JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATION- 2023

B.Tech-III Semester (ECE)

COURSE CODE (CREDITS): 18B11MA314 (4) MAX. MARKS: 35

COURSE NAME: Probability Theory and Random Processes

COURSE INSTRUCTORS: SST MAX. TIME: 2 Hours

Note: ***(a)*** *All questions are compulsory.*

(b) *Marks are indicated against each question in square brackets.*

(c) *The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.*

(d) *Use of scientific calculator is allowed.*

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1. Find the power spectral density of the random telegraph signal $Y(t) = \{z.X(t), t \geq 0\}$ with auto-correlation function $R_{YY}(t, t+\tau) = e^{-4\lambda|\tau|}$. **[5M](CO-5)**

2. Prove that the random process $X(t) = A\cos(\omega t + \theta)$, where A and ω are constants and θ is uniformly distributed random variable in $(0, 2\pi)$, is a mean-ergodic process. **[5M](CO-5)**

3. A random process $\{X(t)\}$ is characterized by four sample functions described as below:

$$X(t) = \begin{cases} -1, s_1 \\ -2, s_2 \\ -3, s_3 \\ t, s_4 \end{cases}$$

   a) Find the marginal distributions of $X(0)$ and $X(2)$.

   b) Construct the joint probability mass function of $X(0)$ and $X(2)$.

   [2M+2M+1M](CO-5)

4. An electrical firm manufactures LED bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800$ hours against the alternative, $\mu \neq 800$ hours, if a random sample of 30 LED bulbs has an average life of 788 hours, at 0.05 level of significance. **[5M](CO-4)**

5. Fuses produced by a company will blow in 12.40 minutes on the average when overloaded. Suppose the mean blow time of 20 fuses subjected to overload is 10.63 minutes and standard deviation 2.48 meters. Does this information tend to support or refute the claim that the population mean blow time is 12.40 meters , at 0.05 level of significance? **[5M](CO-4)**

6. Assume that the length of phone calls made at a particular telephone booth is exponentially distributed with a mean of 3 minutes. If you arrive at the telephone booth just as Chris was about to make a call, find the following:
   a) The probability that you will wait more than 5 minutes before Chris is done with the call.
   b) The probability that Chris' call will last between 2 minutes and 6 minutes.

[2M+3M](CO-3)

7. Find the mathematical expectation and variance from the moment generating function of the random variable X, given by $M_X(t) = \frac{1}{1-2t}$. Also use it to find moment generating functions $M_{X-1}(t)$ and $M_{3X}(t)$. **[3M+1M+1M](CO-2)**

Statistical Tables:

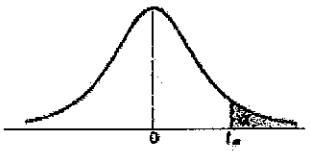

Table A.4 Critical Values of the t-Distribution

	α						
v	**0.40**	**0.30**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086