JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION- 2023

B.Tech.-III Semester (ECE/ ECM)

COURSE CODE (CREDITS): 18B11MA314 (4)   MAX. MARKS: 25

COURSE NAME: Probability Theory and Random Processes

COURSE INSTRUCTOR: Saurabh Srivastava   MAX. TIME: 1 Hour 30 Minutes

Note: (a) All questions are compulsory.

   (b) Marks are indicated against each question in square brackets.

   (c) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.

   (d) Use of scientific calculator is allowed.

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1. The probability that an automobile being filled with gasoline also needs an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and the filter need changing is 0.14.
   a) If the oil has to be changed, what is the probability that a new oil filter is needed?
   b) If a new oil filter is needed, what is the probability that the oil has to be changed?

   [2M+2M](CO-1)

2. A random variable X has a mean of 4 and a variance of 2. Use the Chebyshev inequality to obtain an upper bound for $P[|X - 4| \geq 2]$. **[2M](CO-2)**

3. The probability that a certain kind of component will survive a shock test is 3/4. Find the probability that exactly 2 of the next 4 components tested survive. **[3M](CO-3)**

4. The complexity of arrivals and departures of planes at an airport is such that computer simulation is often used to model the "ideal" conditions. For a certain airport with three runways, it is known that in the ideal setting the following are the probabilities that the individual runways are accessed by a randomly arriving commercial jet:

   Runway 1: p1 = 2/9,
   Runway 2: p2 = 1/6,
   Runway 3: p3 = 11/18.

   What is the probability that 6 randomly arriving airplanes are distributed in the following fashion?

   Runway 1: 2 airplanes,
   Runway 2: 1 airplane,
   Runway 3: 3 airplanes **[3M](CO-3)**

5. The loaves of rye bread distributed to local stores by a certain bakery have an average length of 30 centimetres and a standard deviation of 2 centimetres. Assuming that the lengths are normally distributed, what percentage of the loaves are
   a) longer than 31.7 centimetres?
   b) between 31.3 and 33.5 centimetres in length?
   c) shorter than 30.9 centimetres? **[2M+2M+2M](CO-3)**

6. A professor in the School of Business in a university polled a dozen colleagues about the number of professional meetings they attended in the past five years (x) and the number of papers they submitted to refereed journals (y) during the same period. The summary data are given as follows: $n = 12, \bar{x} = 4, \bar{y} = 12, \sum_{i=1}^{n} x_i^2 = 232, \sum_{i=1}^{n} x_i y_i = 318$. Fit a simple linear regression model between x and y by finding out the estimates of intercept and slope.

[4M](CO-4)

7. Find a linear curve that best fits the following data points $(-2,1), (-1,2), (0,3), (1,3), (2,4)$ by using the method of least squares.

[3M](CO-4)

Standard Normal Values:

0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633