Jaypee University of Information Technology, Waknaghat

Test-3 Examination - December 2023

B.Tech - III Semester (BI/BT)

Course Code/Credits: 18B11MA312/4
Course Title: Probability and Statistical Techniques
Course Instructor: RAD

Max. Marks: 35

Max. Time: 2 Hours

Note: Answer all the questions. Describe random variables along with range where applicable.
Use of scientific calculators is allowed.

---

1. Consider the marks distribution of 58 students tabulated: (6 Marks) [CO-1]

Class	383-387	388-392	393-397	398-402	403-407
Frequency	8	10	15	17	8

   (a) Determine the *variance* via deviation method by taking $\mathcal{A} = 395$.

   (b) What is the *standard deviation*?

2. A biotech company uses 3 high-throughput genotyping machines A, B and C to process a certain number of arrays. Suppose that (3 Marks) [CO-2]

   Machine A processes 50% of the arrays with a genotyping error rate of 3%

   Machine B processes 30% of the arrays with a genotyping error rate of 4%

   Machine C processes 20% of the arrays with a genotyping error rate of 5%

   An array processed by the company is selected randomly.

   (a) Compute the probability that the randomly selected array is erroneous.

   (b) Given that the array is erroneous, find the probability that it was processed by A.

3. For 5 pairs of observations, the following results are obtained: $\sum X = 15$, $\sum Y = 25$, $\sum X^2 = 55$, $\sum Y^2 = 135$, $\sum XY = 83$. (4 Marks) [CO-3]

   (a) Find the equation of regression line of Y on X.

   (b) Estimate the value of Y when $X = 8$.

4. In most colleges, it is desirable for the variances of exam grades to be nearly the same among instructors. Two college instructors are interested in whether there is any variation in the way they grade math exams. They each grade the same set of $n_1 = n_2 = 30$ exams. The first instructor's grades have a variance of 52.3. The second instructor's grades have a variance of 89.9. Test the claim that the first instructor's variance is smaller at $\alpha = 5\%$ *level of significance*. (5 Marks) [CO-3]

   (a) State null and alternative hypotheses.

   (b) Calculate the F-test statistic from the samples given.

   (c) Taking the critical value as $F_\alpha = 1.85$, what conclusion do you arrive?

5. Suppose that a home-television manufacturer advertises that its new TV has a mean size of 50 inches. You take a random sample of $n = 30$ TVs and test their size. You find that the average is $\bar{x} = 47$ inches and std. deviation is $s = 5.5$ inches. (5 Marks) [CO-3]

   (a) State null and alternative hypotheses.

   (b) Find the critical value z_α at $\alpha = 5\%$ and calculate the test-statistic value.

   (c) Is there enough evidence to support the advertised claim?

6. We know that smoking has all sorts of ill effects on people; among other things, there is evidence that it affects fertility. Weinberg and Gladen (1986) examined the effects of smoking and the ease with which women become pregnant. They asked 586 women who had planned pregnancies, and asked them how many menstrual cycles it had taken for them to become pregnant after discontinuing contraception. They also sorted the women into whether they were smokers or non-smokers. Use $\chi^2_{0.05,2} = 5.99$. (6 Marks) [CO-3]

	1 cycle	2 cycle	3 cycle	Total
Smokers	29 (a)	16 (22.70)	55 (40.27)	100
Non-smokers	198 (188.26)	107 (110.30)	181 (195.73)	486
Total	227	123	236	586

   (a) State H_0 and H_1. Calculate a. With critical value, sketch the rejection region.

   (b) Use χ^2-test of independence to test the hypothesis with a significance level of 5%.

7. A fisheries researcher wishes to test for a difference in mean weights of a single species of fish caught by fishermen in three different lakes in Nova Scotia. The significance level for the test will be 0.05. Consider the following partial ANOVA table: (6 Marks) [CO-5]

Source	df	SS	MS	F-statistic	F_α
Between	?	17.04	?	?	4.2565
Within	9	?	?		
Total	?	31.23			

   (a) State null and alternative hypotheses.

   (b) Write down the formulae for SS_W, MS_B, MS_W and complete the ANOVA table.

(Standard) Normal Probability Table:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451