Jaypee University of Information Technology, Waknaghat

Test-2 Examination - October 2023

B.Tech - III Semester (BI/BT)

Course Code/Credits: 18B11MA312/4   Max. Marks: 25

Course Title: Probability and Statistical Techniques

Course Instructor: RAD   Max. Time: 1 Hour 30 min.

Note: Answer all the questions. Describe random variables along with range where applicable. Use of scientific calculators is allowed.

1. Let **A** and **B** be *mutually exclusive* events: $\mathbb{P}(A) = 0.29$; $\mathbb{P}(B) = 0.43$. (3 Marks) [CO-2]

   (a) Find $\mathbb{P}(A^c)$.

   (b) Compute $\mathbb{P}(A \cup B)$.

2. A radar system is designed such that the probability of detecting the presence of an aircraft in its range is 98%. However, if no aircraft is present in its range it still reports (falsely) that an aircraft is present with a probability of 5%. At any time, the probability that an aircraft is present within the range of the radar is 7%. (4 Marks) [CO-2]

   (a) What is the probability that an aircraft is detected by the radar system?

   (b) What is the probability that an aircraft is present in the range of the radar given that an aircraft is detected?

3. The probability distribution of the number of imperfections X per 10 meters of a synthetic fabric in continuous rolls of uniform width is given by (4 Marks) [CO-2]

x	0	1	2	3	4
f(x)	0.41	0.37	0.16	0.05	0.01

   Construct the *cumulative distribution function* of **X**.

4. A continuous random variable X has the *distribution function* (CDF): (4 Marks) [CO-2]

$$\mathbb{F}_X(x) = \begin{cases} 0 & , \quad x < 0 \\ kx^2 & , \quad 0 \leq x \leq 6 \\ 1 & , \quad x > 6 \end{cases}$$

   (a) Determine the value of the constant k.

   (b) Calculate $\mathbb{P}\left(\frac{1}{2} \leq X \leq 1\right)$.

5. Probability that a medical surgery being success is 75%. Suppose that the surgery is performed on 3 patients. Let X be the number of successes. (5 Marks) [CO-2]

   (a) What are the possible values of **X**?

   (b) What is the probability that the surgery is successful on exactly 2 patients?

   (c) What is the expected number of successes?

6. *Carbon monoxide* (CO) emissions for a certain kind of car vary with a mean 2.9 g/mi and standard deviation 0.4 g/mi. CO emissions follow a normal distribution. (5 Marks) [CO-2]

   (a) What is the probability that the CO emissions for a randomly selected car fall between 2.9 and 3.5 g/mi?

   (b) Find x_0: only 3.6% of cars will be expected to have CO emissions above x_0 g/mi.

(Standard) Normal probability table to compute $\mathbb{P}(Z \le z)$:
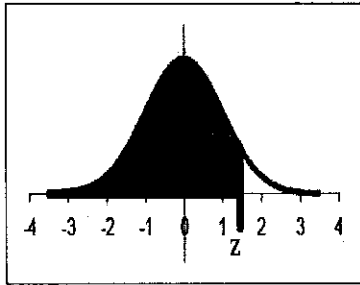
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936