JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT
TEST-3 EXAMINATION- 2024
B. Tech-IV Semester (CSE/IT)

COURSE CODE (CREDITS): 18B11CI414 (3) MAX. MARKS: 35
COURSE NAME: Discrete Computational Mathematics
COURSE INSTRUCTORS: RKB, NKT, AVA* MAX. TIME: 2 Hours

Note: *(a) All questions are compulsory.*
(b)Marks are indicated against each question in square brackets.
(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems.

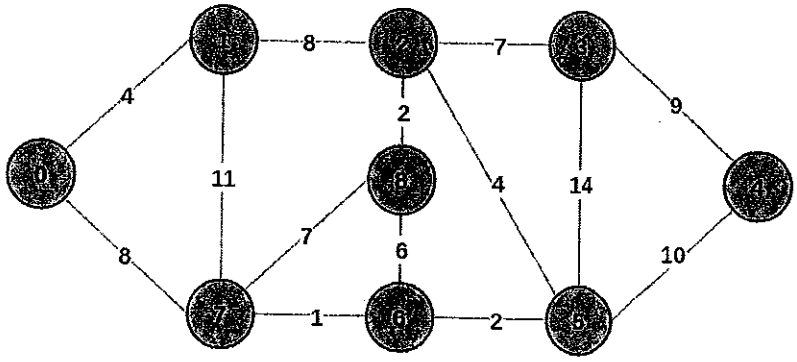
1. [CO-1] Test the validity of the following argument using the concept of truth table:
If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore, there was no ball game. **(3)**

2. [CO-2] Using principle of mathematical induction, prove that $(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)\ \forall n,\ n \in N$. **(3)**

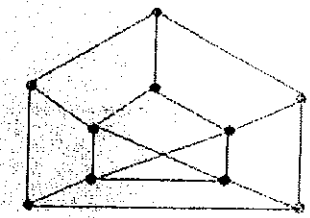
3. [CO-3] Let $A = \{3, 4, 5, 6, 7, 8\}$ and define a binary relation R on A as follows:
For all $x, y \in A$, $x\,R\,y \Leftrightarrow 2 \mid (x - y)$. Draw the directed graph of R. **(2)**

4. [CO-3] Draw Hasse diagram for the poset $(P(S), \subseteq)$, where $P(S)$ is power set of S & $S = \{a, b, c\}$. Is it a complemented lattice? Justify your answer. **(2)**

5. [CO-4] Using Prim's algorithm, find the minimum spanning tree for the following weighted labeled graph: **(4)**



6. [CO-4] With the help of Kuratowski's theorem, show that the following graph (label the graph as per your convenience) is a non-planar graph: **(3)**



7. [CO-4] Eight chemicals are to be shipped across country by air express. The cost of doing this depends on the number of containers shipped. The cost of shipping one container is \$125. For each additional container the cost increases by \$85. Some chemicals interact with one another and it is too risky to ship them in the same container. The chemicals are labeled by $c_1, c_2, c_3, \ldots, c_8$ and chemicals that interact with a given chemical are given below: **(3)**

$c_1: c_2, c_5, c_6$ $c_2: c_1, c_3, c_5, c_7$ $c_3: c_2, c_4, c_7$ $c_4: c_3, c_6, c_7, c_8$
$c_5: c_1, c_2, c_6, c_7, c_8$ $c_6: c_1, c_4, c_5, c_8$ $c_7: c_2, c_3, c_4, c_5, c_8$ $c_8: c_4, c_5, c_6, c_7$

What is the minimum cost of shipping the chemicals and how should the chemicals be packed into containers?

8. [CO-4] A certain tree T with 21 vertices has only vertices of degree 1, 3, 5 and 6. If T has exactly 15 end-vertices and 1 vertex of degree 6, how many vertices of T have degree 5? **(2)**

9. [CO-5]

(a) Show that the set $G = \{A, B, C, D\}$; where $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $D = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$; forms a multiplicative abelian group. **(3)**

(b) Find generators of the cyclic group $(Z_4, \oplus_4)$. **(2)**

10. [CO-5] Suppose $R = \{a, b, c, d, e\}$ is a set with operation of addition and multiplication defined as follows: **(4)**

+	**a**	**b**	**c**	**d**	**e**
a	a	b	c	d	e
b	b	c	d	e	a
c	c	d	e	a	b
d	d	e	a	b	c
e	e	a	b	c	d

×	**a**	**b**	**c**	**d**	**e**
a	a	a	a	a	a
b	a	b	c	d	e
c	a	c	e	b	d
d	a	d	b	e	c
e	a	e	d	c	b

Show that $(R, +, \times)$ is a commutative ring with unity.

11. [CO-6]

(a) Define a phase structure grammar. **(1)**

(b) Discuss the language generated by the grammar $G = (T, N, S, P)$; where **(3)**

$T = \{a, b, c\}$, $N = \{S, B, C\}$, S is the starting symbol; and

$P = [S \longrightarrow aSBc, S \longrightarrow aBc, cb \longrightarrow Bc, aB \longrightarrow ab, bB \longrightarrow bb, bc \longrightarrow bc, cc \longrightarrow cc]$.

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