

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATION- 2023

M. Tech.-I Semester (CSE: Data Science)

COURSE CODE (CREDITS): 22M11MA111 (3)

MAX. MARKS: 35

COURSE NAME: Mathematical Foundations for Data Science

COURSE INSTRUCTORS: SST

MAX. TIME: 2 Hours

Note: (a) All questions are compulsory.

(b) Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.

(d) Use of a scientific calculator is allowed.

1. Assume that people in a particular society can be classified as belonging to the upper class (U), middle class (M), and lower class (L). Membership in any class is inherited in the following probabilistic manner. Given that a person is raised in an upper-class family, he or she will have an upper-class family with a probability of 0.7, a middle-class family with a probability of 0.2, and a lower-class family with a probability of 0.1. Similarly, given that a person is raised in a middle-class family, he or she will have an upper-class family with a probability of 0.1, a middle-class family with a probability of 0.6, and a lower-class family with a probability of 0.3. Finally, given that a person is raised in a lower-class family, he or she will have a middle-class family with a probability of 0.3 and a lower-class family with a probability of 0.7.

Determine:

- the transition probability matrix
- the state-transition diagram for this problem.
- The limiting state probabilities.

[2M+1M+3M](CO-4)

2. Write the gradient descent algorithm and use it to find the minimum value of the function $f(x,y) = x^2 + 5y^2$ after 2 iterations with learning rate 0.4 and initial point (1,2).

[2M+3M](CO-4)

3. A random sample of eight people yields the following (ordered) counts of the number of times they swam in the past month: 0 1 2 2 4 6 6 7. Use the Kolmogorov-Smirnov test to test if there is any evidence to suggest that the data were sampled from a Uniform distribution.

[4M](CO-3)

4. From a mathematics class of 12 equally capable students using programmed materials, 5 students are selected at random and given additional instruction by the teacher. The results of the final examination are as follows:

	Grade							
Additional Instruction	87	69	78	91	80			
No Additional Instruction	75	88	64	82	93	79	67	

Use the Mann-Whitney U-test with $\alpha = 0.05$ to determine if the additional instruction affects the average grade. [5M](CO-3)

5. Suppose the weights of randomly selected college students are normally distributed with unknown mean μ and standard deviation σ . A random sample of 10 college students yielded the following weights (in pounds): 115 122 130 127 149 160 152 138 149 180. Based on the definitions given above, identify the likelihood function and the maximum likelihood estimator of μ . [5M](CO-3)

6. Fit a linear model for the expansion of gelatin, using the method of least squares for the following data:

Humidity of air x [%]	10	20	30	40
Expansion of gelatin y [%]	0.8	1.6	2.3	2.8

[5M](CO-1)

7. Find the extreme values of the function $f(x, y) = xy$ subject to the constraint given by $g(x, y) = x^2 + y^2 - 10 = 0$. [5M](CO-1)

Statistical Tables: One-Sample Kolmogorov-Smirnov Table

$n \backslash \alpha$	0.001	0.01	0.02	0.05	0.1	0.15	0.2
1		0.99500	0.99000	0.97500	0.95000	0.92500	0.90000
2	0.97764	0.92930	0.90000	0.84189	0.77639	0.72614	0.68377
3	0.92063	0.82900	0.78456	0.70760	0.63604	0.59582	0.56481
4	0.85046	0.73421	0.68887	0.62394	0.56522	0.52476	0.49265
5	0.78137	0.66855	0.62718	0.56327	0.50945	0.47439	0.44697
6	0.72479	0.61660	0.57741	0.51926	0.46799	0.43526	0.41035
7	0.67930	0.57580	0.53844	0.48343	0.43607	0.40497	0.38145
8	0.64098	0.54180	0.50654	0.45427	0.40962	0.38062	0.35828

Table A.17 (continued) Critical Values for the Wilcoxon Rank-Sum Test

One-Tailed Test at $\alpha = 0.025$ or Two-Tailed Test at $\alpha = 0.05$

n_1	n_2																			
	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20			
1																				
2					0	0	0	0	1	1	1	1	1	2	2	2	2			
3		0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8			
4	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	13			
5		2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20			
6			5	6	8	10	11	13	14	16	17	19	21	22	24	25	27			