. [3M] [CO1]